CS 2770: Computer Vision

Grouping & Transformations

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February 4, 2020

Plan for this lecture

- Group pixels into:
  - Edges: Extract gradients and threshold
  - Lines: Find which edge points are collinear or belong to another shape
  - Segments: Find which pixels form a consistent region, e.g. via clustering
- Transform pixels:
  - Find relationships between multiple views of the same world point
  - *Both parts rely on finding geometric relationships between pixels*

Edge detection

- **Goal**: map image from 2d array of pixels to a set of curves or line segments or contours.
- **Why?**

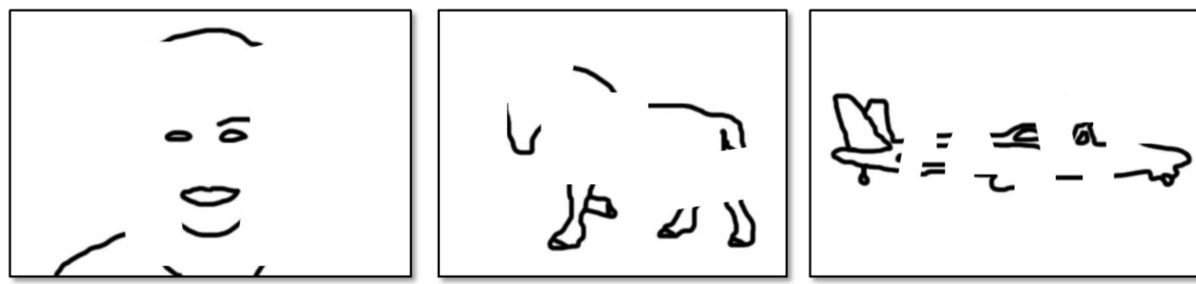


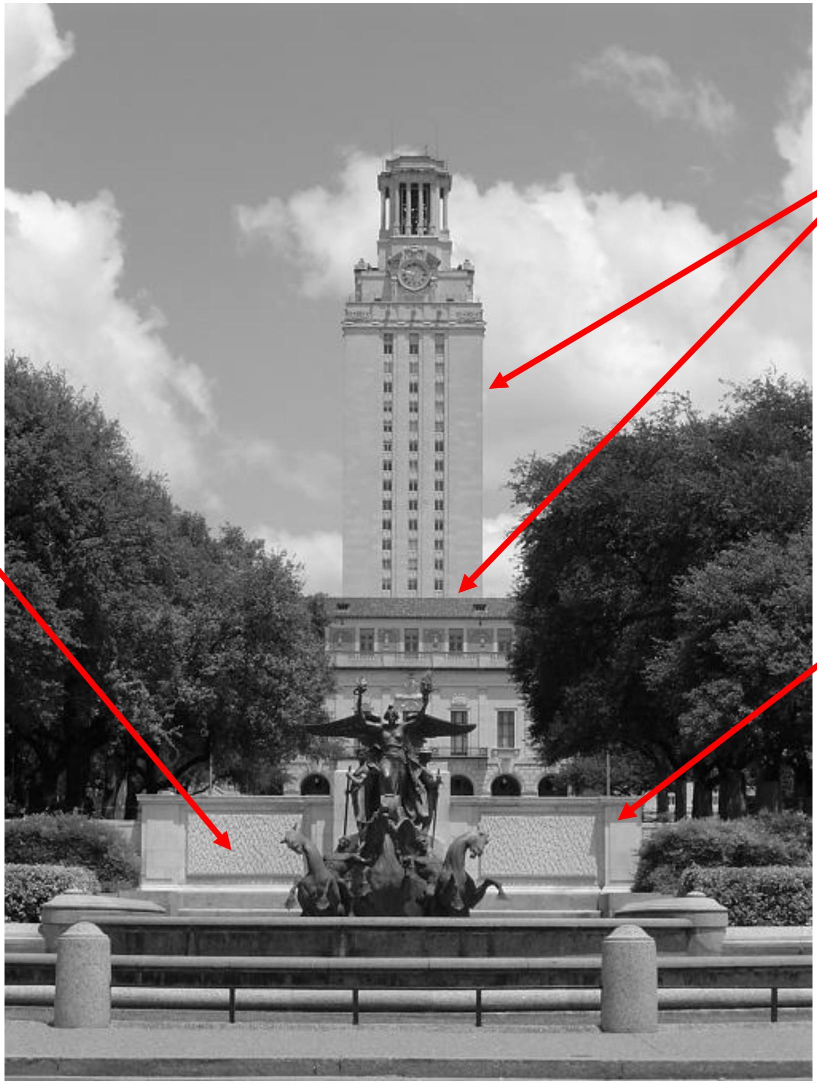
Figure from J. Shotton et al., PAMI 2007

- **Main idea**: look for differences in intensity, i.e. find strong gradients, then post-process

Adapted from K. Grauman

What causes an edge?

Reflectance change: appearance information, texture



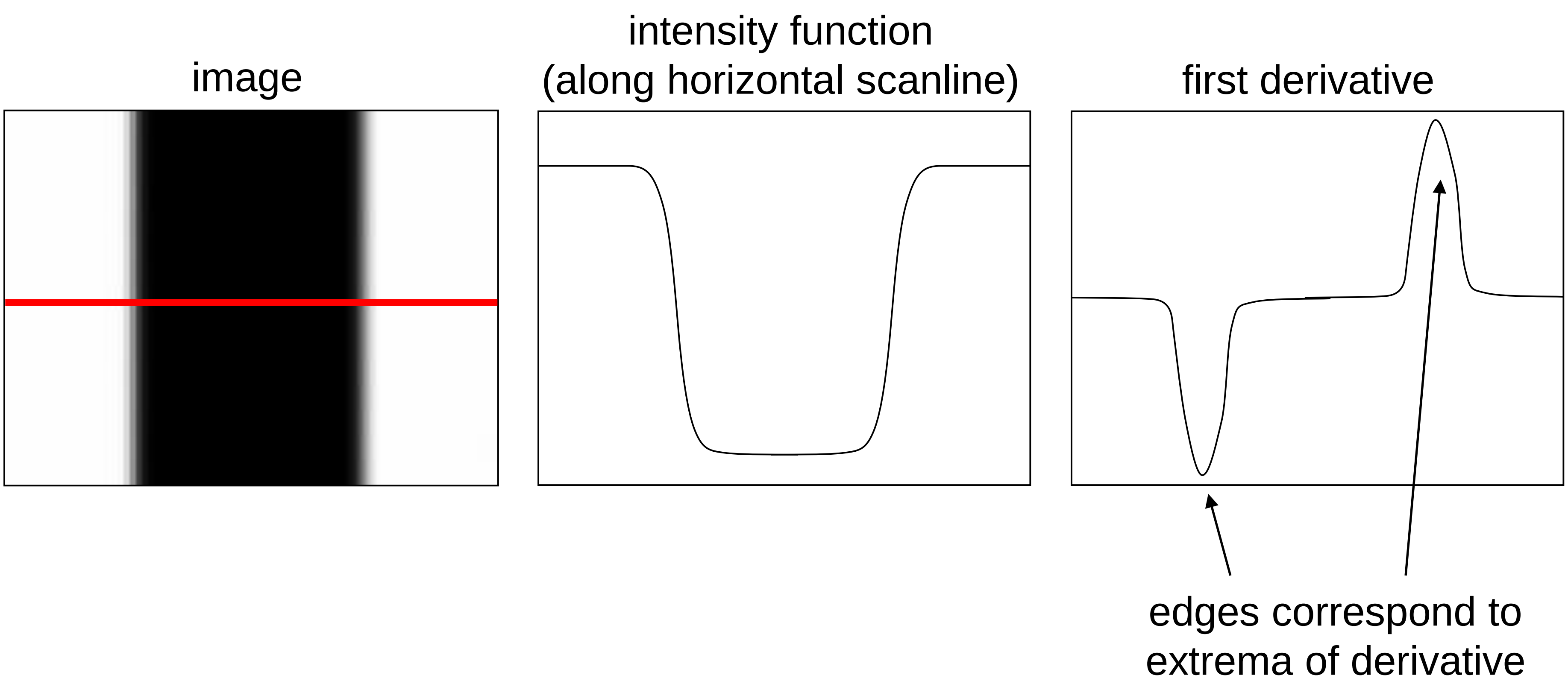
Depth discontinuity: object boundary

Cast shadows

Adapted from K. Grauman

Characterizing edges

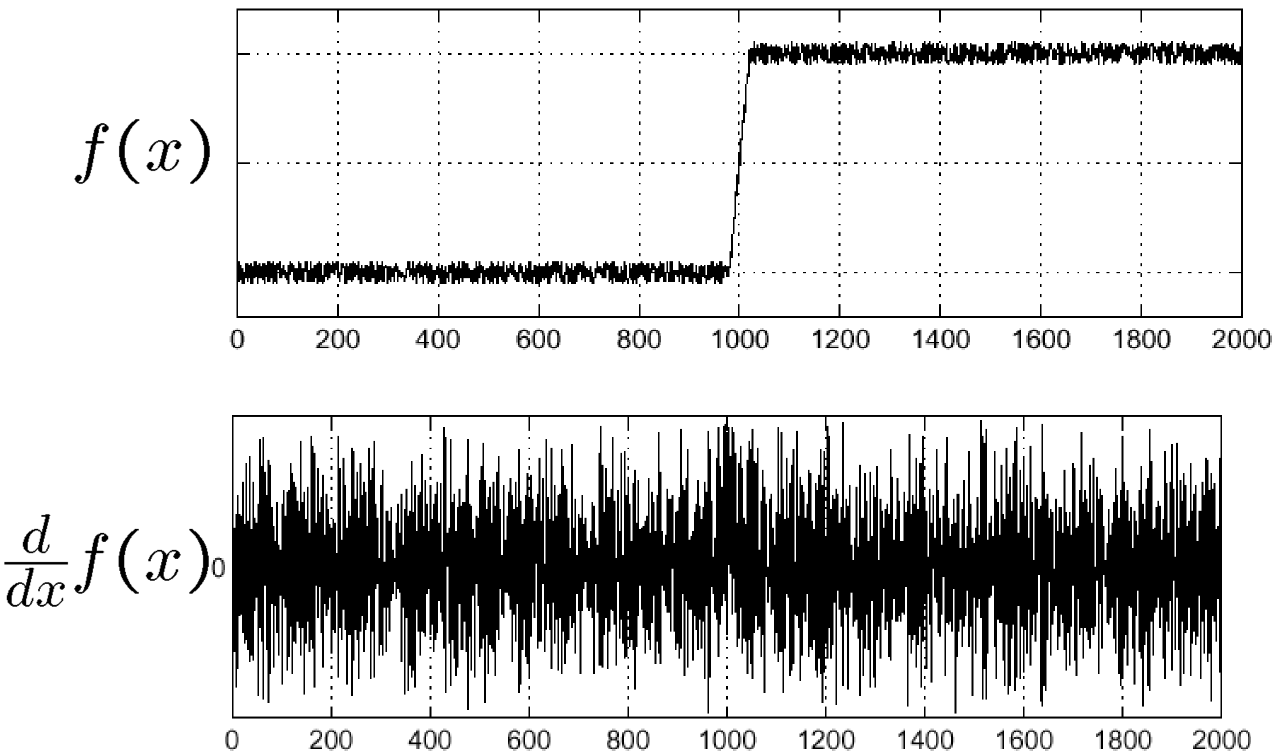
- An edge is a place of rapid change in the image intensity function



Source: L. Lazebnik

Now with a little noise…

- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal

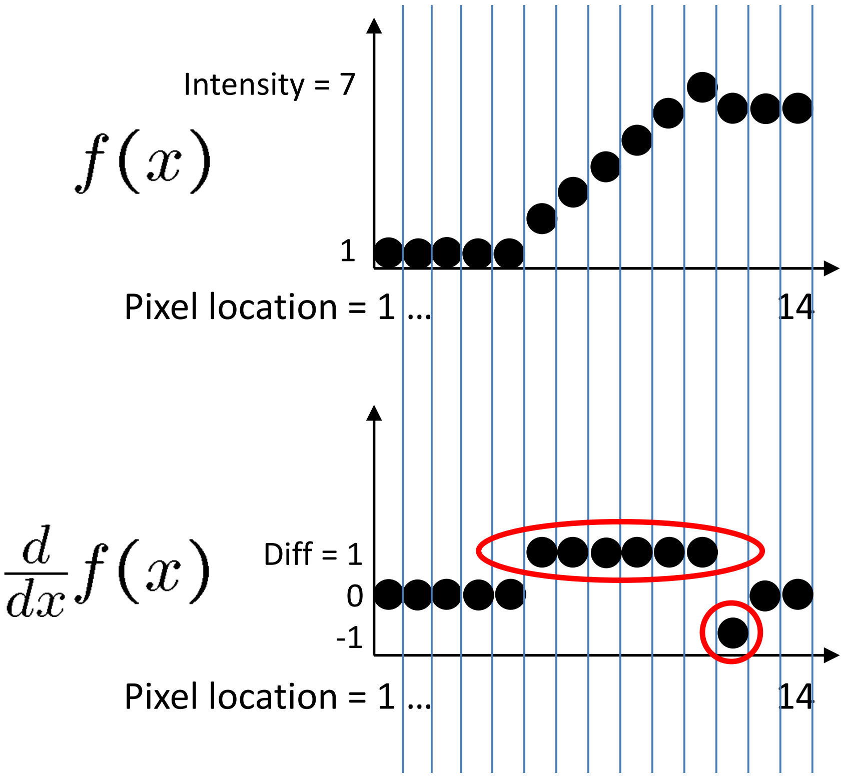


Where is the edge?

Source: S. Seitz

Without noise

- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal



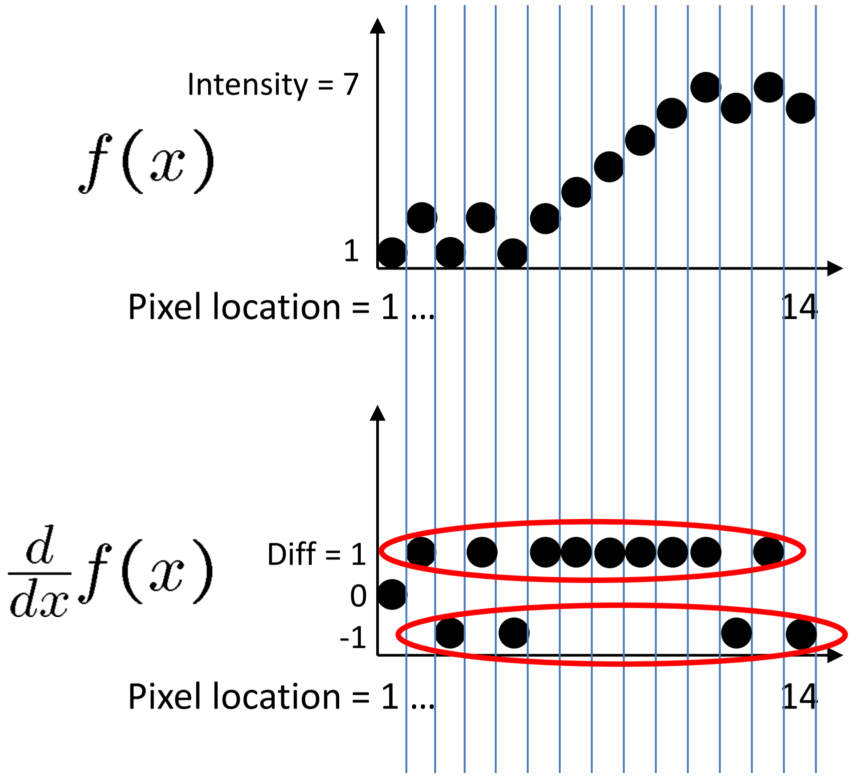
$$= \frac{\Delta f(a)}{\Delta a} = \frac{f(a+h) - f(a)}{(a+h) - (a)} = \frac{f(a+h) - f(a)}{h}$$

Where is the edge?

https://en.wikipedia.org/wiki/Derivative

With noise

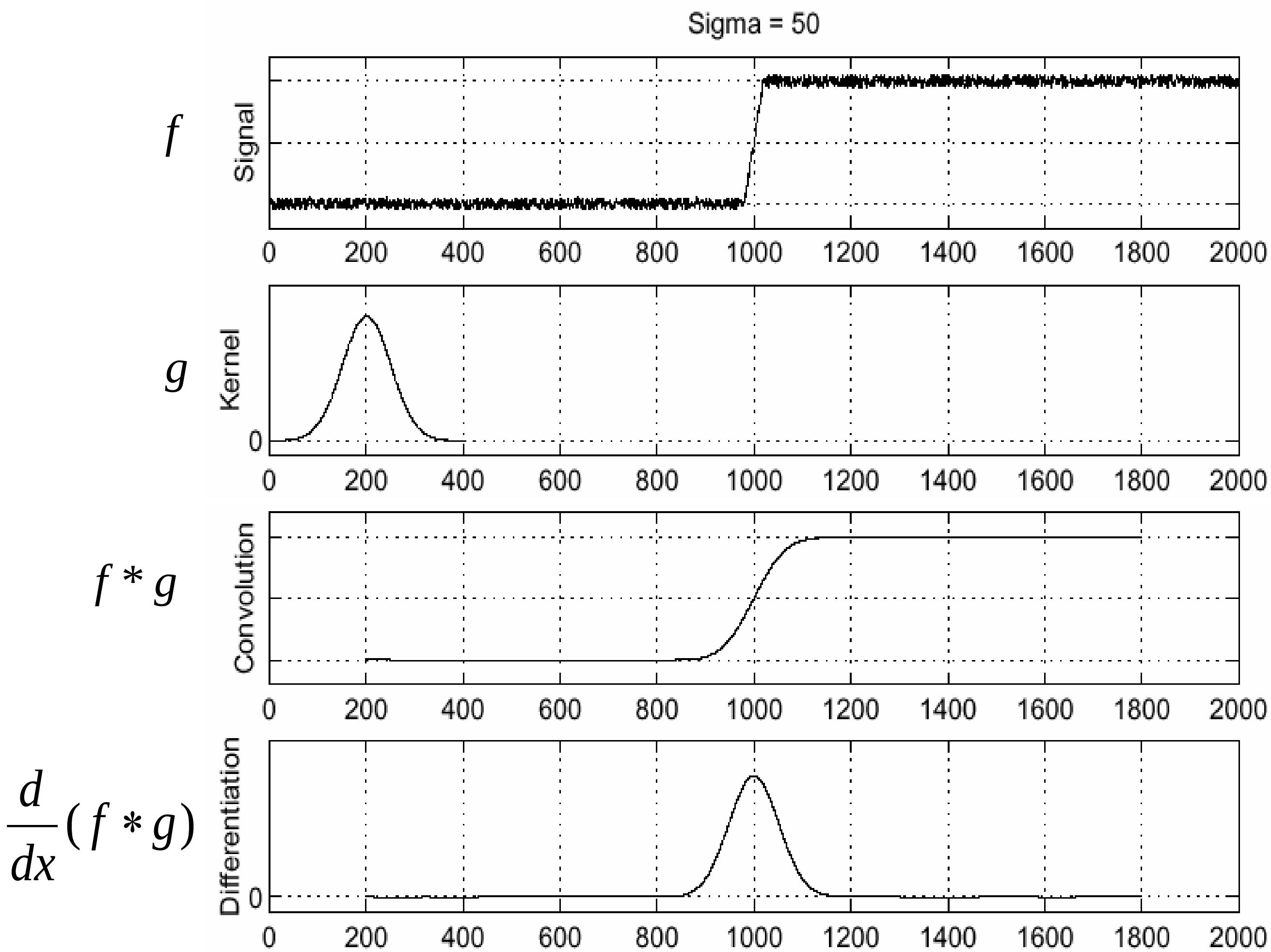
- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal



$$= \frac{\Delta f(a)}{\Delta a} = \frac{f(a+h)-f(a)}{(a+h)-(a)} = \frac{f(a+h)-f(a)}{h}$$

Where is the edge?

Solution: smooth first



- To find edges, look for peaks in $\frac{d}{dx}(f * g)$

Source: S. Seitz

Derivative theorem of convolution

- Differentiation is convolution, and convolution is associative:

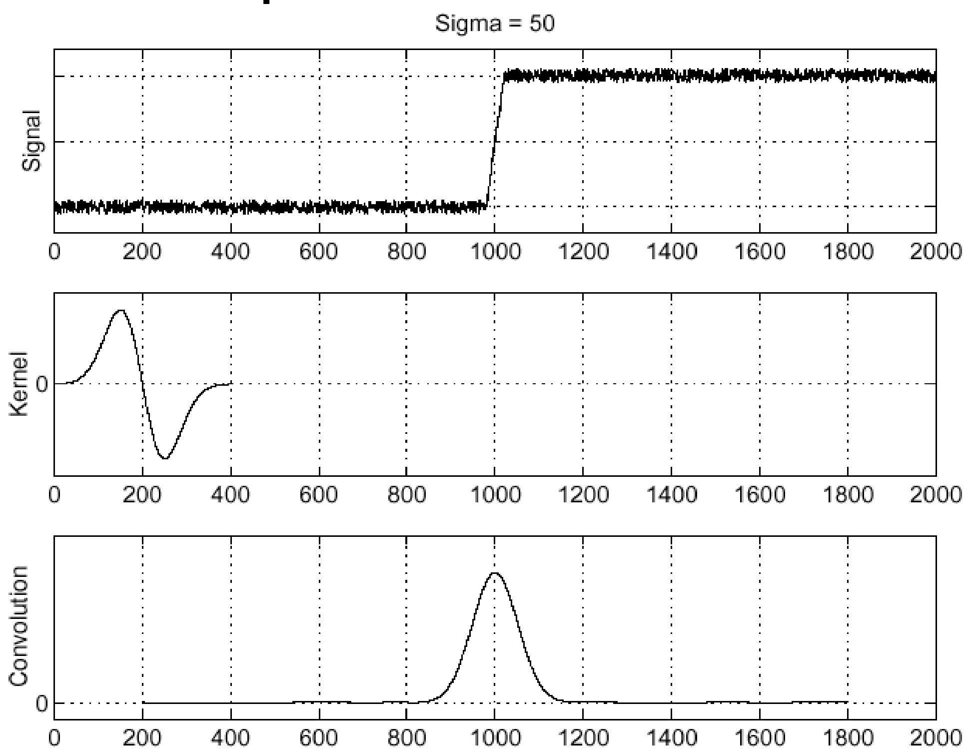
$$\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$$

- This saves us one operation:

f — Image with edge

$\frac{d}{dx} g$ — Derivative of Gaussian

$f * \frac{d}{dx} g$ — Edge = max of derivative



Source: S. Seitz

Canny edge detector

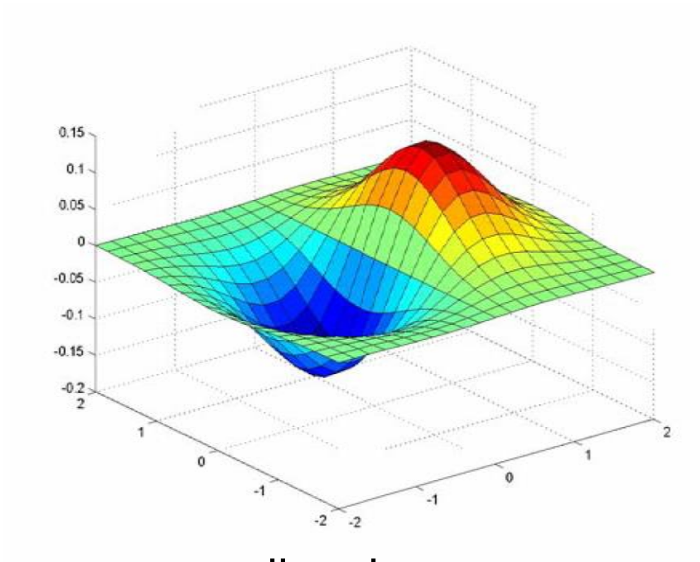
- Filter image with derivative of Gaussian
- Find magnitude and orientation of gradient
- Threshold: Determine which local maxima from filter output are actually edges
- **Non-maximum suppression**:
  - Thin wide “ridges” down to single pixel width
- **Linking and thresholding** (**hysteresis**):
  - Define two thresholds: low and high
  - Use the high threshold to start edge curves and the low threshold to continue them

Adapted from K. Grauman, D. Lowe, L. Fei-Fei

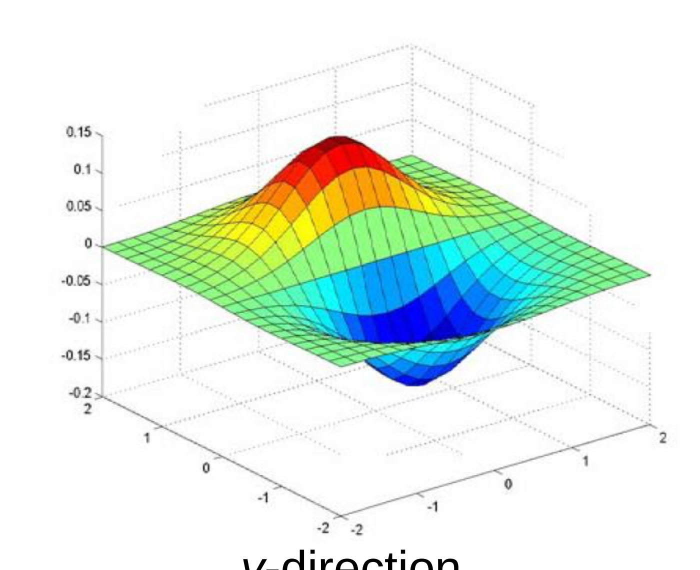
Example

input image (“Lena”)

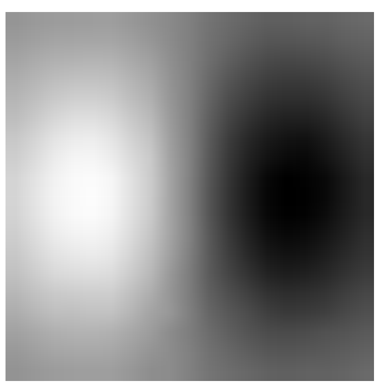
Derivative of Gaussian filter

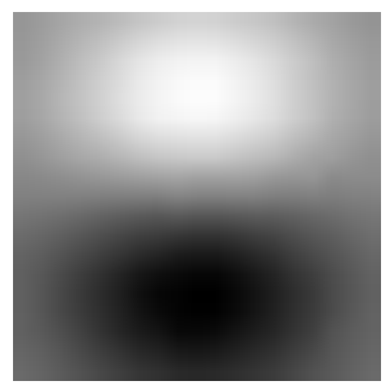


x-direction



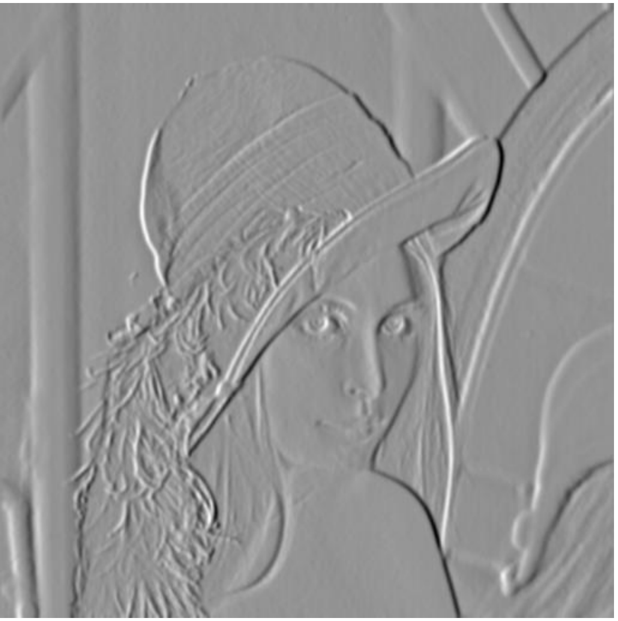
y-direction





Source: L. Lazebnik

Compute Gradients


X-Derivative of Gaussian


Y-Derivative of Gaussian


Gradient Magnitude

Source: D. Hoiem

Thresholding

- Choose a threshold value t
- Set any pixels less than t to 0 (off)
- Set any pixels greater than or equal to t to 1 (on)

Source: K. Grauman

The Canny edge detector

norm of the gradient (magnitude)

Source: K. Grauman

The Canny edge detector

thresholding

Source: K. Grauman

Another example: Gradient magnitudes

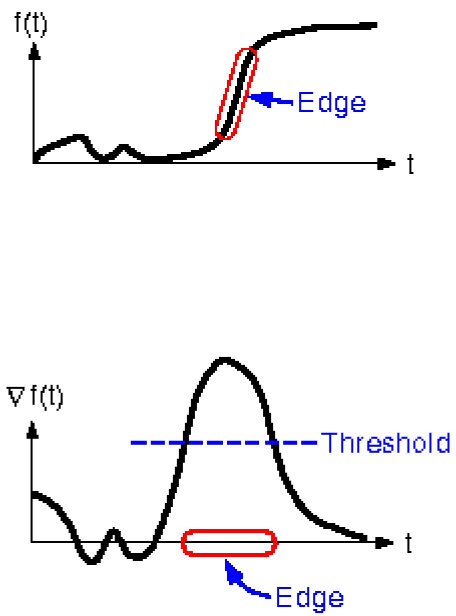
K. Grauman

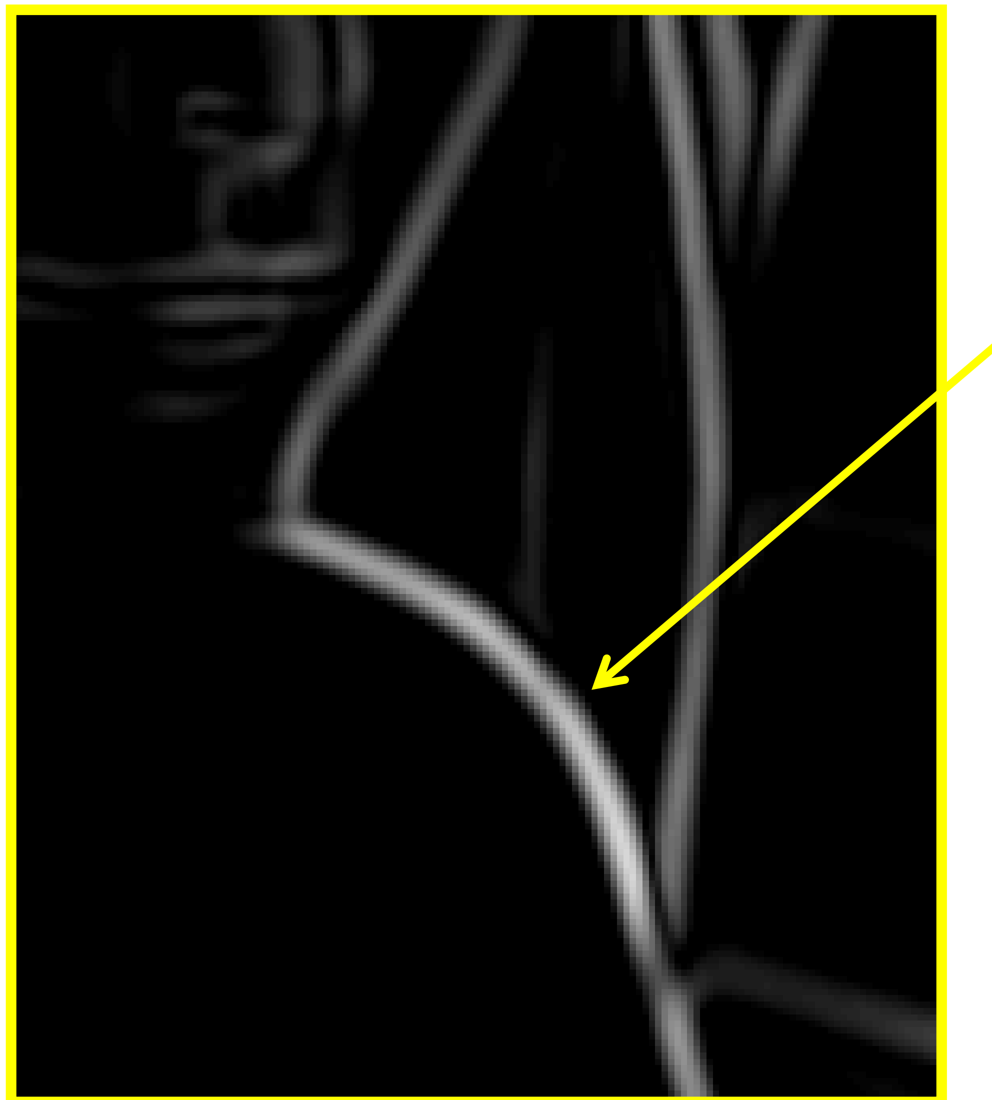
Thresholding gradient with a lower threshold

Thresholding gradient with a higher threshold

Source: K. Grauman

The Canny edge detector

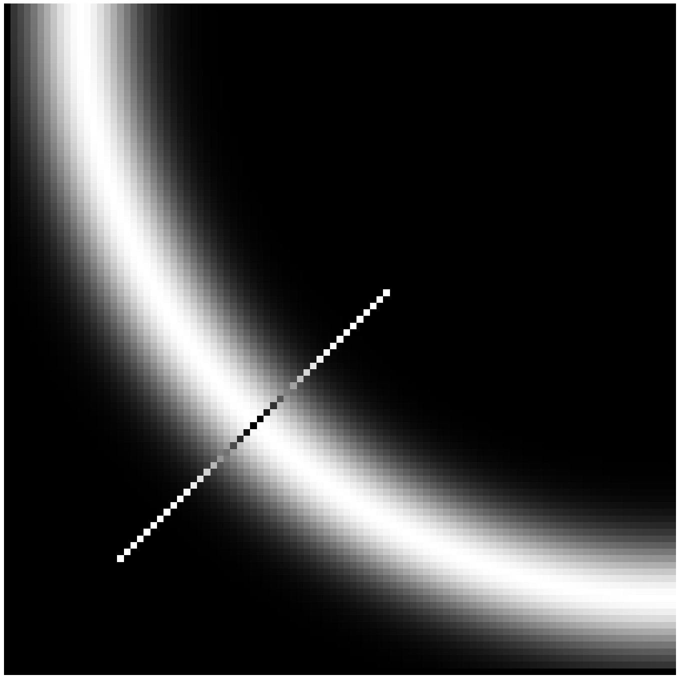


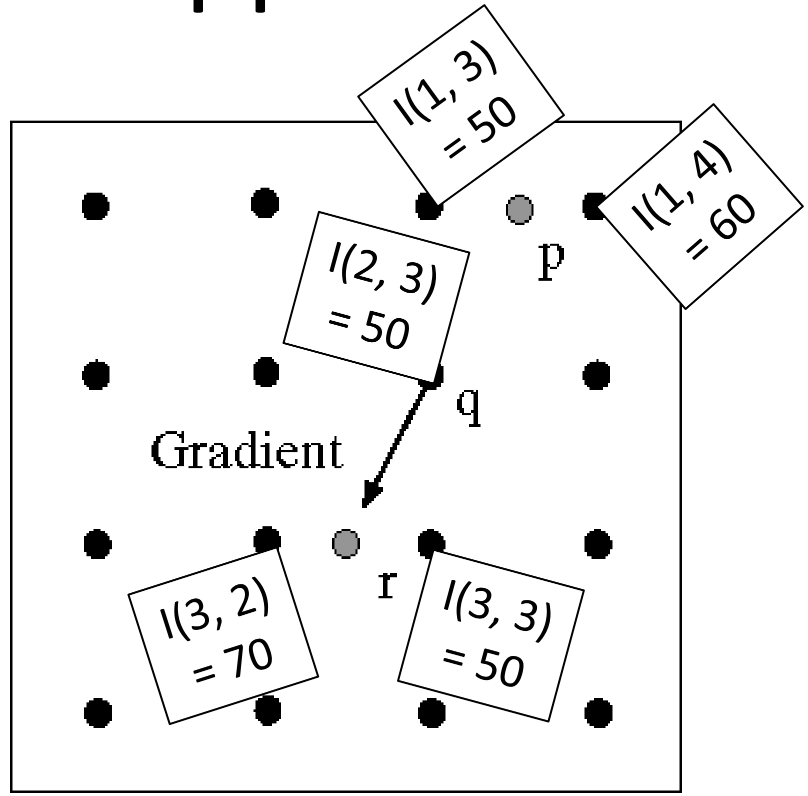


How to turn these thick regions of the gradient into curves?

Source: K. Grauman

Non-maximum suppression



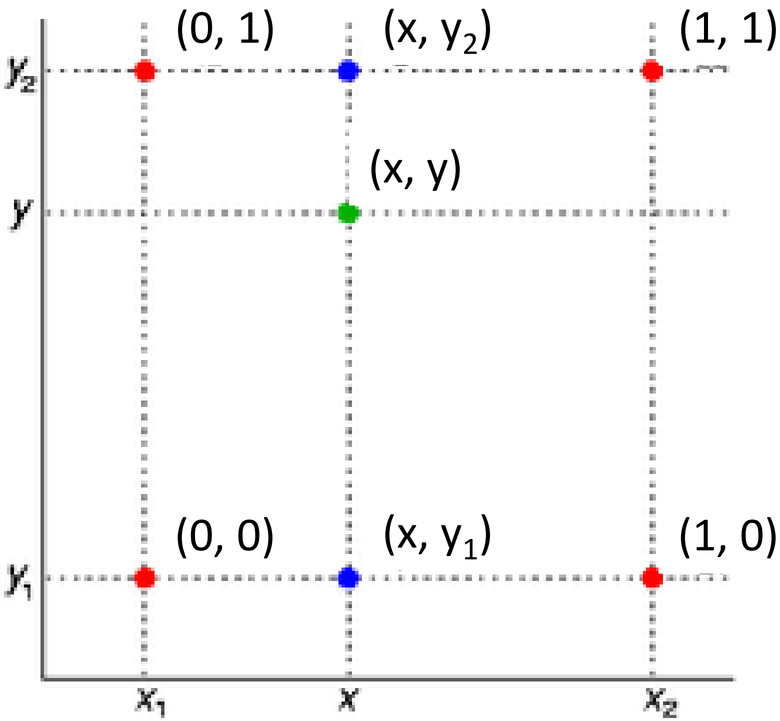


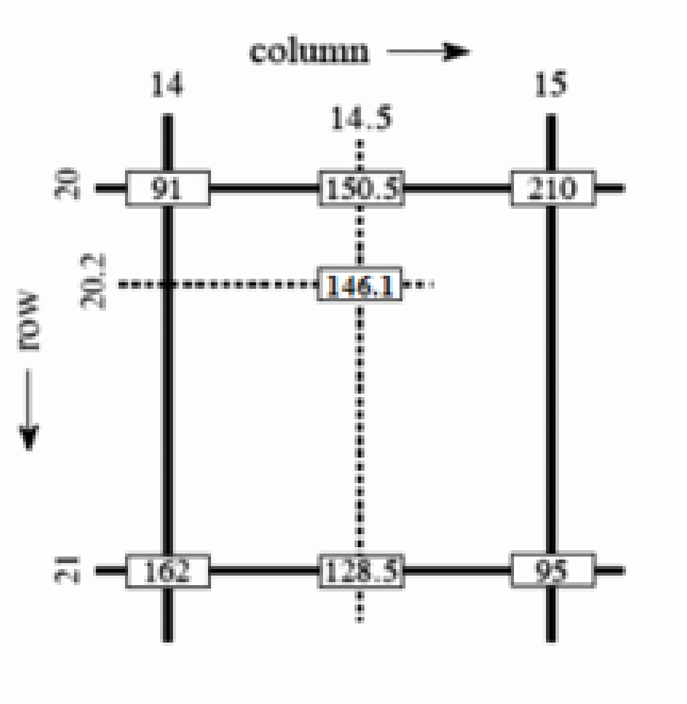
- Check if pixel is local maximum along gradient direction
- Compare to pixels immediately neighboring on both sides
  - i.e. compare **q** to **p** and **r**
- Requires checking interpolated "pixels" p and r (at non-integer locations, so no intensity information) – *bilinear interpolation*

Adapted from K. Grauman

Bilinear interpolation

$$f(x,y) \approx \begin{bmatrix}1-x & x\end{bmatrix} \begin{bmatrix}f(0,0) & f(0,1) \\ f(1,0) & f(1,1)\end{bmatrix} \begin{bmatrix}1-y \\ y\end{bmatrix}.$$

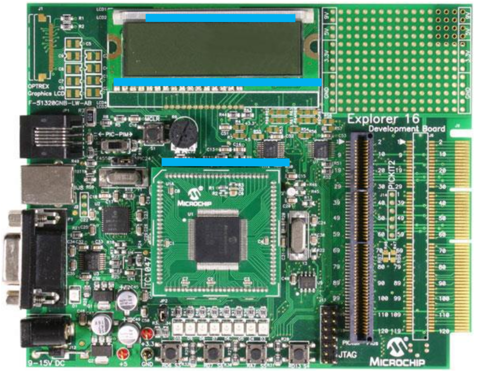




http://en.wikipedia.org/wiki/Bilinear_interpolation

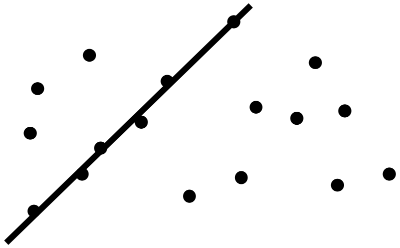
Related: Line detection (fitting)

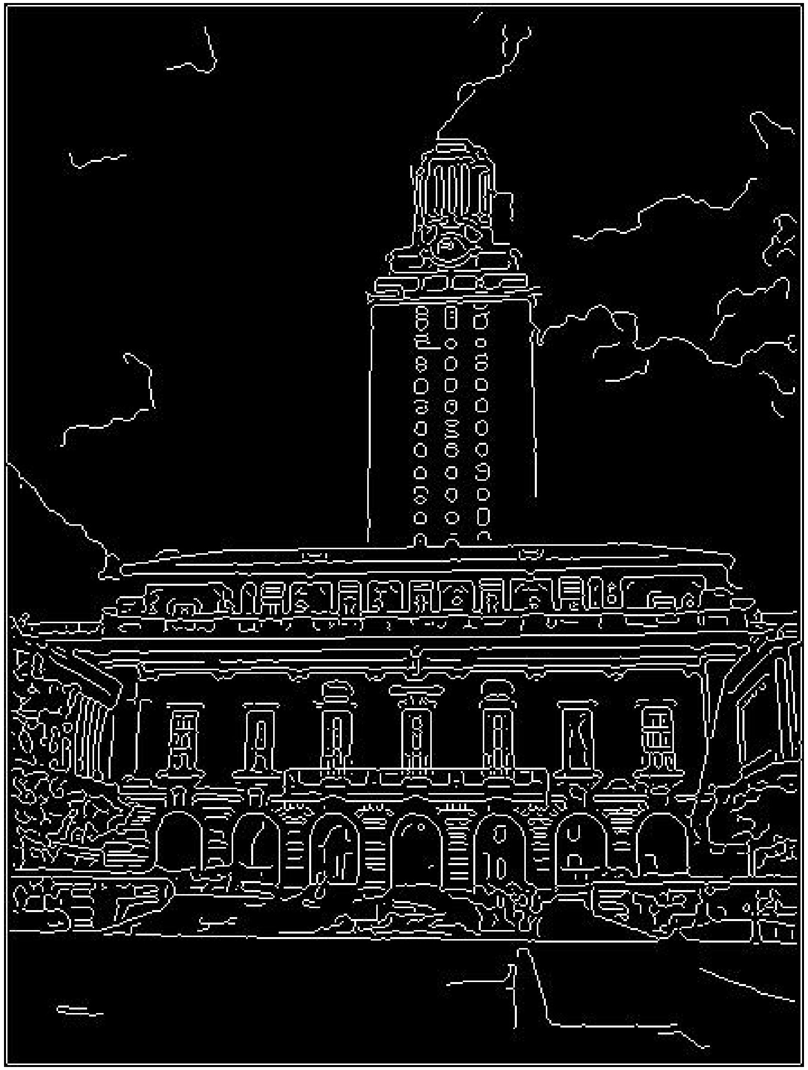
- Why fit lines?
  Many objects characterized by presence of straight lines


- Why aren’t we done just by running edge detection?

Difficulty of line fitting





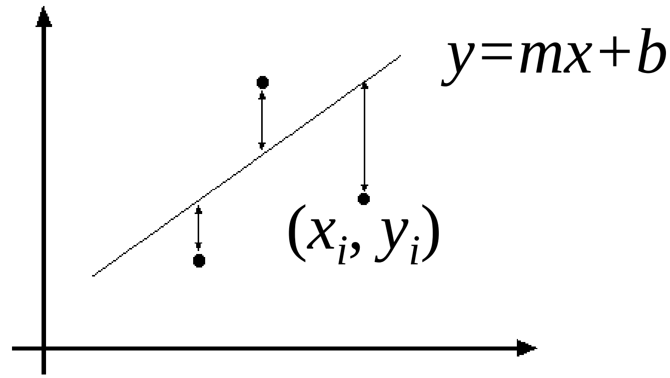
- **Noise** in measured edge points, orientations:
  - e.g. edges not collinear where they should be
  - how to detect true underlying parameters?
- **Extra** edge points (clutter):
  - which points go with which line, if any?
- Only some parts of each line detected, and some parts are **missing:**
  - how to find a line that bridges missing evidence?

Adapted from Kristen Grauman

Least squares line fitting

- Data: $(x_1, y_1), \ldots, (x_n, y_n)$
- Line equation: $y_i = m\,x_i + b$

- Find (m, b) to minimize



$$\boxed{E = \sum_{i=1}^{n} (\underbrace{mx_i + b}_{\text{where line you found tells you point is along y axis}} - \underbrace{y_i}_{\text{where point really is along y axis}})^2}$$

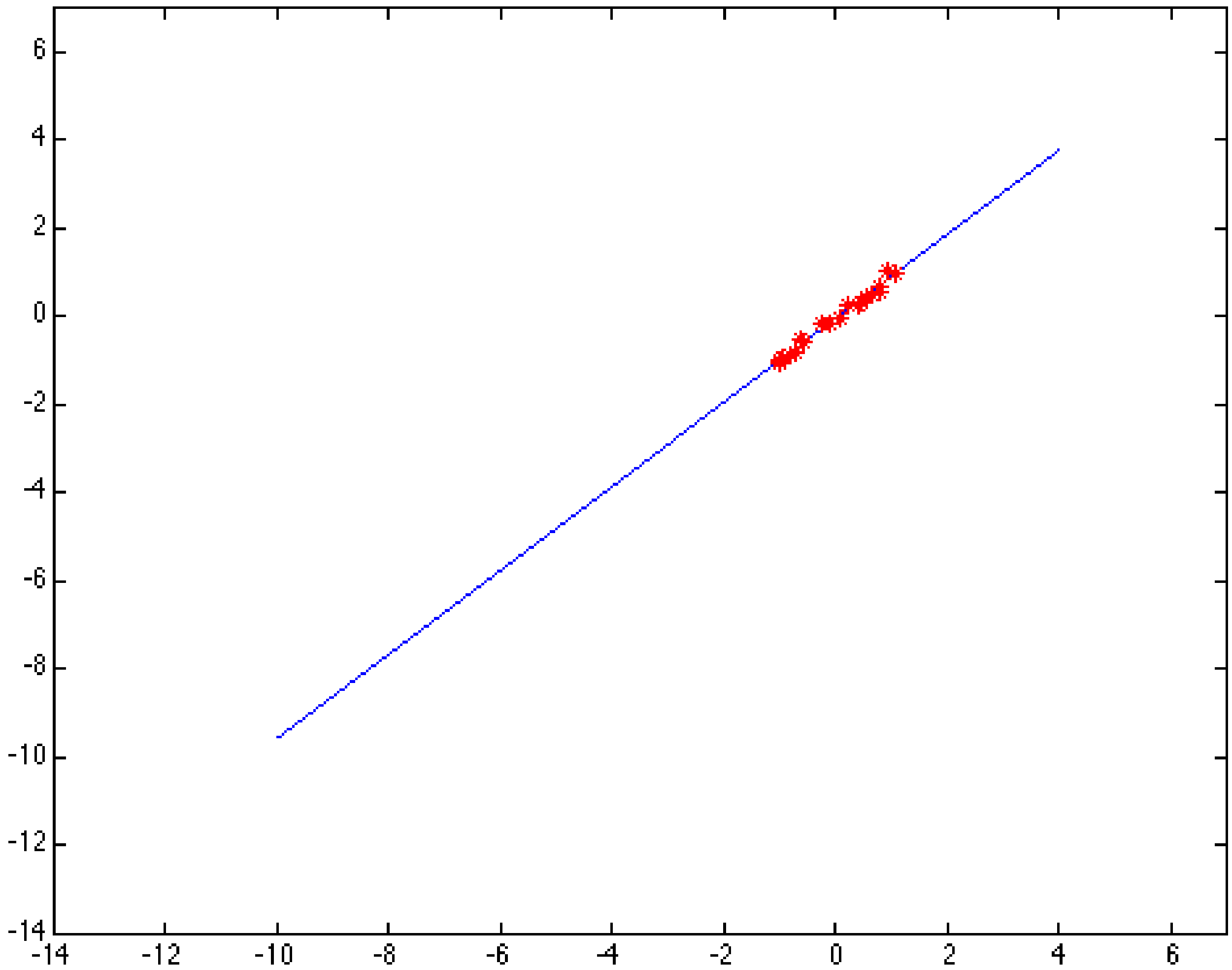
You want to find a single line that "explains" all of the points in your data, but data may be noisy!

$$E = \sum_{i=1}^{n}\left(\begin{bmatrix} x_i & 1 \end{bmatrix}\begin{bmatrix} m \\ b \end{bmatrix} - y_i\right)^2 = \left\| \begin{bmatrix} x_1 & 1 \\ & \\ x_n & 1 \end{bmatrix}\begin{bmatrix} m \\ b \end{bmatrix} - \begin{bmatrix} y_1 \\ \\ y_n \end{bmatrix} \right\|^2 = \left\| \mathbf{Ap} - \mathbf{y} \right\|^2$$

Matlab: `p = A \ y;`

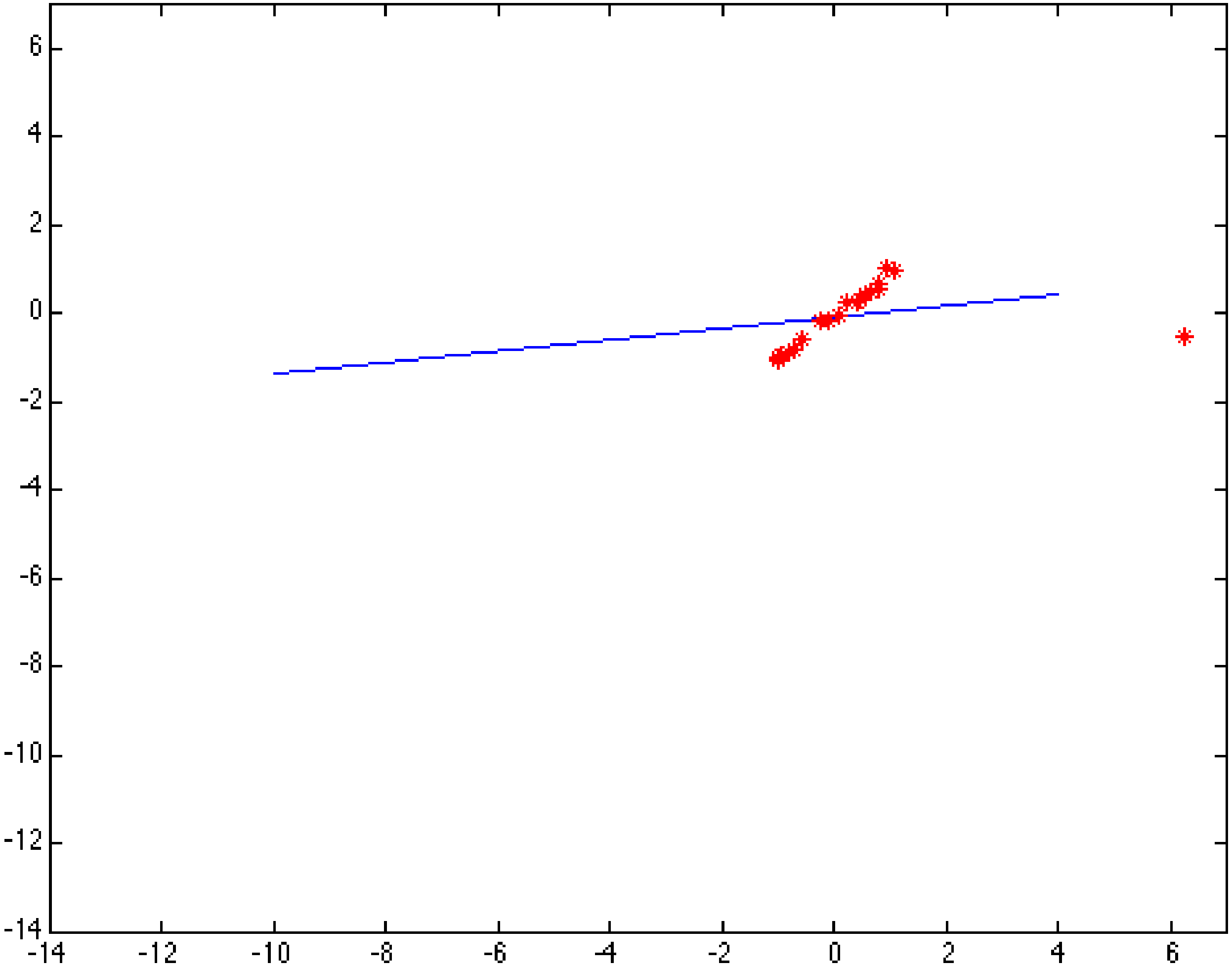
Adapted from Svetlana Lazebnik

Outliers affect least squares fit



Kristen Grauman

Outliers affect least squares fit



Kristen Grauman

Dealing with outliers: Voting

- **Voting** is a general technique where we let the features *vote for all models that are compatible with it*.
  - Cycle through features, cast votes for model parameters.
  - Look for model parameters that receive *a lot of votes*.
- Noise & clutter features?
  - They will cast votes too, *but* typically their votes should be inconsistent with the majority of “good” features.
- Two common techniques
  - Hough transform
  - RANSAC

Adapted from Kristen Grauman

Finding lines in an image: Hough space

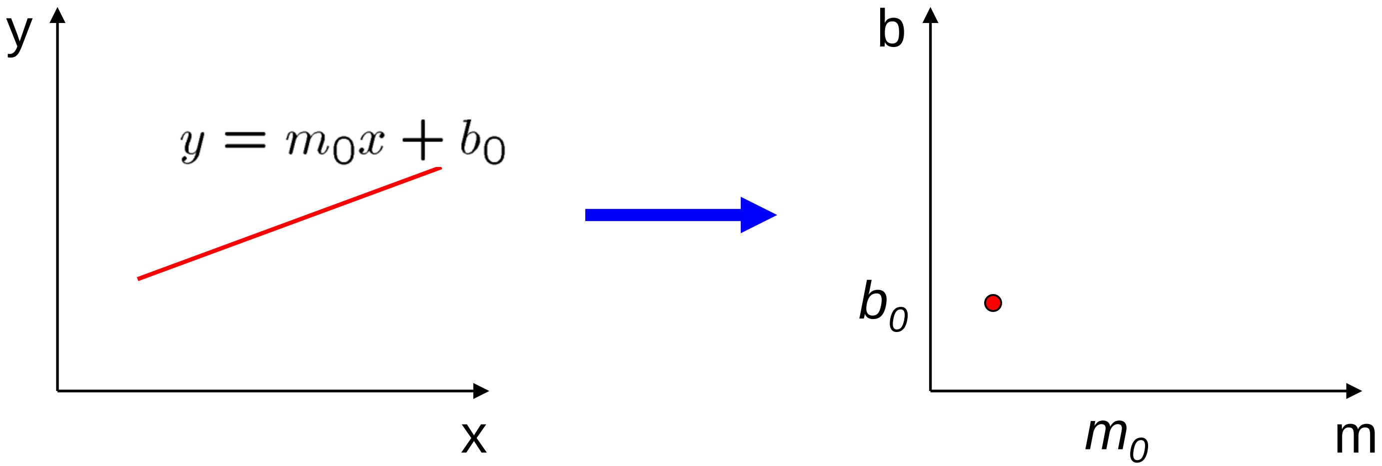


image space

Hough (parameter) space

Connection between image (x,y) and Hough (m,b) spaces

$y = m_0 x + b_0$
- A line in the image corresponds to a point in Hough space

Steve Seitz

Finding lines in an image: Hough space

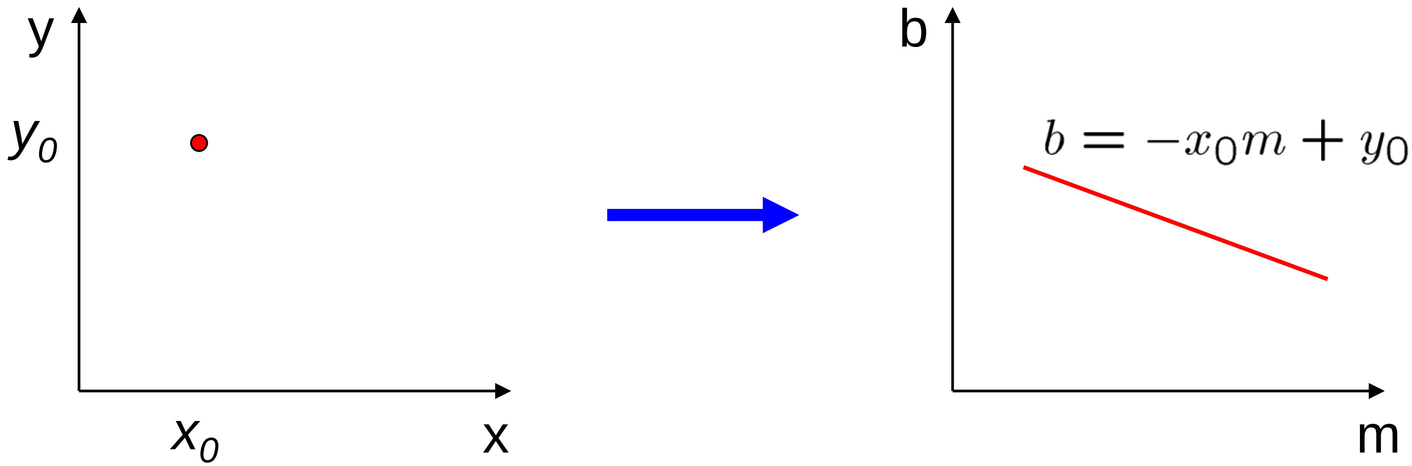


image space

Hough (parameter) space

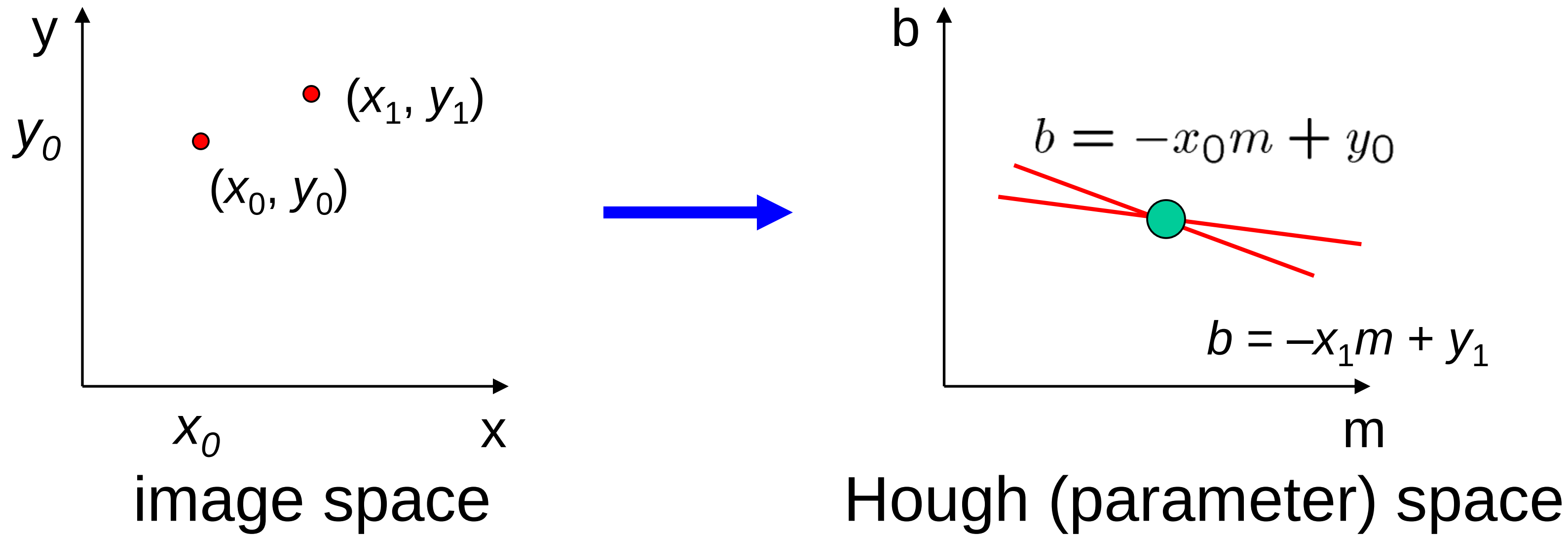
Connection between image (x,y) and Hough (m,b) spaces

$y = m_0 x + b_0$

- A line in the image corresponds to a point in Hough space
- What does a point (x_0, y_0) in the image space map to?
  - Answer: the solutions of $b = -x_0 m + y_0$
  - This is a line in Hough space
  - Given a pair of points (x,y), find all (m,b) such that y = mx + b

Adapted from Steve Seitz

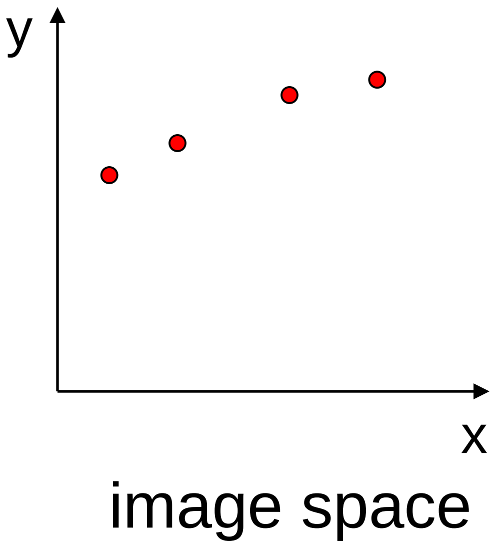
Finding lines in an image: Hough space

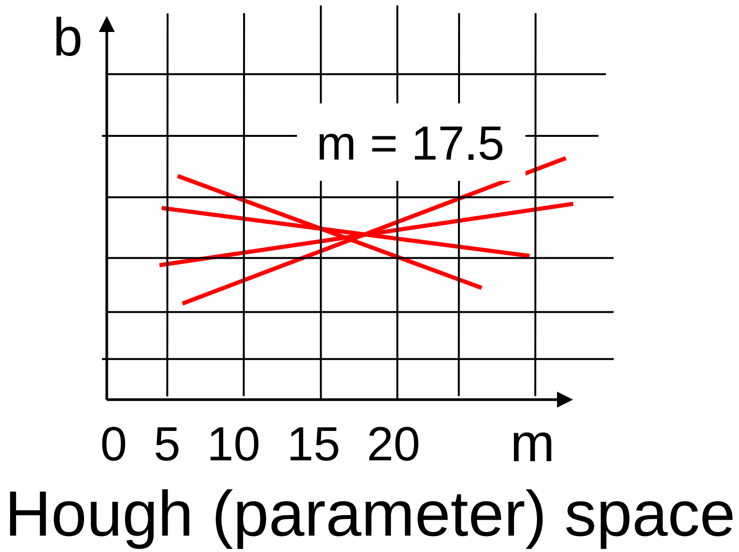


What are the line parameters for the line that contains both (x_0, y_0) and (x_1, y_1)?

- It is the intersection of the lines $b = -x_0 m + y_0$ and $b = -x_1 m + y_1$

Finding lines in an image: Hough space



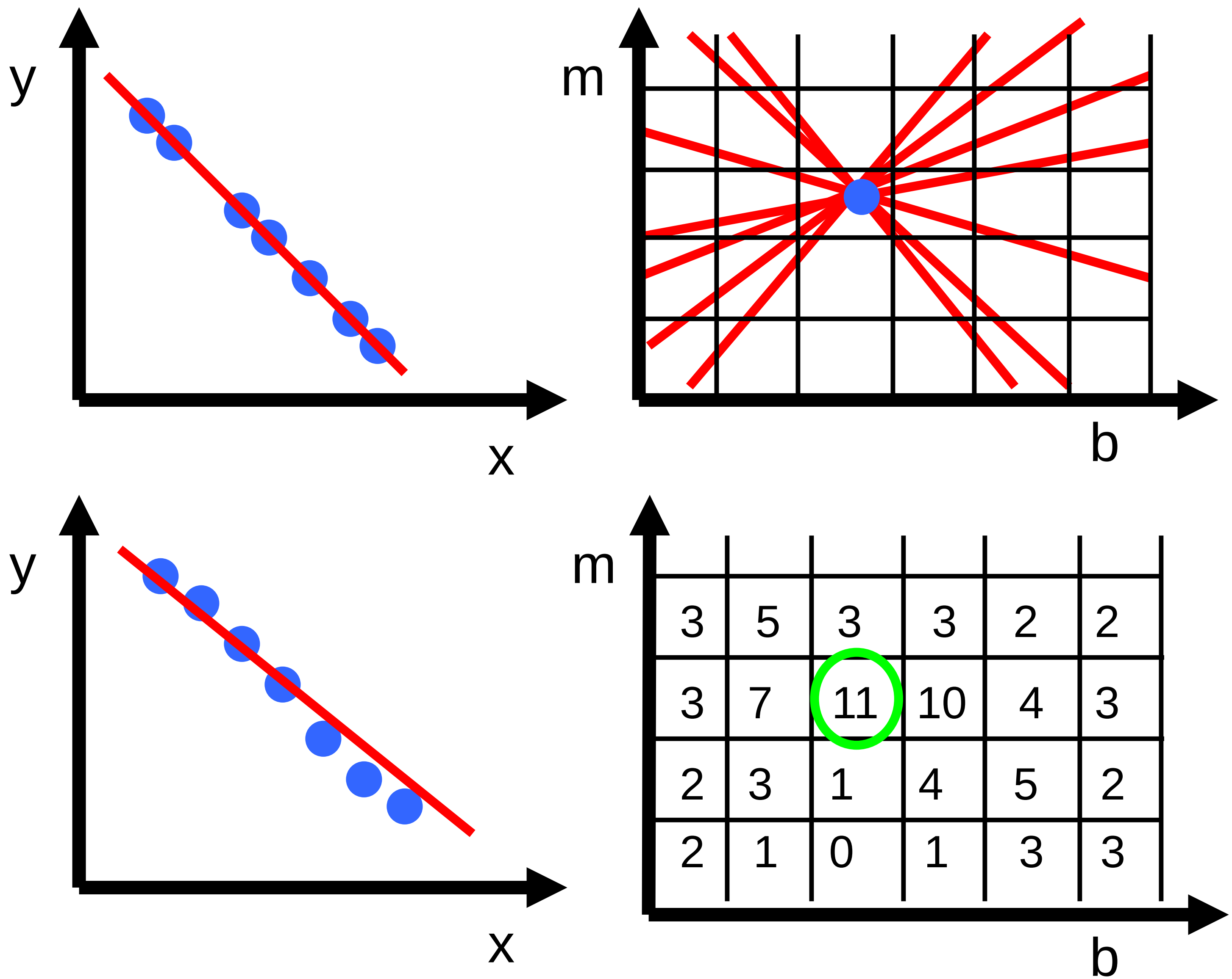


How can we use this to find the most likely parameters (m,b) for the most prominent line in the image space?

- Let each edge point in image space *vote* for a set of possible parameters in Hough space
- Accumulate votes in discrete set of bins; parameters with the most votes indicate line in image space.

Steve Seitz

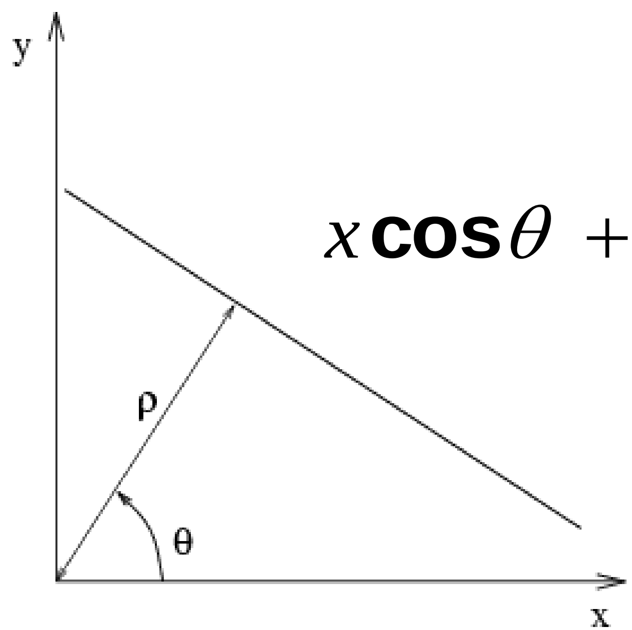
Finding lines in an image: Hough space



Adapted from Silvio Savarese

Parameter space representation

- Problems with the (m,b) space:
  - Unbounded parameter domains
  - Vertical lines require infinite m
- Alternative: *polar representation*



$$x\cos\theta + y\sin\theta = \rho$$

Each point (x,y) will add a sinusoid in the (θ,ρ) parameter space

Svetlana Lazebnik

Parameter space representation

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  - Unbounded parameter domains
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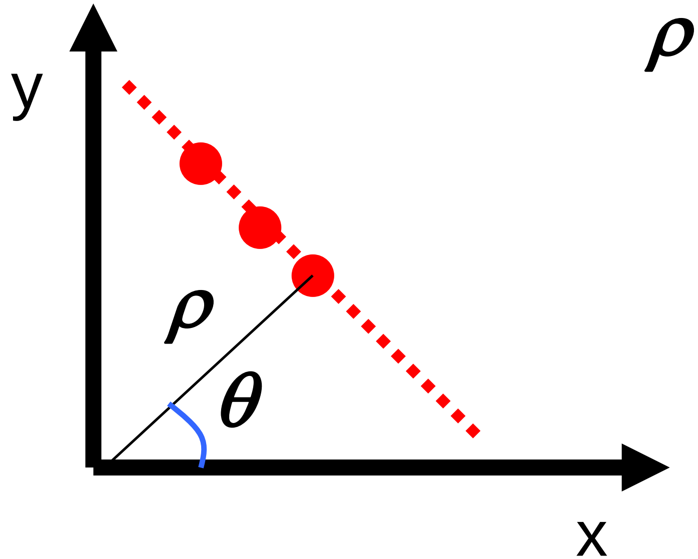
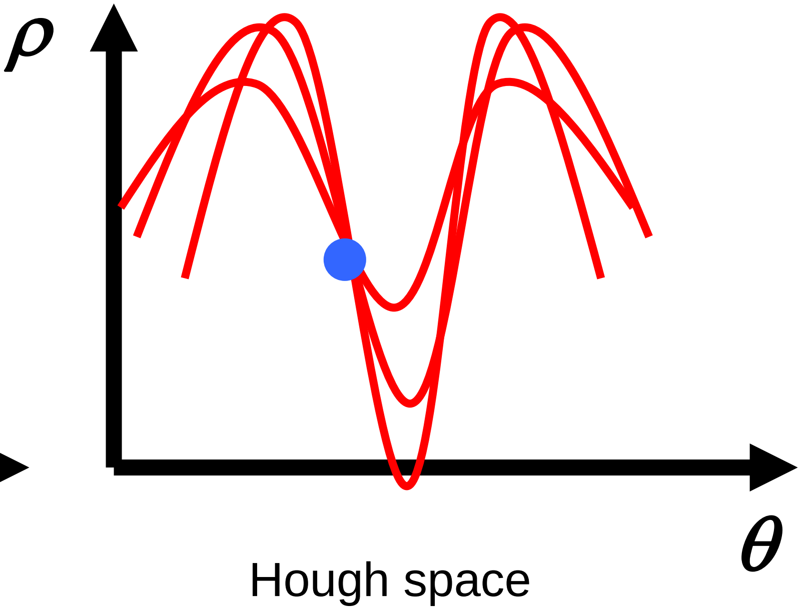


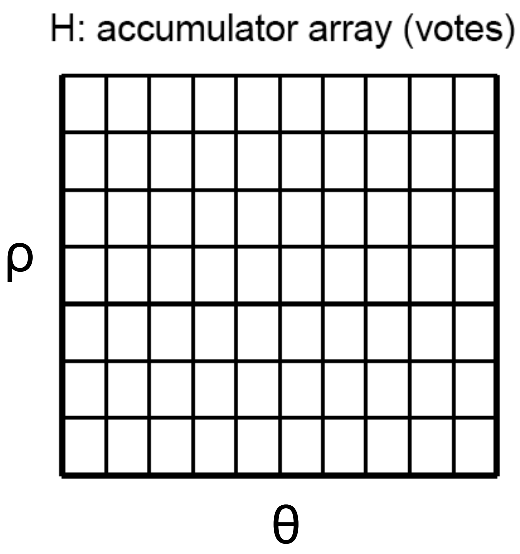


Each point (x,y) will add a sinusoid in the (θ,ρ) parameter space

Svetlana Lazebnik

Algorithm outline: Hough transform

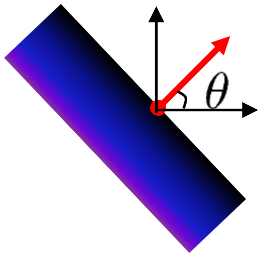
- Initialize accumulator H to all zeros
- For each edge point (x,y) in the image
  For θ = 0 to 180
  $\rho = x \cos\theta + y \sin\theta$
  $H(\theta, \rho) = H(\theta, \rho) + 1$
  end
  end
- Find the value(s) of (θ^*, ρ^*) where $H(\theta, \rho)$ is a local maximum
  - The detected line in the image is given by $\rho^* = x \cos\theta^* + y \sin\theta^*$



Svetlana Lazebnik

Incorporating image gradients

- Recall: when we detect an edge point, we also know its gradient direction
- But this means that the line is uniquely determined!



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

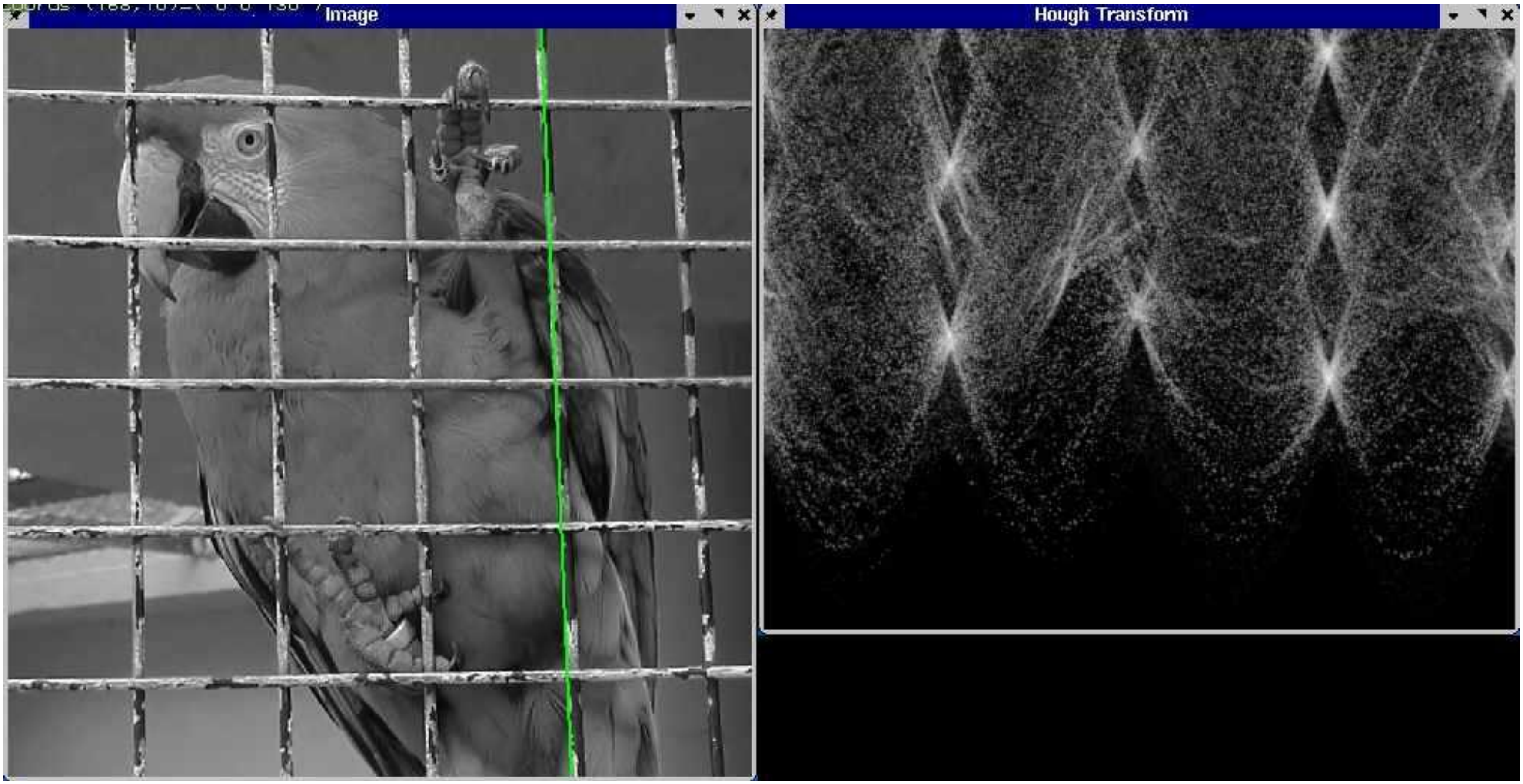
$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)$$

- Modified Hough transform:

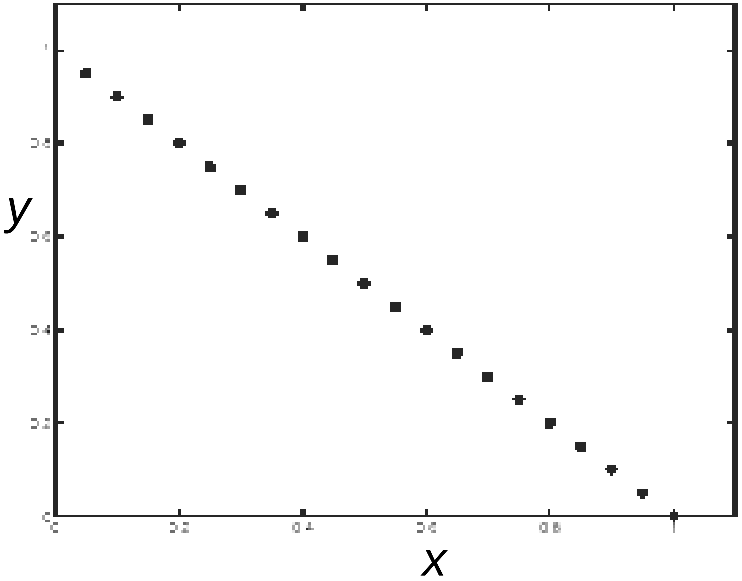
For each edge point (x,y) in the image
    θ = gradient orientation at (x,y)
    ρ = x cos θ + y sin θ
    H(θ, ρ) = H(θ, ρ) + 1
end

Svetlana Lazebnik

Hough transform example



Impact of noise on Hough



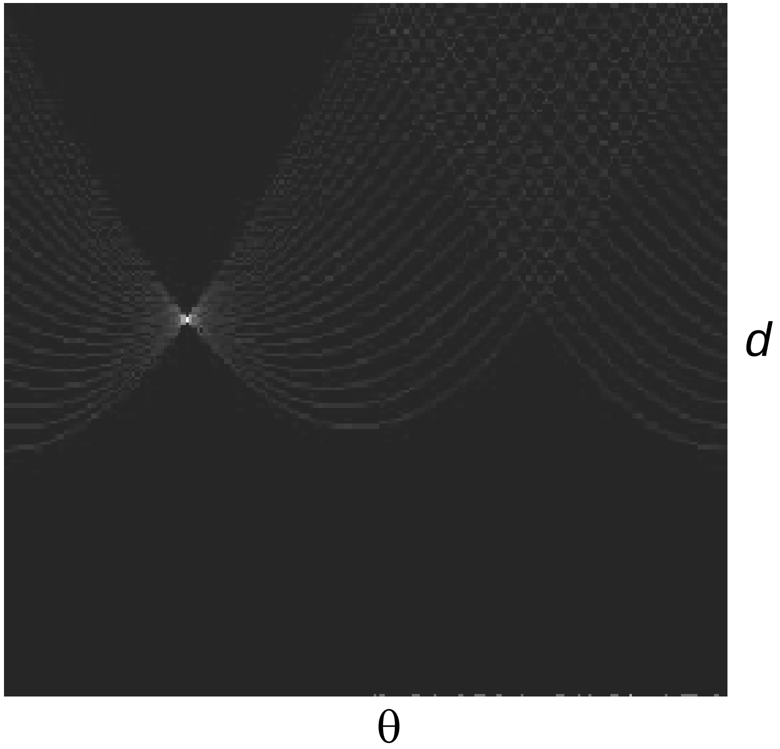
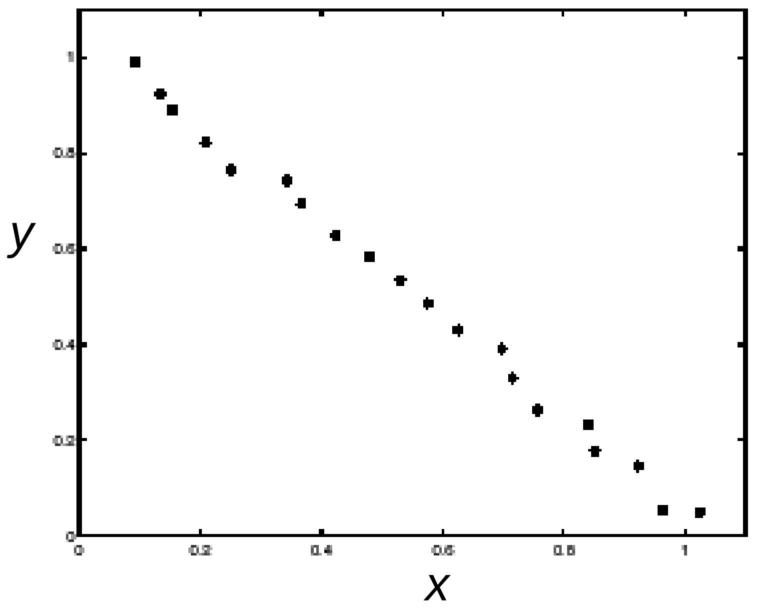


Image space edge coordinates

Votes

Impact of noise on Hough



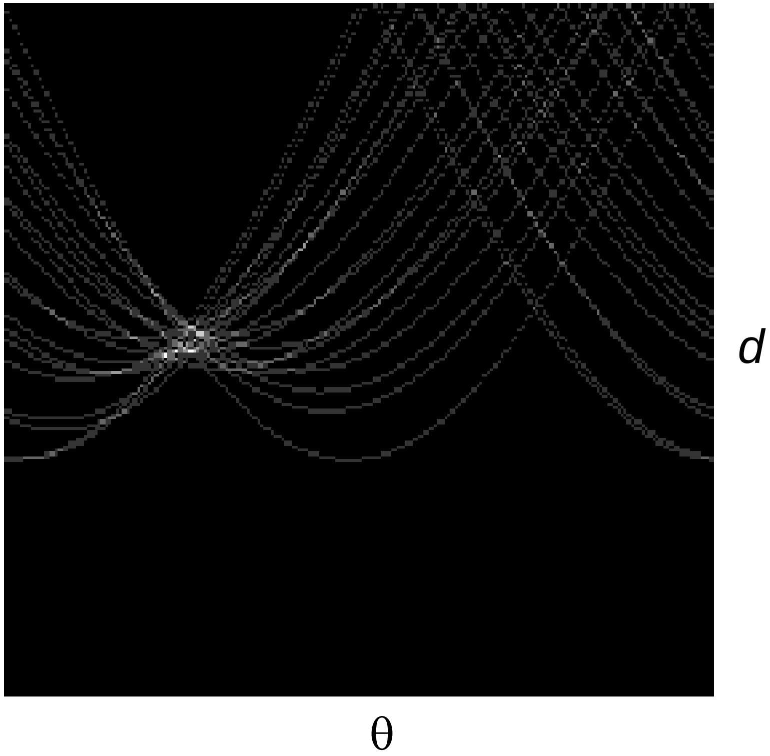


Image space edge coordinates

Votes

What difficulty does this present for an implementation?

Kristen Grauman

Voting: practical tips

- Minimize irrelevant tokens first (reduce noise)
- Choose a good grid / discretization

Too fine ? Too coarse

  - **Too coarse:** large votes obtained when too many different lines correspond to a single bucket
  - **Too fine:** miss lines because points that are not exactly collinear cast votes for different buckets

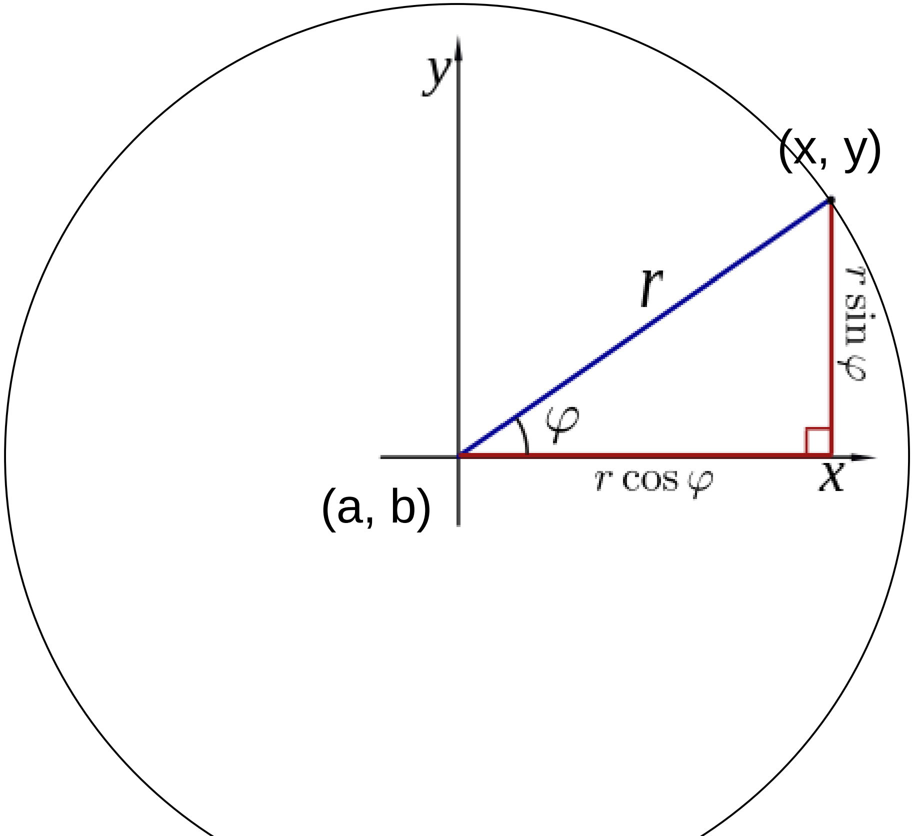
- Vote for neighbors (smoothing in accumulator array)
- Use direction of edge to reduce parameters by 1
- To read back which points voted for “winning” peaks, keep tags on the votes

Hough transform for circles

- A circle with radius *r* and center (*a, b*) can be described as:

$x = a + r\cos(\theta)$

$y = b + r\sin(\theta)$



Hough transform for circles

- Circle: center (*a, b*) and radius r

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- For a fixed radius r, unknown gradient direction

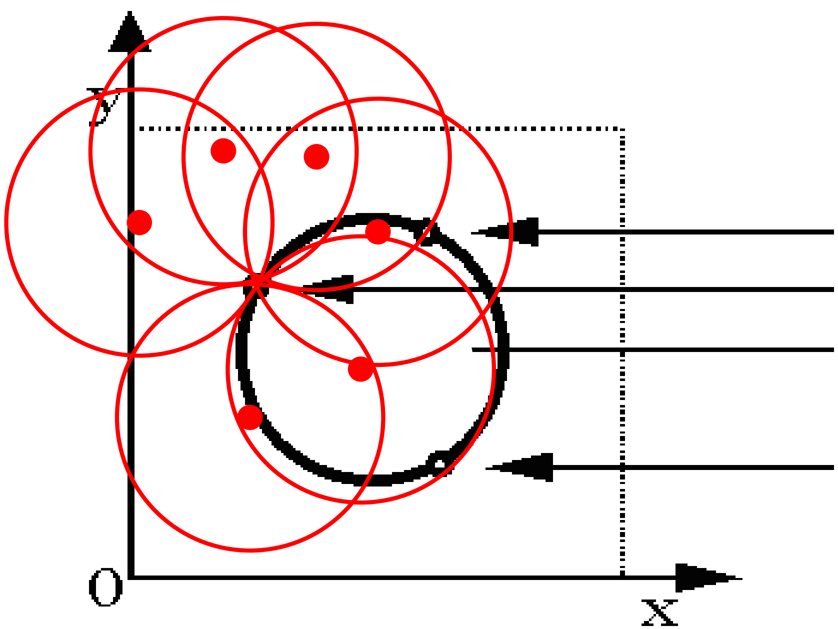
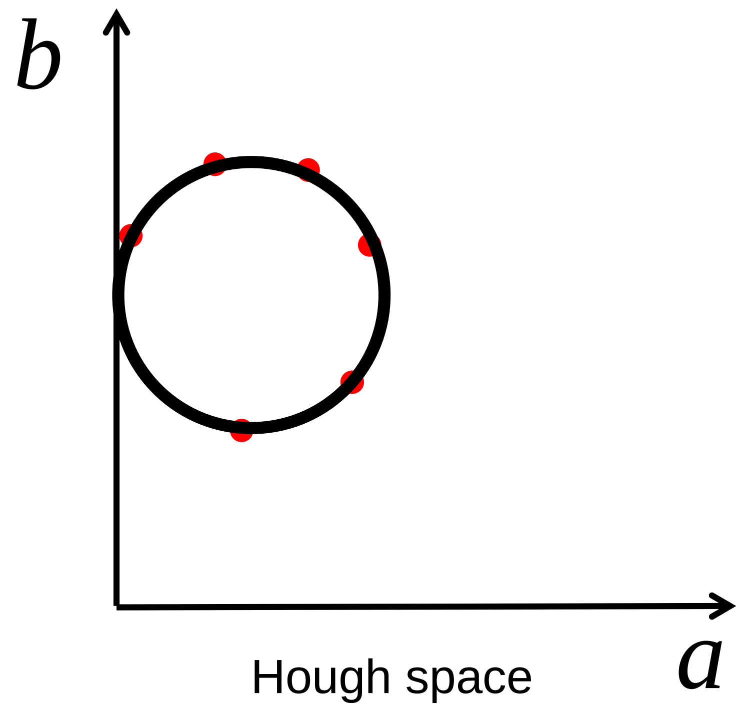


Image space



Hough space

Kristen Grauman

Hough transform for circles

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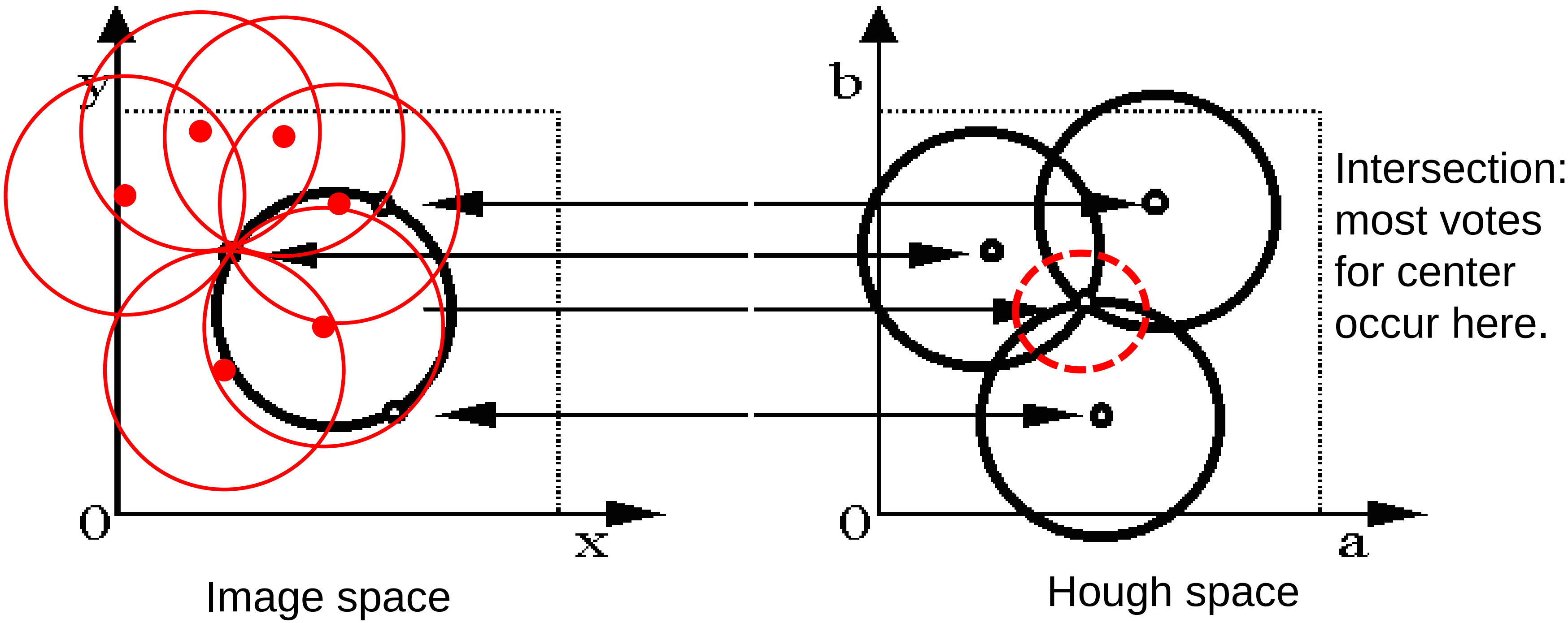


Kristen Grauman

Hough transform for circles

$x = a + r\cos(\theta)$
$y = b + r\sin(\theta)$

For every edge pixel (x,y) :

  For each possible radius value r:

    For each possible gradient direction θ:

    // or use estimated gradient at (x,y)

      $a = x - r\cos(\theta)$ // column

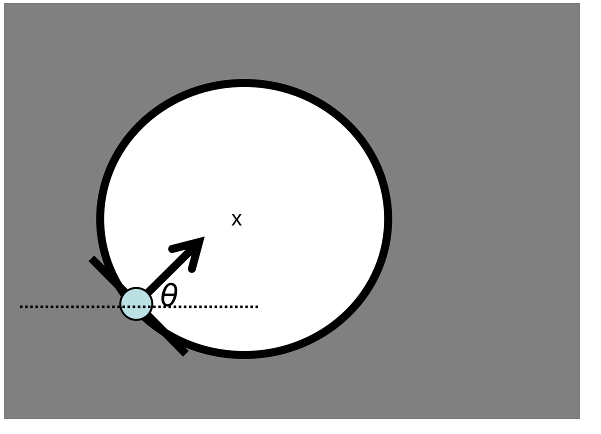
      $b = y - r\sin(\theta)$ // row

      H[a,b,r] += 1

    end

  end

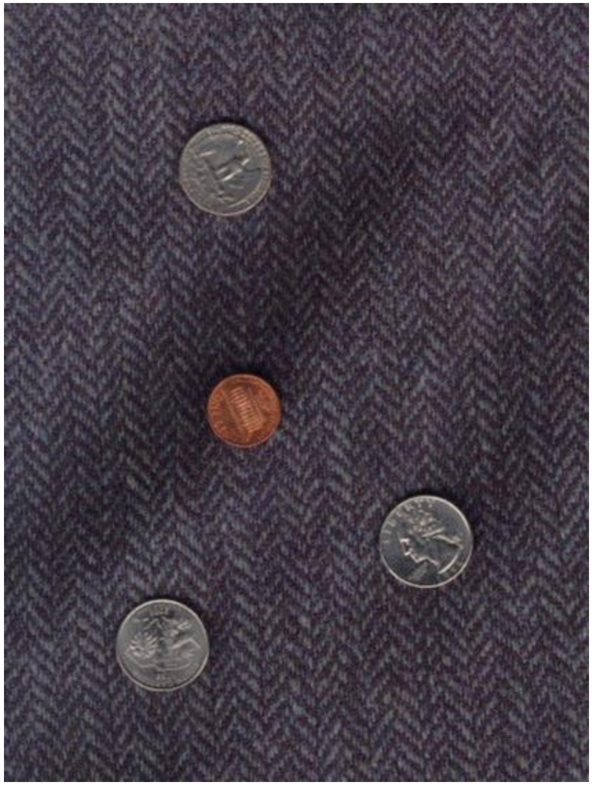
end



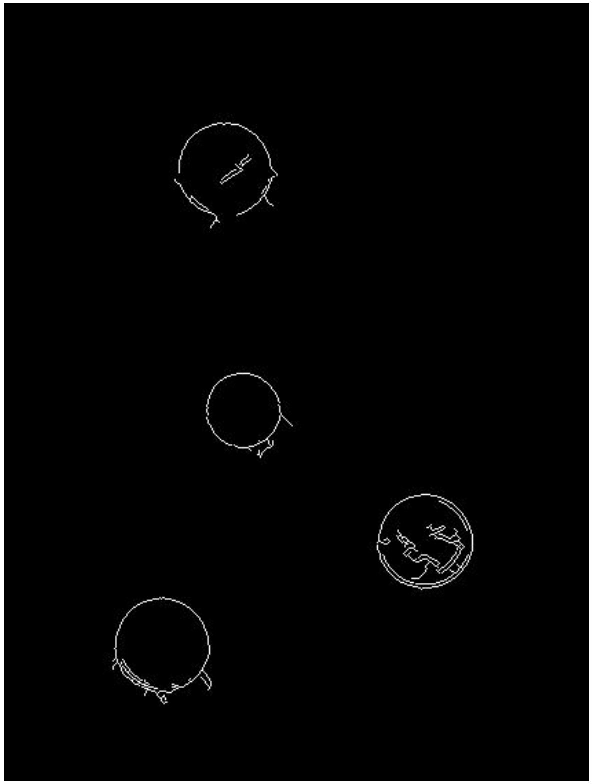
Modified from Kristen Grauman

Example: detecting circles with Hough

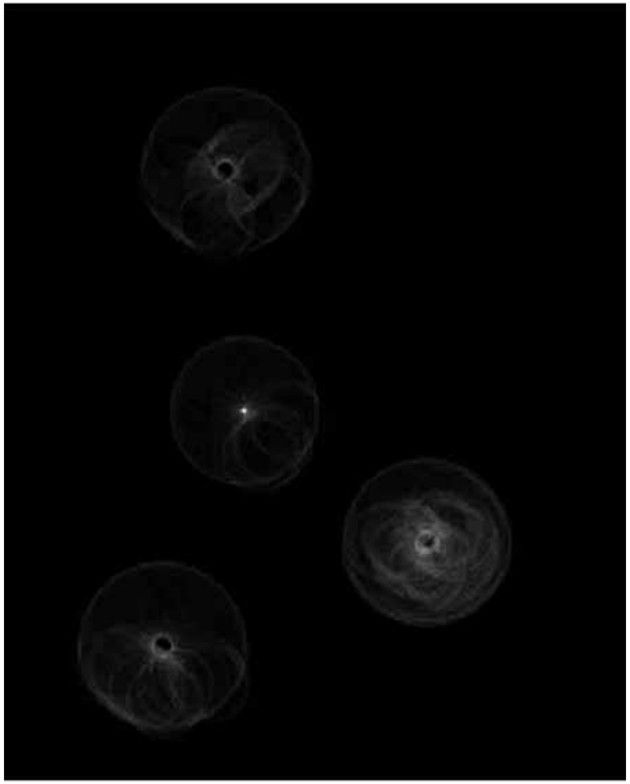
Original



Edges



Votes: Penny



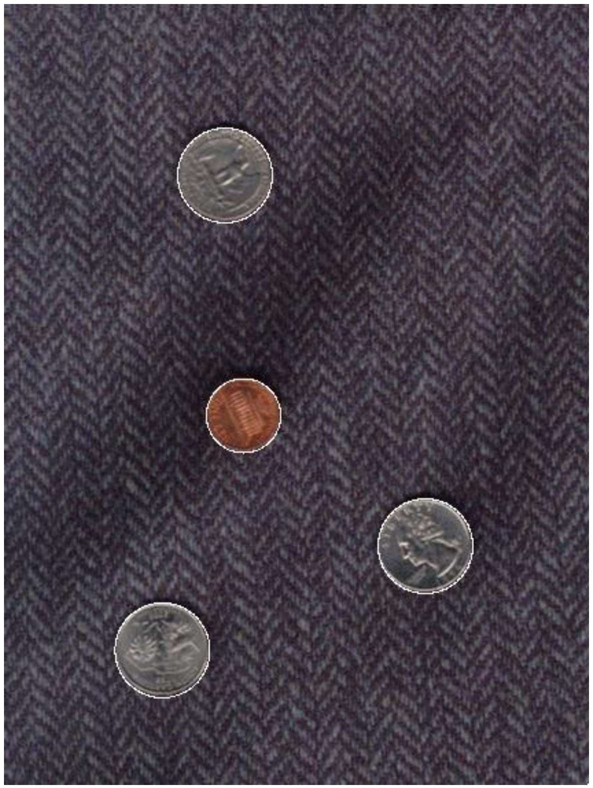
Note: a different Hough transform (with separate accumulators) was used for each circle radius (quarters vs. penny).

Kristen Grauman, images from Vivek Kwatra

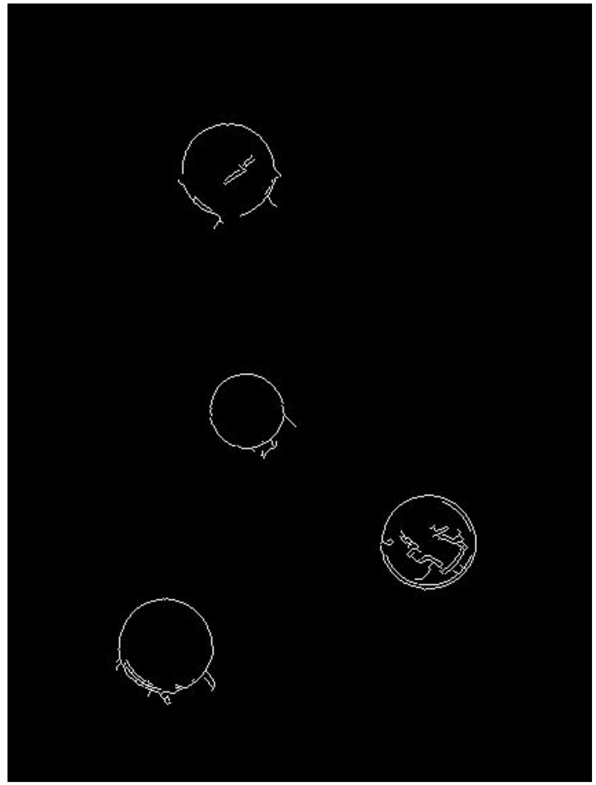
Example: detecting circles with Hough

Combined detections

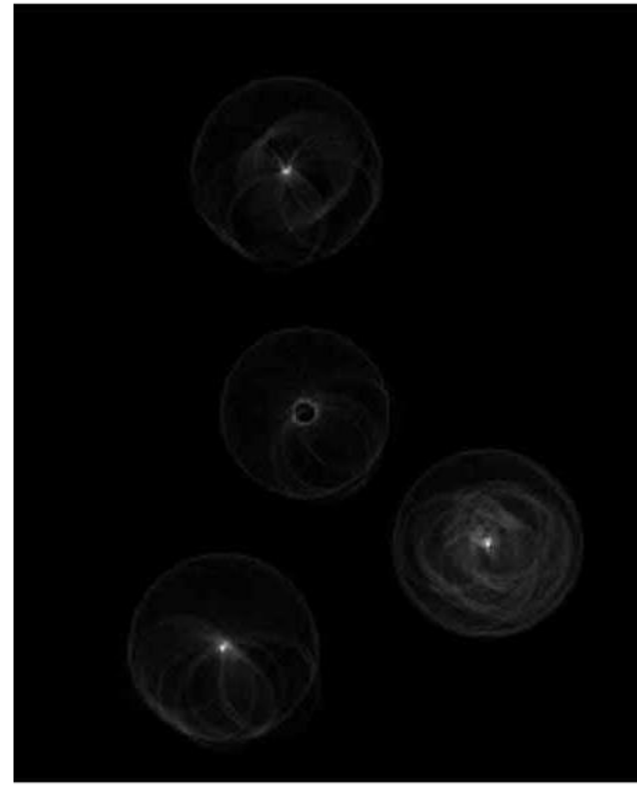
Original



Edges



Votes: Quarter



Note: a different Hough transform (with separate accumulators) was used for each circle radius (quarters vs. penny).

Kristen Grauman, images from Vivek Kwatra

Hough transform: pros and cons

Pros

- All points are processed independently, so can cope with occlusion, gaps
- Some robustness to noise: noise points *unlikely* to contribute *consistently* to any single bin
- Can detect multiple instances of a model in a single pass

Cons

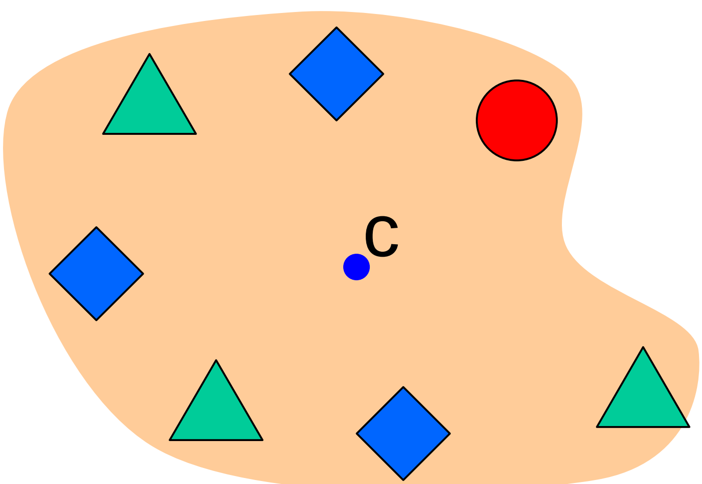
- Complexity of search time for maxima increases exponentially with the number of model parameters
  - If 3 parameters and 10 choices for each, search is $O(10^3)$
- Quantization: can be tricky to pick a good grid size

Adapted from Kristen Grauman

Generalized Hough transform

- We want to find a template defined by its reference point (center) and several distinct types of landmark points in stable spatial configuration

Template

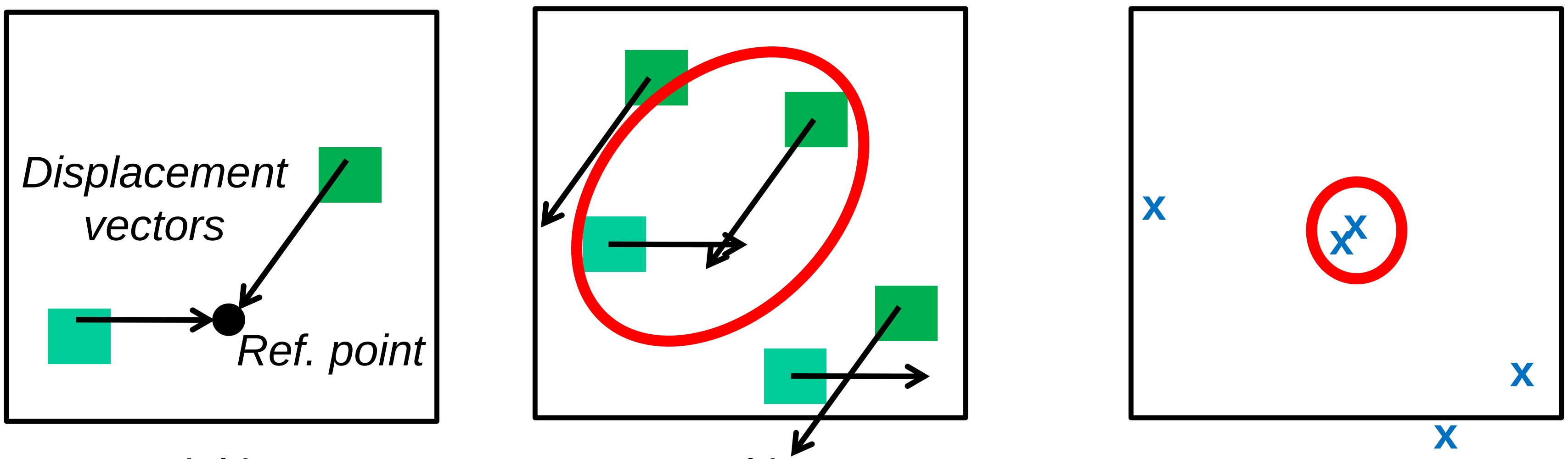


Triangle, circle, diamond: some *type* of visual token, e.g. feature or edge point

Adapted from Svetlana Lazebnik

Generalized Hough transform

Intuition:



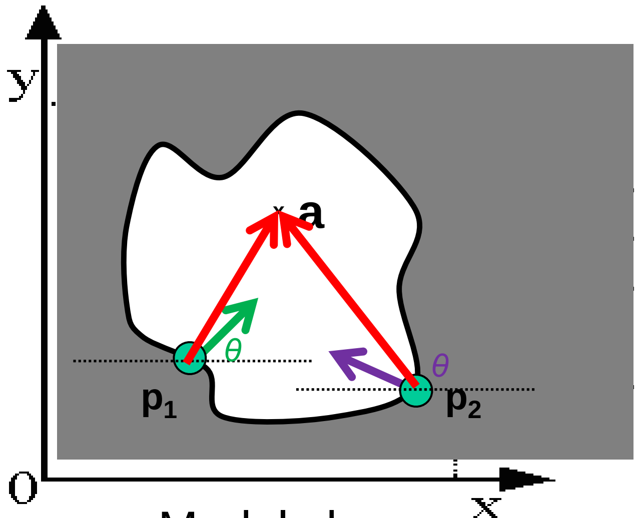
Model image | Novel image | Vote space

Now suppose those colors encode gradient directions…

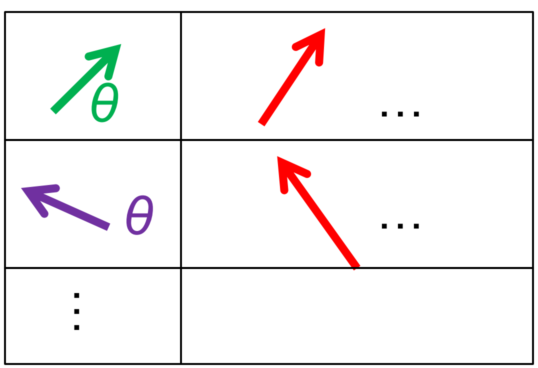
Adapted from Kristen Grauman

Generalized Hough transform

Define a model shape by its boundary points and a reference point.



Model shape



Offline procedure:

At each boundary point, compute displacement vector: $\mathbf{r} = \mathbf{a} - \mathbf{p}_i$.

Store these vectors in a table indexed by gradient orientation θ.

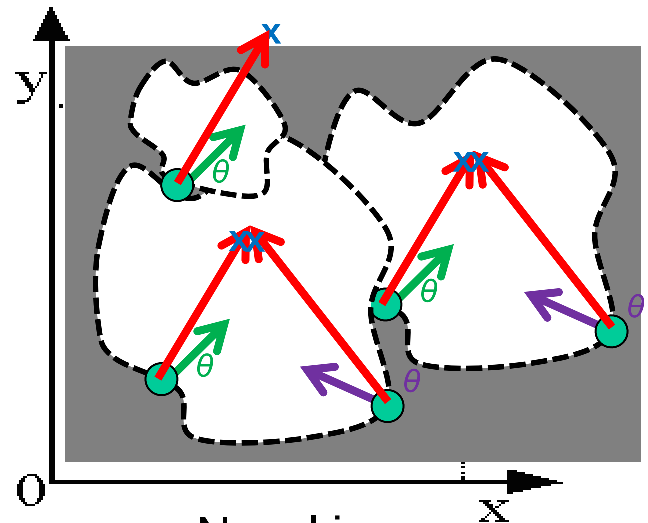
[Dana H. Ballard, Generalizing the Hough Transform to Detect Arbitrary Shapes, 1980]

Generalized Hough transform

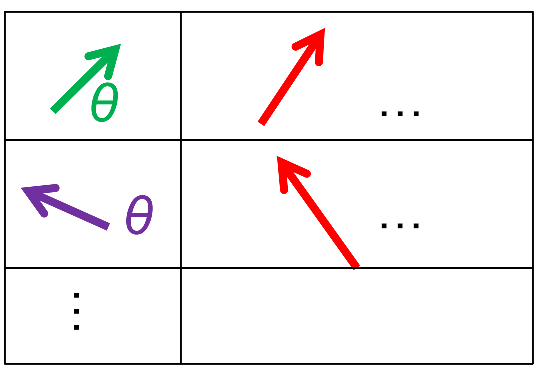
Detection procedure:

For each edge point:

- Use its gradient orientation θ to index into stored table
- Use retrieved **r** vectors to vote for reference point



Novel image



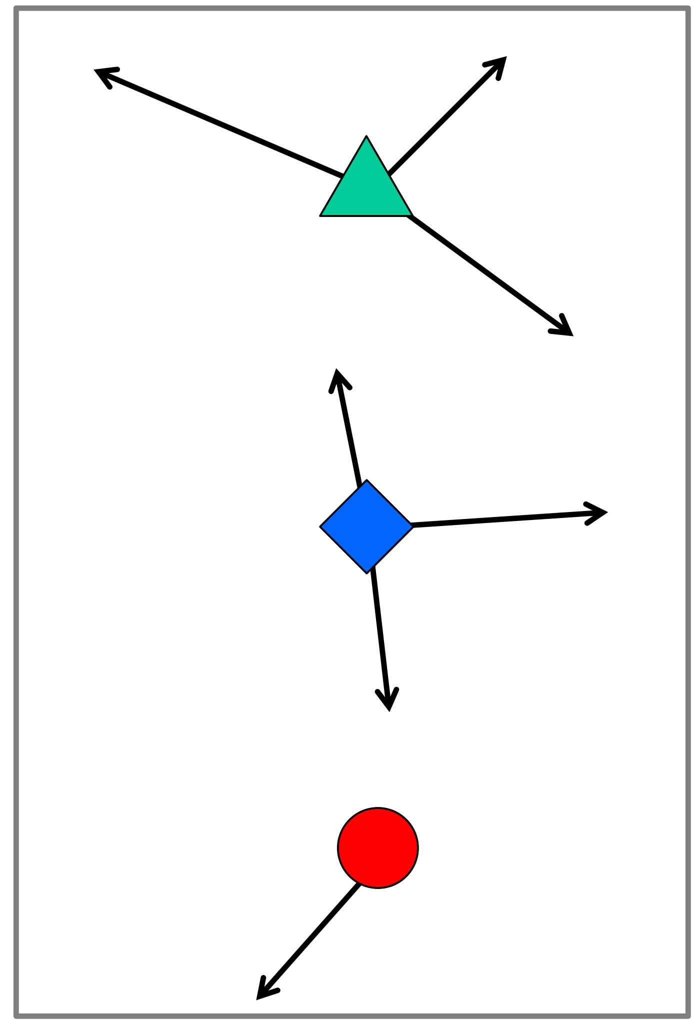
Assuming translation is the only transformation here, i.e., orientation and scale are fixed.

Kristen Grauman

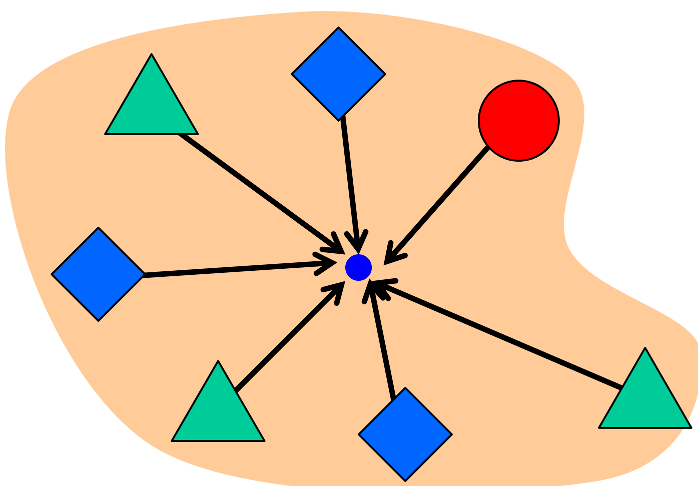
Generalized Hough transform

- Template representation
  - For each type of landmark point, store all possible displacement vectors towards the center

Model



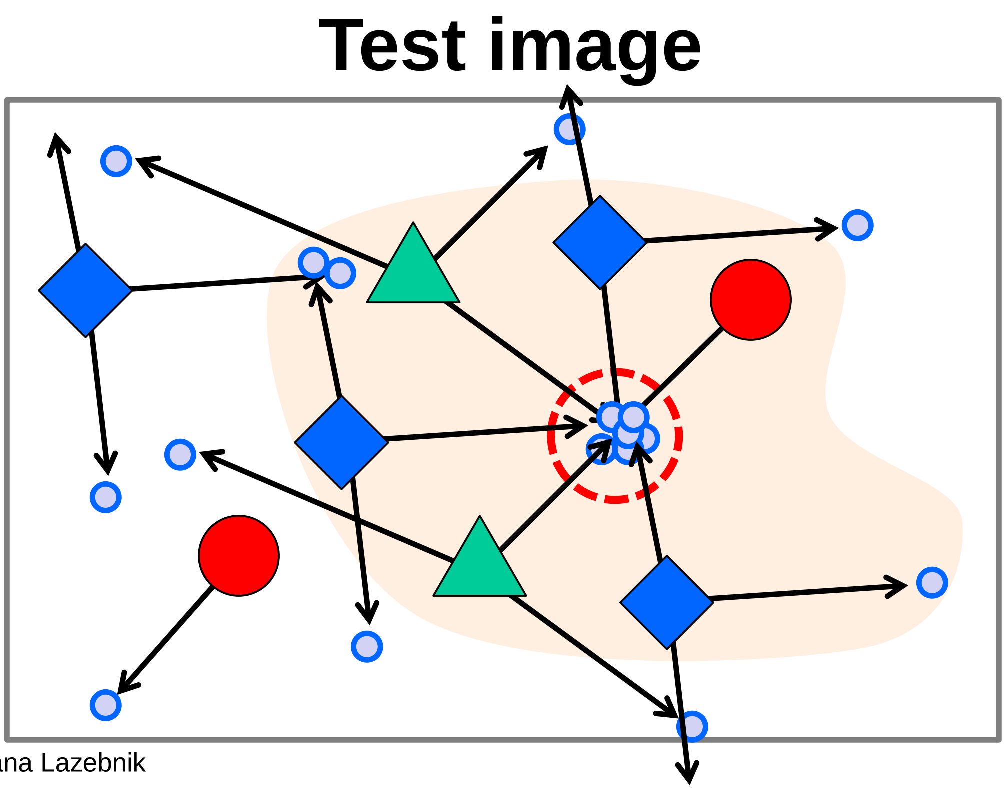
Template

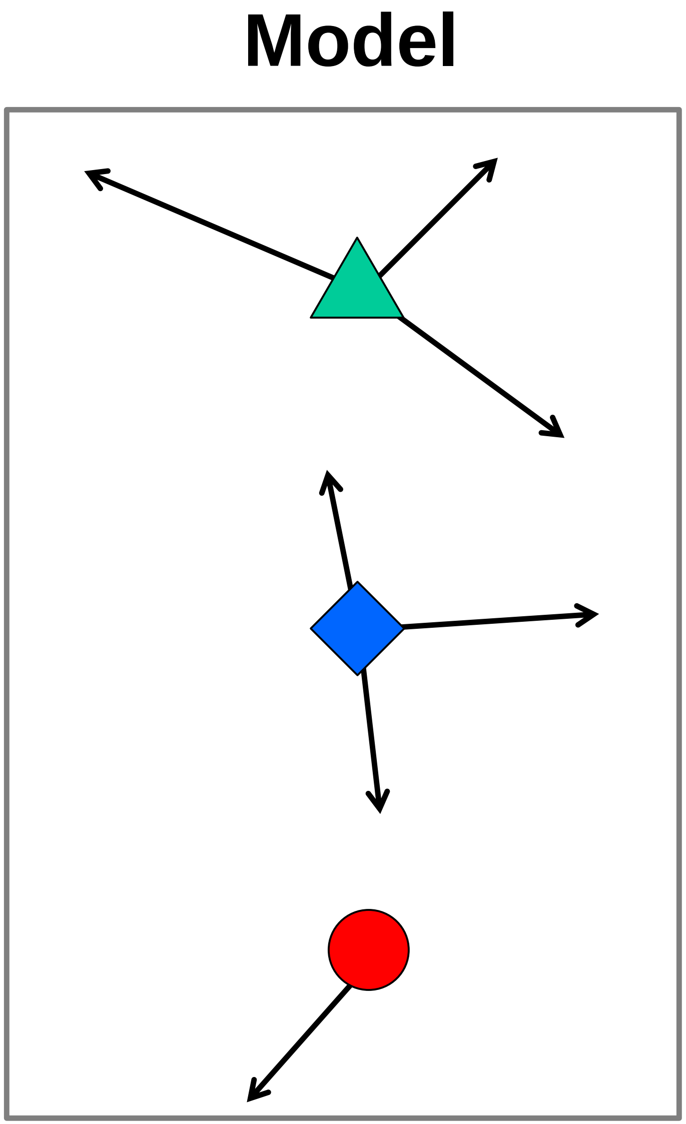


Svetlana Lazebnik

Generalized Hough transform

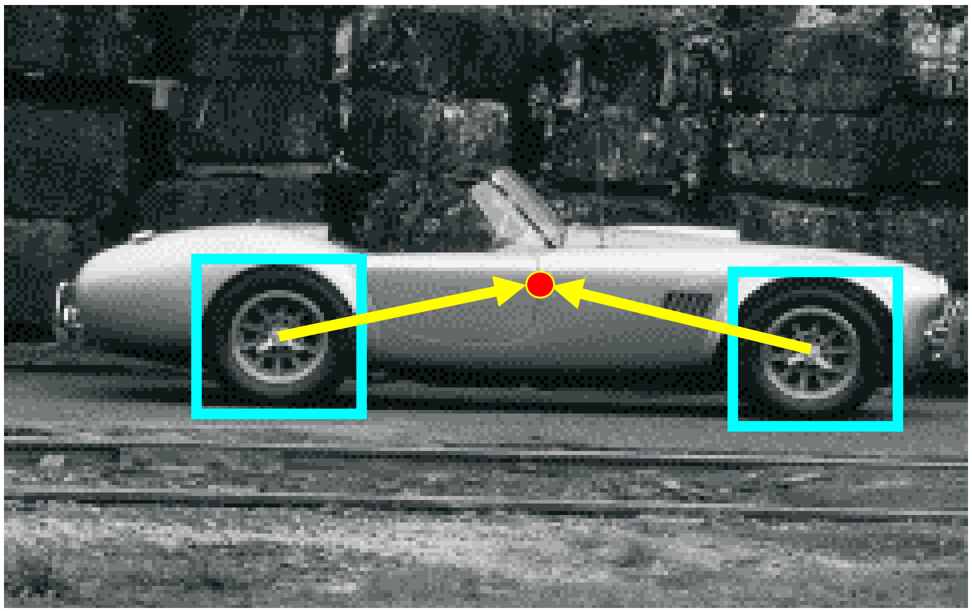
- Detecting the template
  - For each feature in a new image, look up that feature type in the model and vote for the possible center locations associated with that type in the model



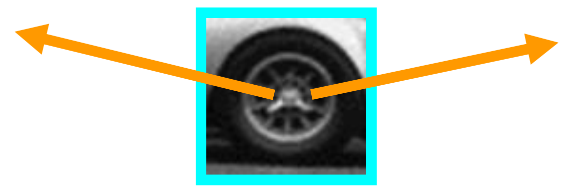


Application: Hough for object detection

- Index displacements by “visual codeword”



training image



“visual codeword” with displacement vectors

B. Leibe, A. Leonardis, and B. Schiele, [Combined Object Categorization and Segmentation with an Implicit Shape Model](), ECCV Workshop on Statistical Learning in Computer Vision 2004

Svetlana Lazebnik

RANSAC

- RANdom Sample Consensus

- **Approach**: we want to avoid the impact of outliers, so let’s look for “inliers”, and use those only.

- **Intuition**: if an outlier is chosen to compute the current fit, then the resulting line won’t have much support from rest of the points.

RANSAC: General form

- RANSAC loop:

1. Randomly select a *seed group* of ***s*** points on which to base model estimate (e.g. ***s***=2 for a line)
2. Fit model to these ***s*** points
3. Find *inliers* to this model (i.e., points whose distance from the line is less than ***t***)
4. Repeat ***N*** times

- Keep the model with the largest number of inliers

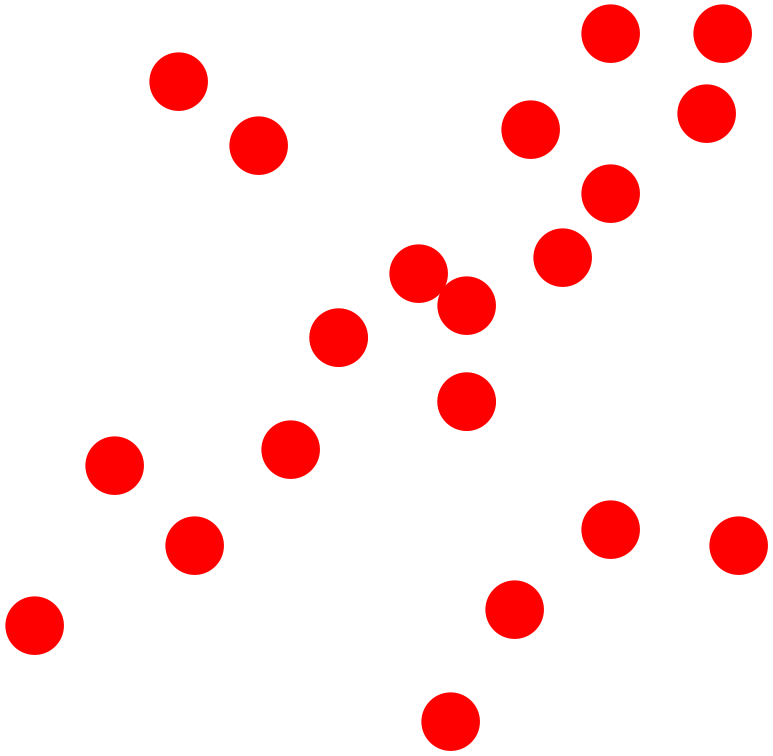
Adapted from Kristen Grauman and Svetlana Lazebnik

RANSAC

(RANdom SAmple Consensus) :

Fischler & Bolles in '81.

Line fitting example



Algorithm:

1. **Sample** (randomly) the number of points required to fit the model
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

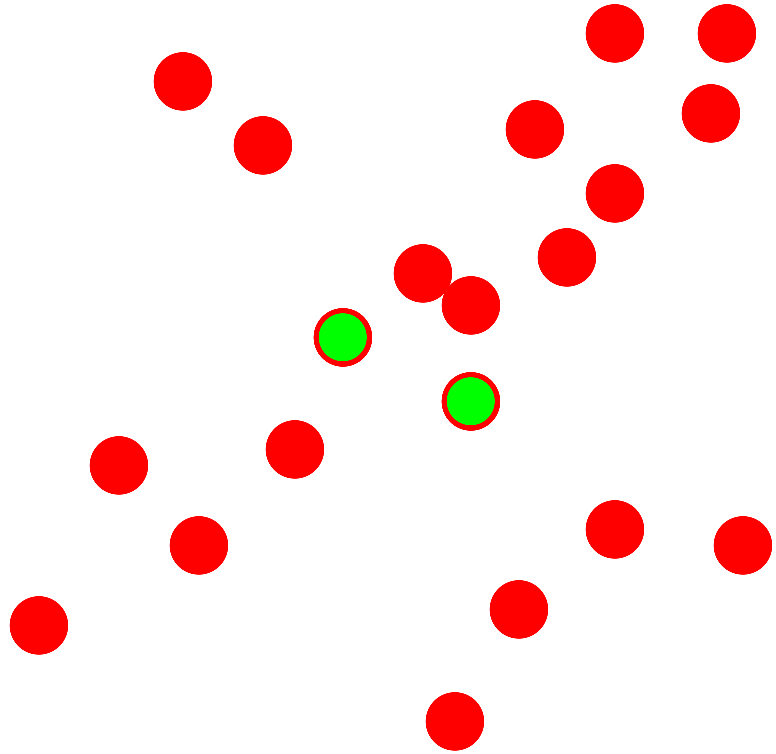
Repeat 1-3 until the best model is found with high confidence

RANSAC

(RANdom SAmple Consensus) :

Fischler & Bolles in ‘81.

Line fitting example



Algorithm:

1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

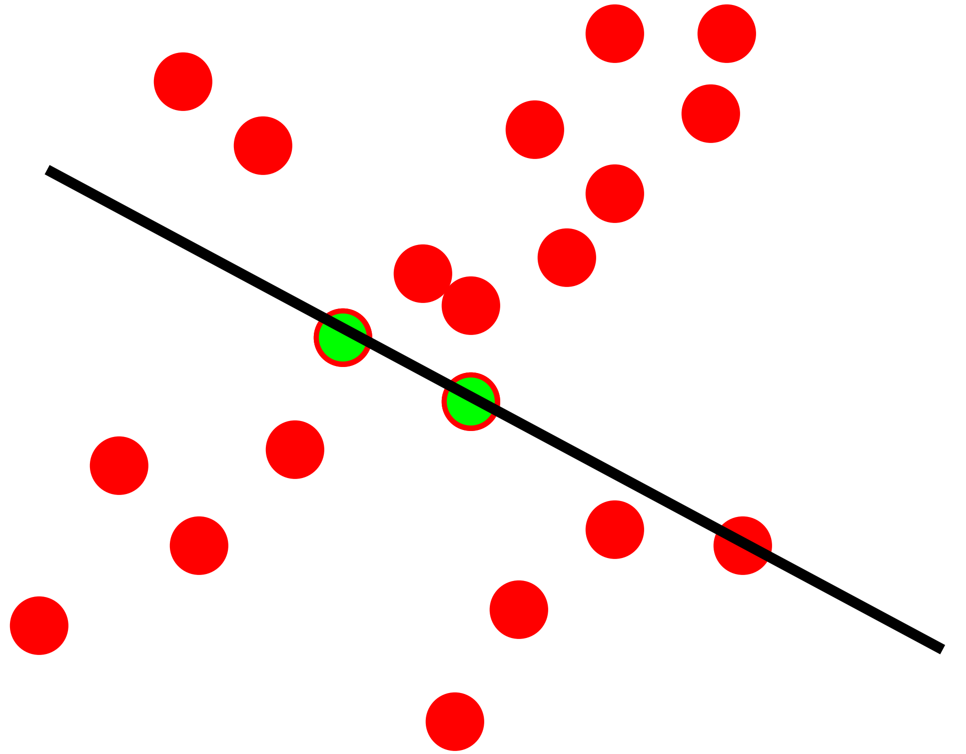
Repeat 1-3 until the best model is found with high confidence

RANSAC

(RANdom SAmple Consensus) :

Fischler & Bolles in ‘81.

Line fitting example



Algorithm:

1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
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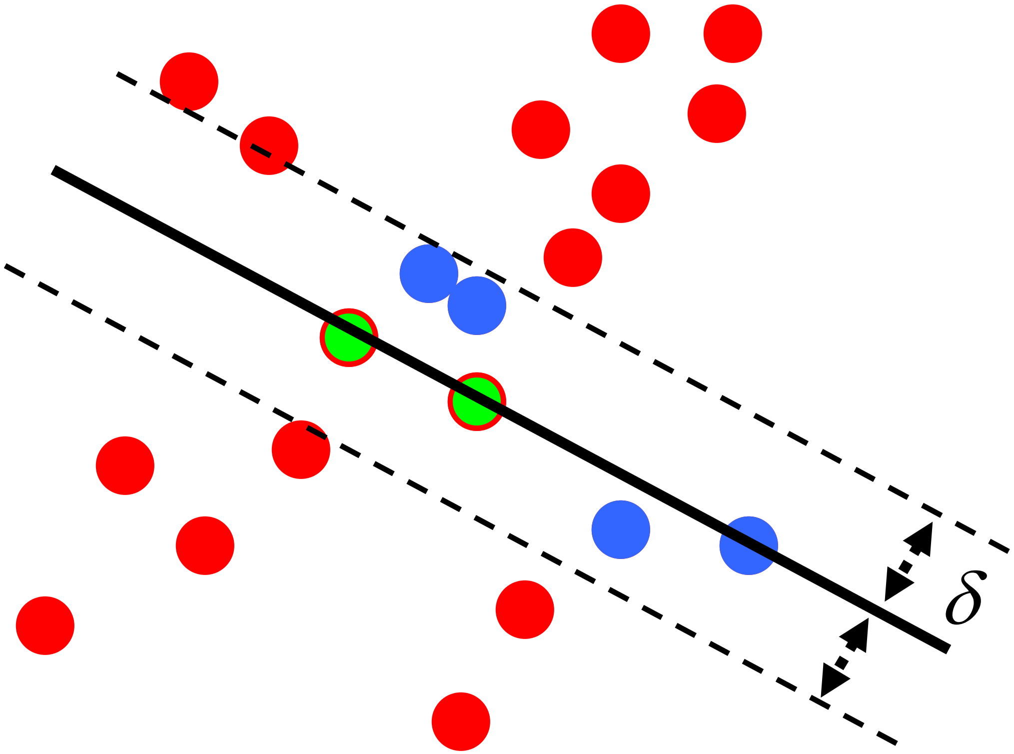
Repeat 1-3 until the best model is found with high confidence

RANSAC

(RANdom SAmple Consensus) :

Fischler & Bolles in '81.

Line fitting example



$$N_I = 6$$

Algorithm:

1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

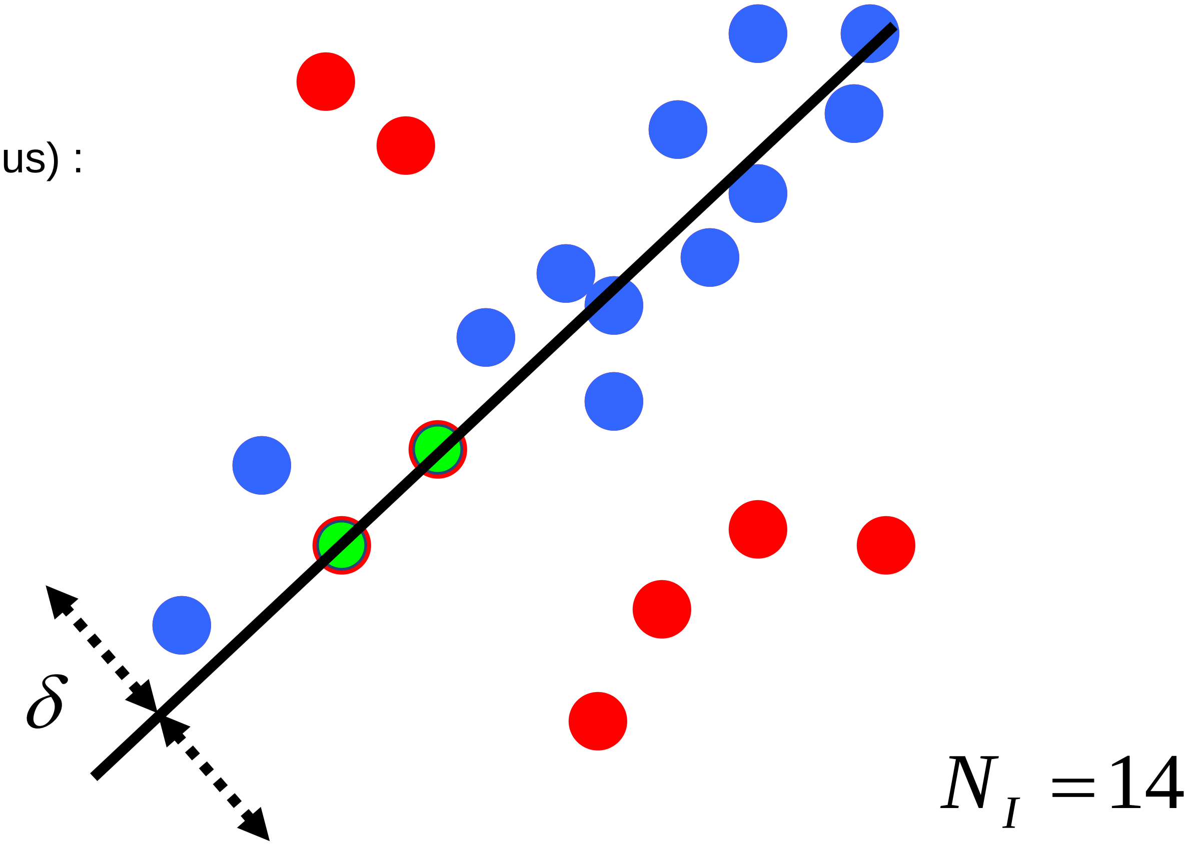
Repeat 1-3 until the best model is found with high confidence

RANSAC

(RANdom SAmple Consensus) :

Fischler & Bolles in '81.

Line fitting example



Algorithm:

1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

How to choose parameters?

- Number of samples *N*
  - Choose *N* so that, with probability *p*, at least one random sample is free from outliers (e.g. *p*=0.99) (outlier ratio: *e*)
- Number of sampled points *s*
  - Minimum number needed to fit the model
- Distance threshold δ
  - Choose δ so that a good point with noise is likely (e.g., prob=0.95) within threshold
  - E.g. for zero-mean Gaussian noise with std. dev. σ: $\delta^2 = 3.84\sigma^2$

$$\boxed{N = \log(1-p) / \log\left(1-(1-e)^s\right)}$$

Explanation in Szeliski 6.1.4

	proportion of outliers *e*						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

RANSAC pros and cons

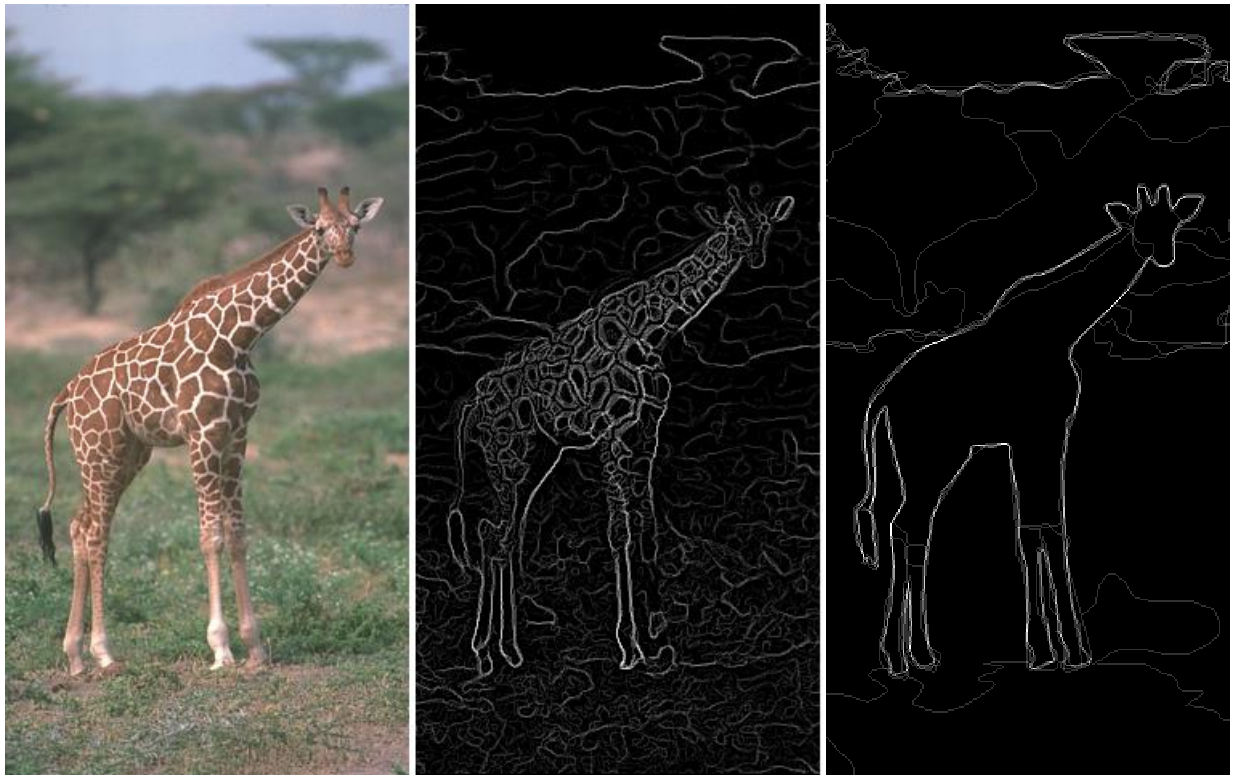
- Pros
  - Applicable to many different problems, e.g. image stitching, relating two views
  - Often works well in practice
- Cons
  - Lots of parameters to tune (see previous slide)
  - Doesn't work well for low inlier ratios (too many iterations, or can fail completely)

Adapted from Svetlana Lazebnik

Plan for this lecture

- Group pixels into:
  - Edges: Extract gradients and threshold
  - Lines: Find which edge points are collinear or belong to another shape
  - Segments: Find which pixels form a consistent region, e.g. via clustering
- Transform pixels:
  - Find relationships between multiple views of the same world point
  - *Both parts rely on finding geometric relationships between pixels*

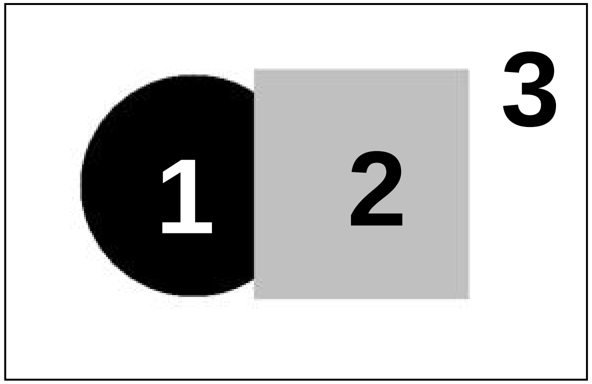
Edges vs Segments



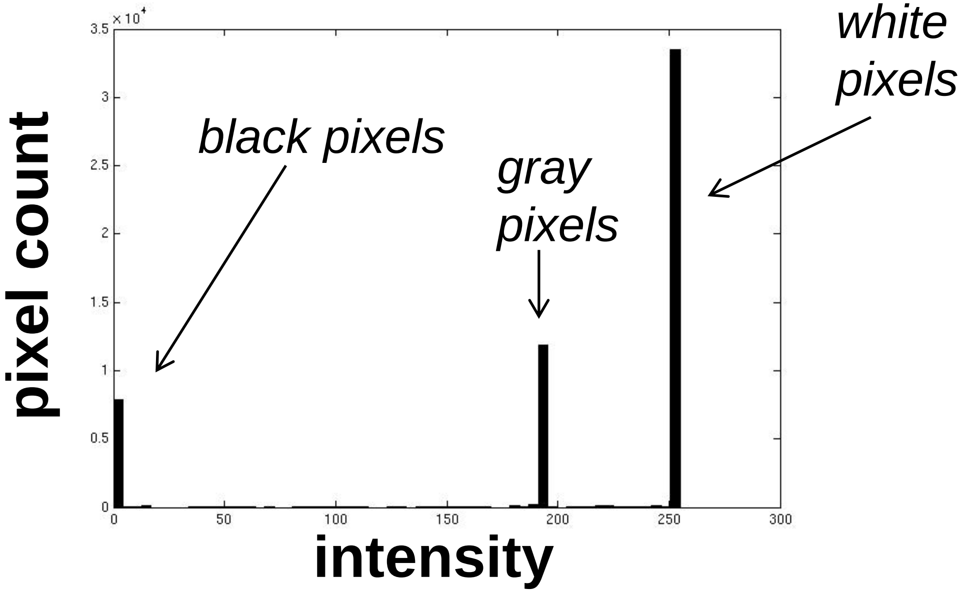
- Edges: More low-level; don't need to be closed
- Segments: Ideally one segment for each semantic group/object; should include closed contours

Figure adapted from J. Hays

Image segmentation: toy example

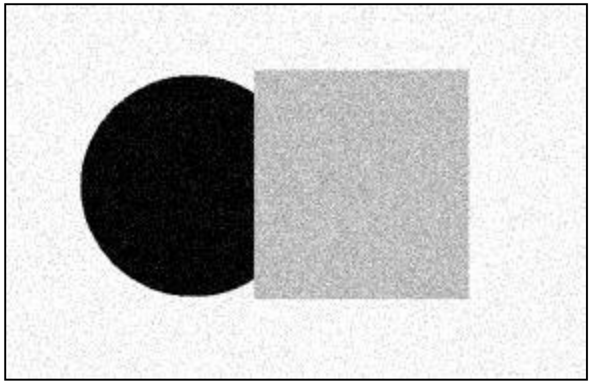


input image

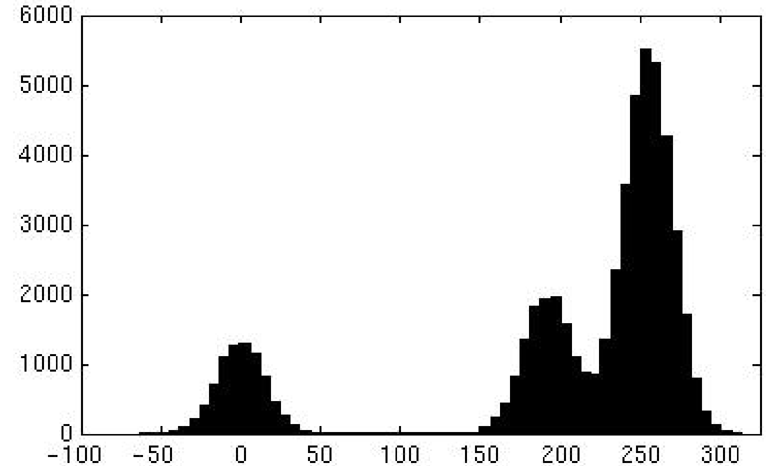


- These intensities define the three groups.
- We could label every pixel in the image according to which of these primary intensities it is.
  - i.e., *segment* the image based on the intensity feature.
- What if the image isn't quite so simple?

Source: K. Grauman



input image

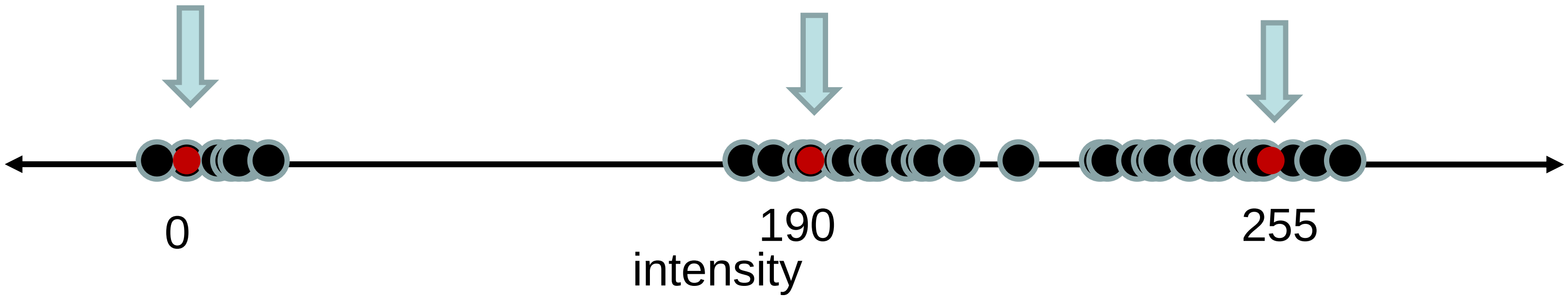


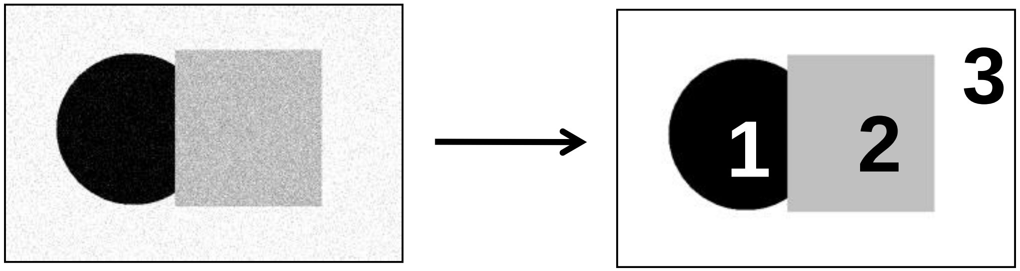
pixel count

intensity

- Now how to determine the three main intensities that define our groups?
- We need to ***cluster.***

Source: K. Grauman





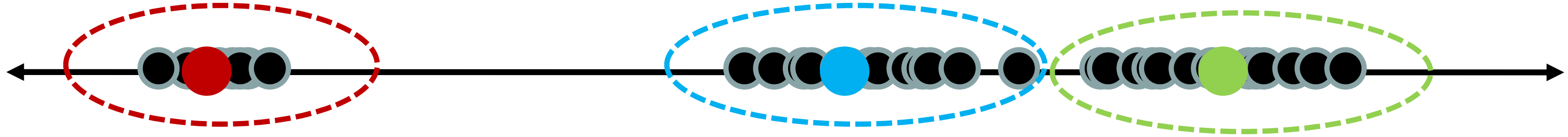
- Goal: choose three “centers” as the representative intensities, and label every pixel according to which of these centers it is nearest to.
- Best cluster centers are those that minimize *sum of squared differences* (SSD) between all points and their nearest cluster center c_i:

$$\sum_{\text{clusters } i} \; \sum_{\text{points p in cluster } i} ||p - c_i||^2$$

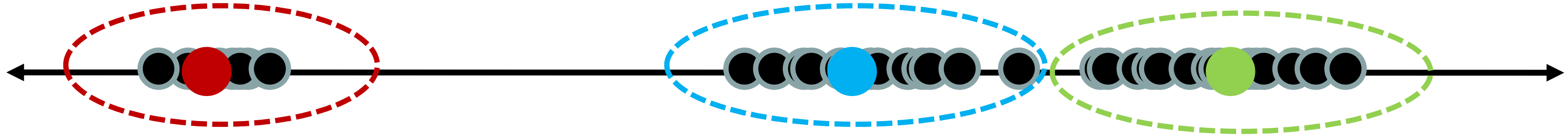
Source: K. Grauman

Clustering

- With this objective, it is a "chicken and egg" problem:
  - If we knew the **cluster centers**, we could allocate points to groups by assigning each to its closest center.



  - If we knew the **group memberships**, we could get the centers by computing the mean per group.



Source: K. Grauman

K-means clustering

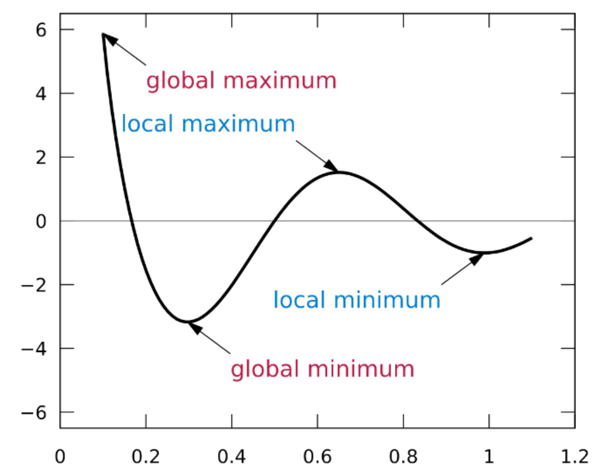
- Basic idea: randomly initialize the *k* cluster centers, and iterate between the two steps we just saw.

  1. *Randomly* initialize the cluster centers, $c_1, \ldots, c_K$
  2. *Given cluster centers*, determine points in each cluster
     - For each point p, find the closest c_i. Put p into cluster i
  3. *Given points in each cluster*, solve for c_i
     - Set c_i to be the mean of points in cluster i
  4. If c_i have changed, repeat Step 2

Properties

- Will always converge to *some* solution
- Can be a "local minimum" of objective:

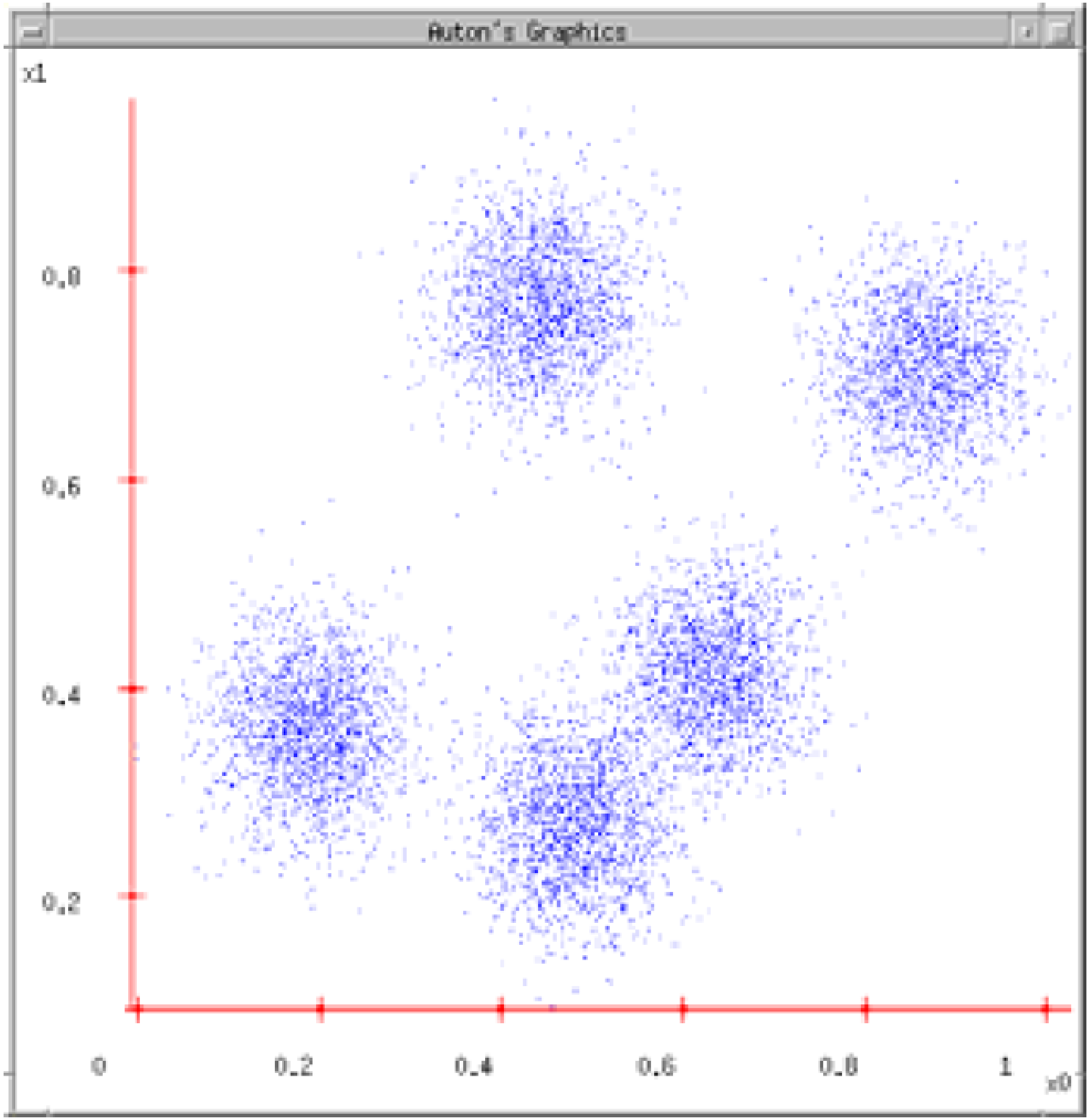
$$\sum_{\text{clusters } i} \sum_{\text{points p in cluster } i} \|p - c_i\|^2$$



Slide: Steve Seitz, image: Wikipedia

K-means

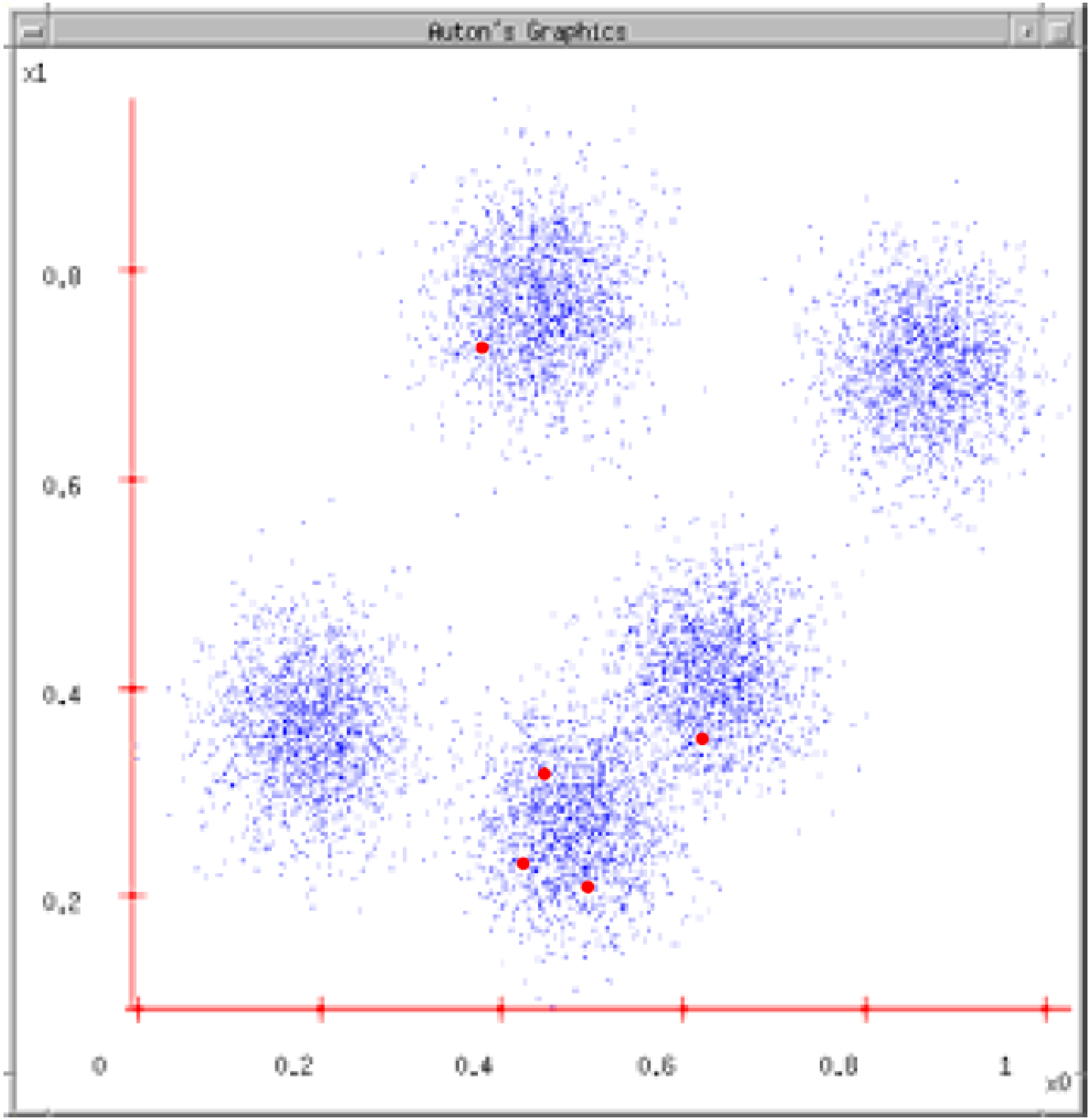
1. Ask user how many clusters they'd like. *(e.g. k=5)*



Source: A. Moore

K-means

1. Ask user how many clusters they'd like. *(e.g. k=5)*
2. Randomly guess k cluster Center locations



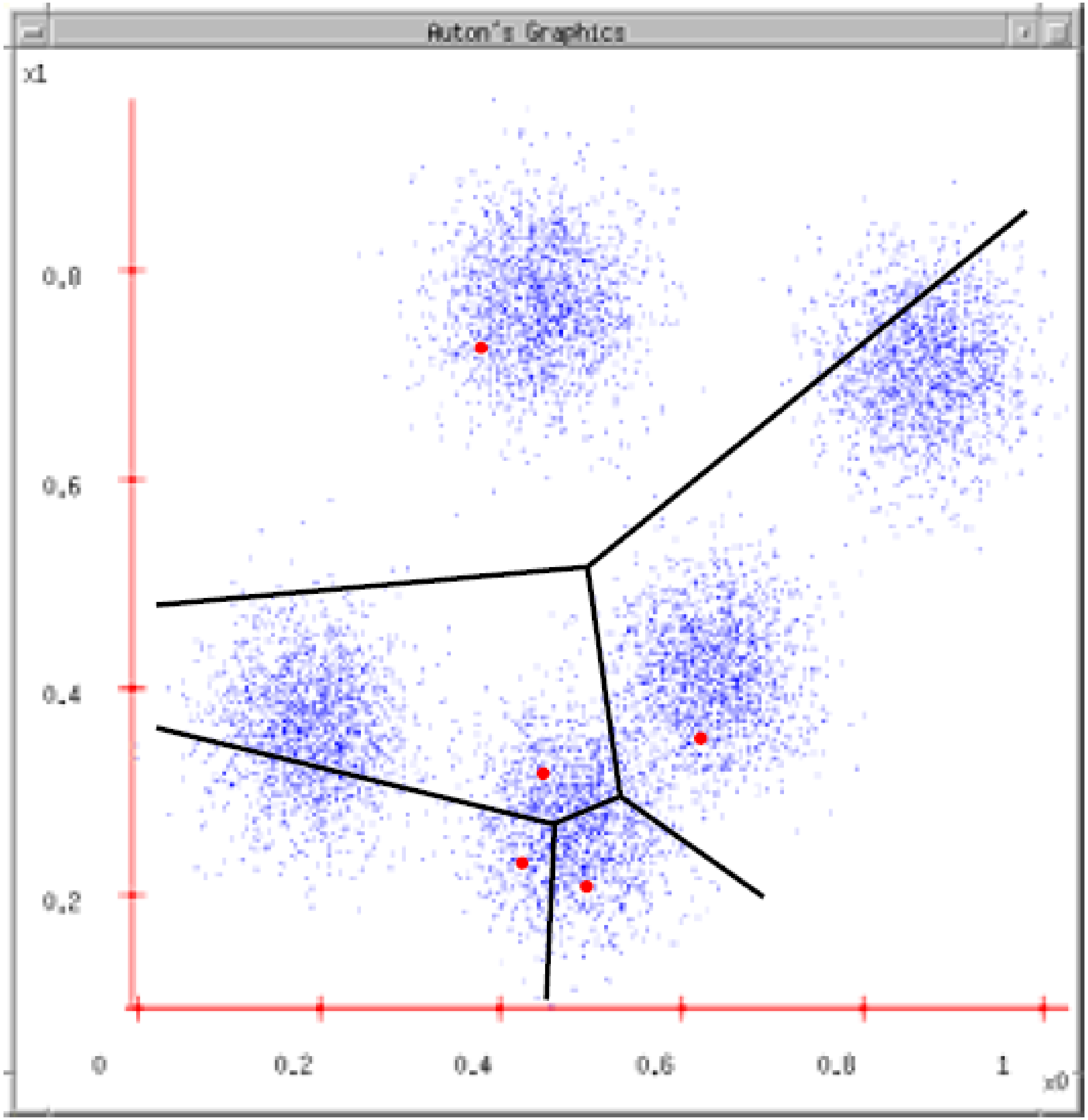
Source: A. Moore

K-means

1. Ask user how many clusters they'd like. *(e.g. k=5)*

2. Randomly guess k cluster Center locations

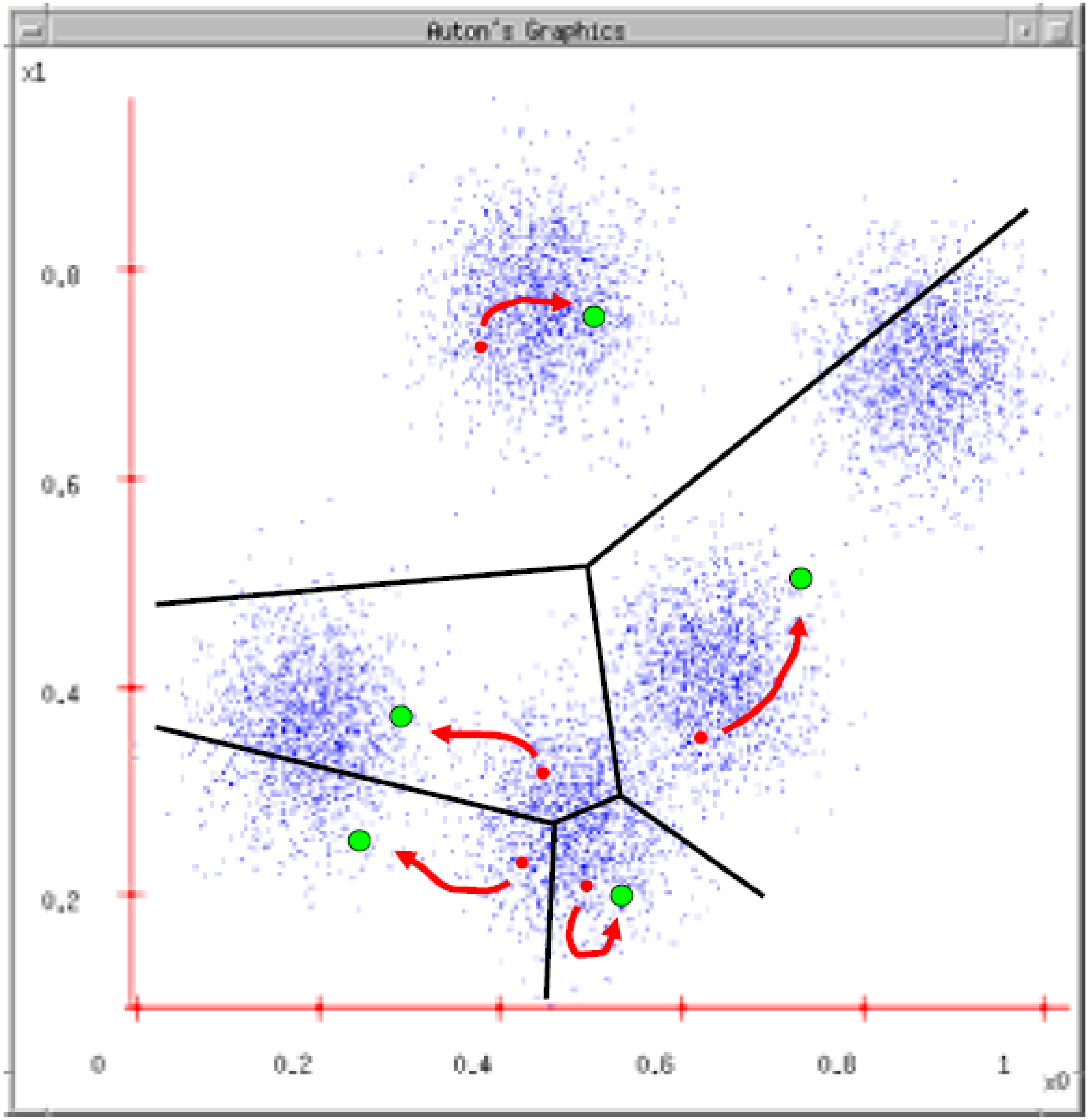
3. Each datapoint finds out which Center it's closest to. (Thus each Center "owns" a set of datapoints)



Source: A. Moore

K-means

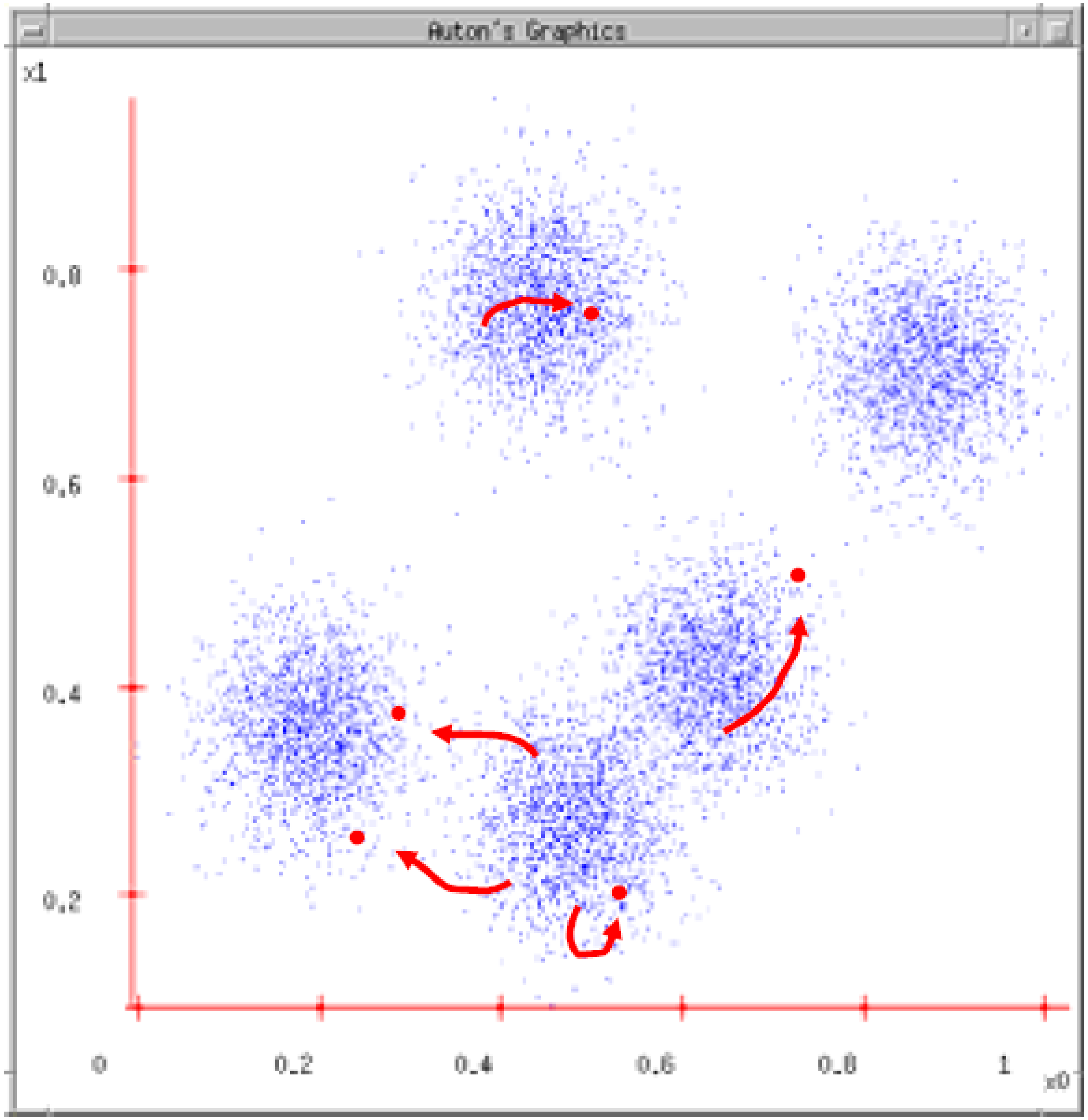
1. Ask user how many clusters they'd like. *(e.g. k=5)*
2. Randomly guess k cluster Center locations
3. Each datapoint finds out which Center it's closest to.
4. Each Center finds the centroid of the points it owns



Source: A. Moore

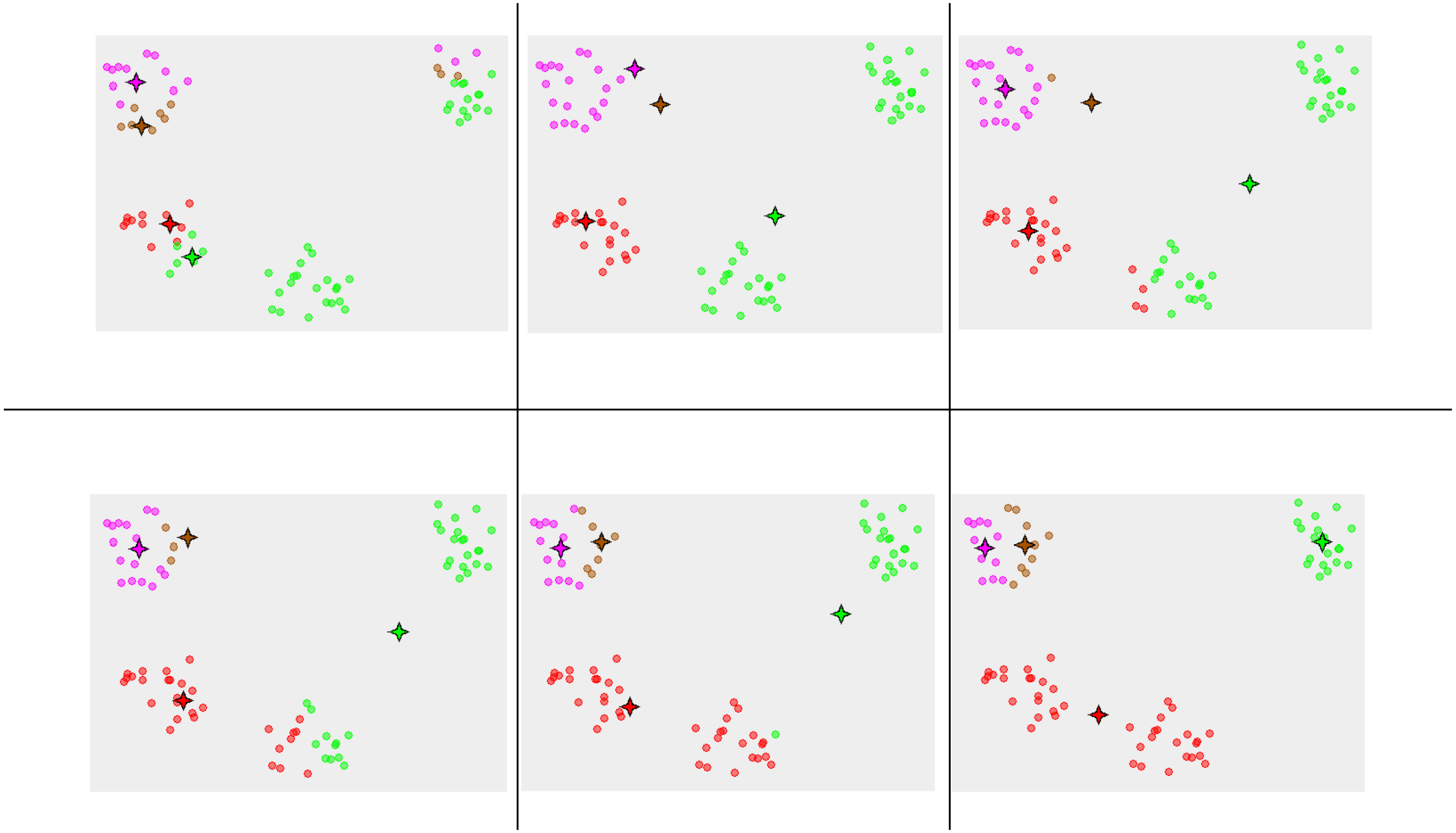
K-means

1. Ask user how many clusters they'd like. *(e.g. k=5)*
2. Randomly guess k cluster Center locations
3. Each datapoint finds out which Center it's closest to.
4. Each Center finds the centroid of the points it owns…
5. …and jumps there
6. …Repeat until terminated!



Source: A. Moore

K-means converges to a local minimum



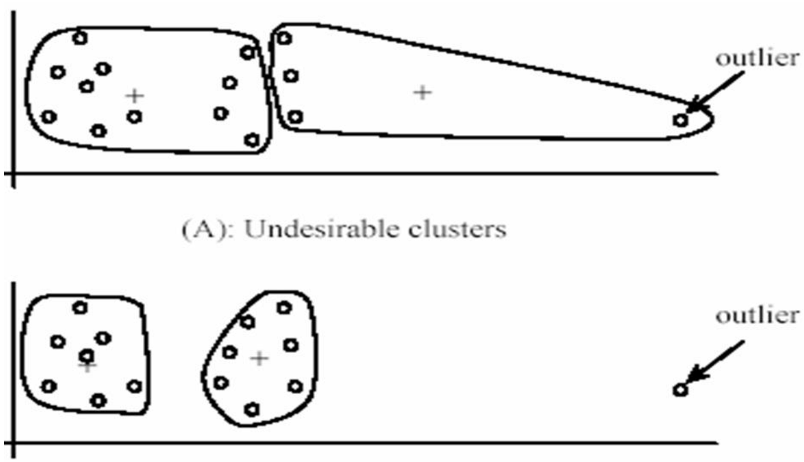
How can I try to fix this problem?

Adapted from James Hays

K-means: pros and cons

Pros

- Simple, fast to compute
- Converges to local minimum of within-cluster squared error

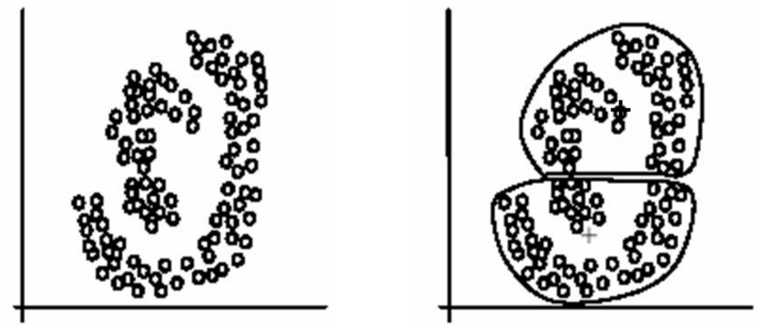


(A): Undesirable clusters

(B): Ideal clusters

Cons/issues

- Setting k?
  - One way: silhouette coefficient
- Sensitive to initial centers
  - Use heuristics or output of another method
  - Try different initializations
- Sensitive to outliers
- Detects spherical clusters



(A): Two natural clusters (B): *k*-means clusters

ALTERNATIVES?

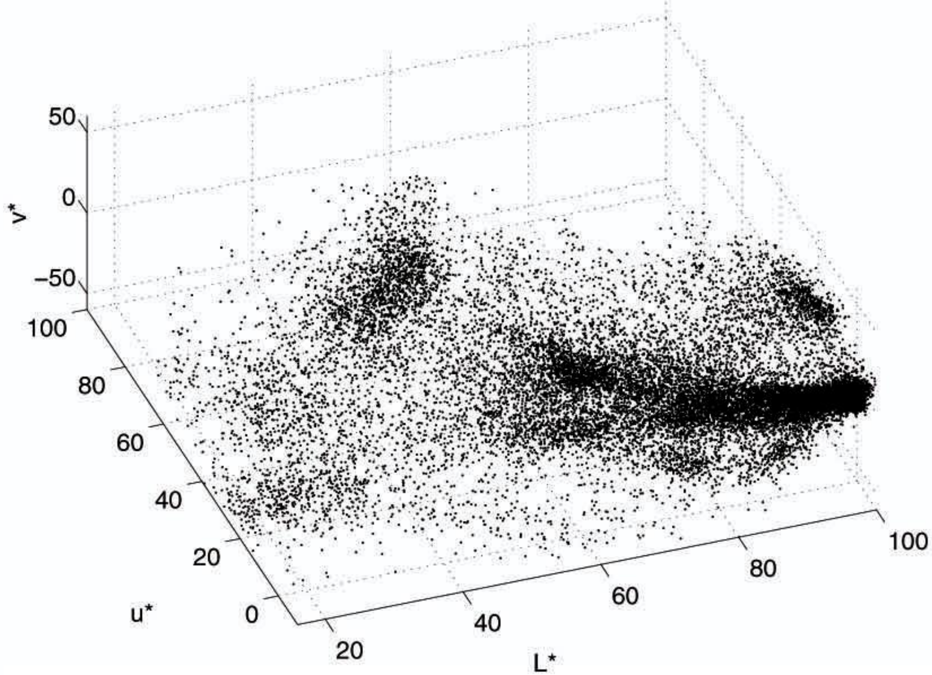
Adapted from K. Grauman

Mean shift algorithm

- The mean shift algorithm seeks *modes* or local maxima of density in the feature space

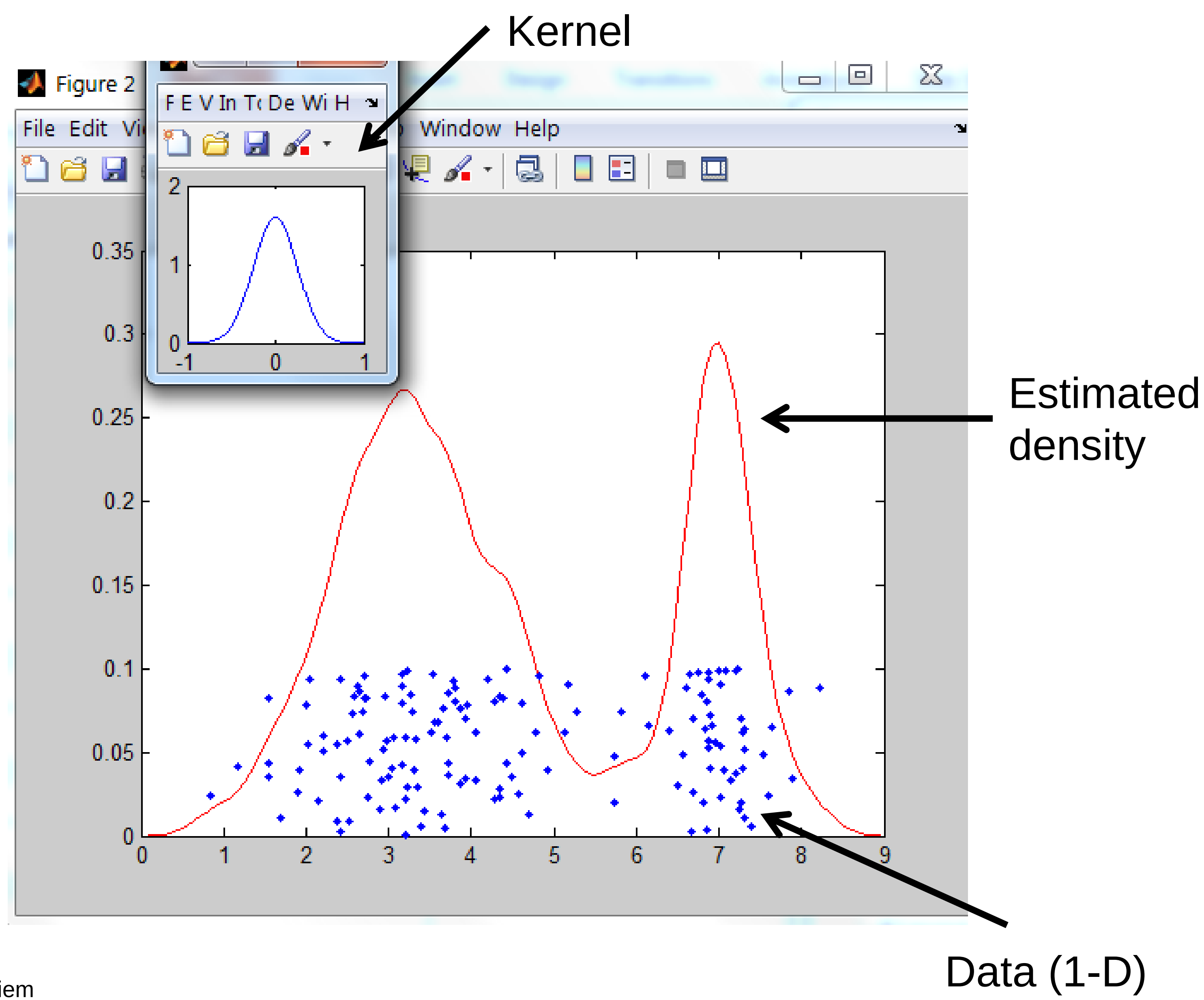
image

Feature space (L*u*v* color values)



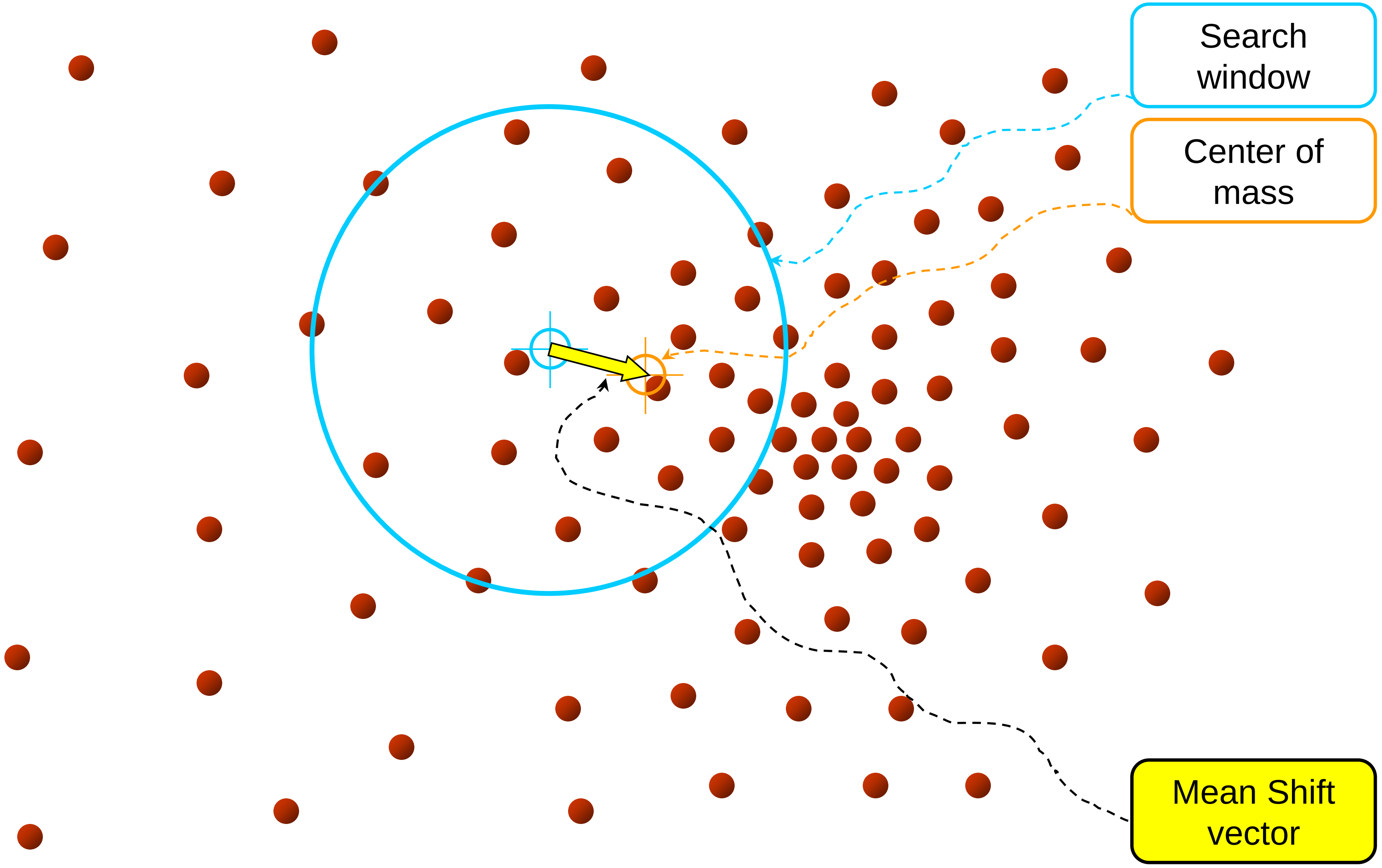
Source: K. Grauman

Kernel density estimation



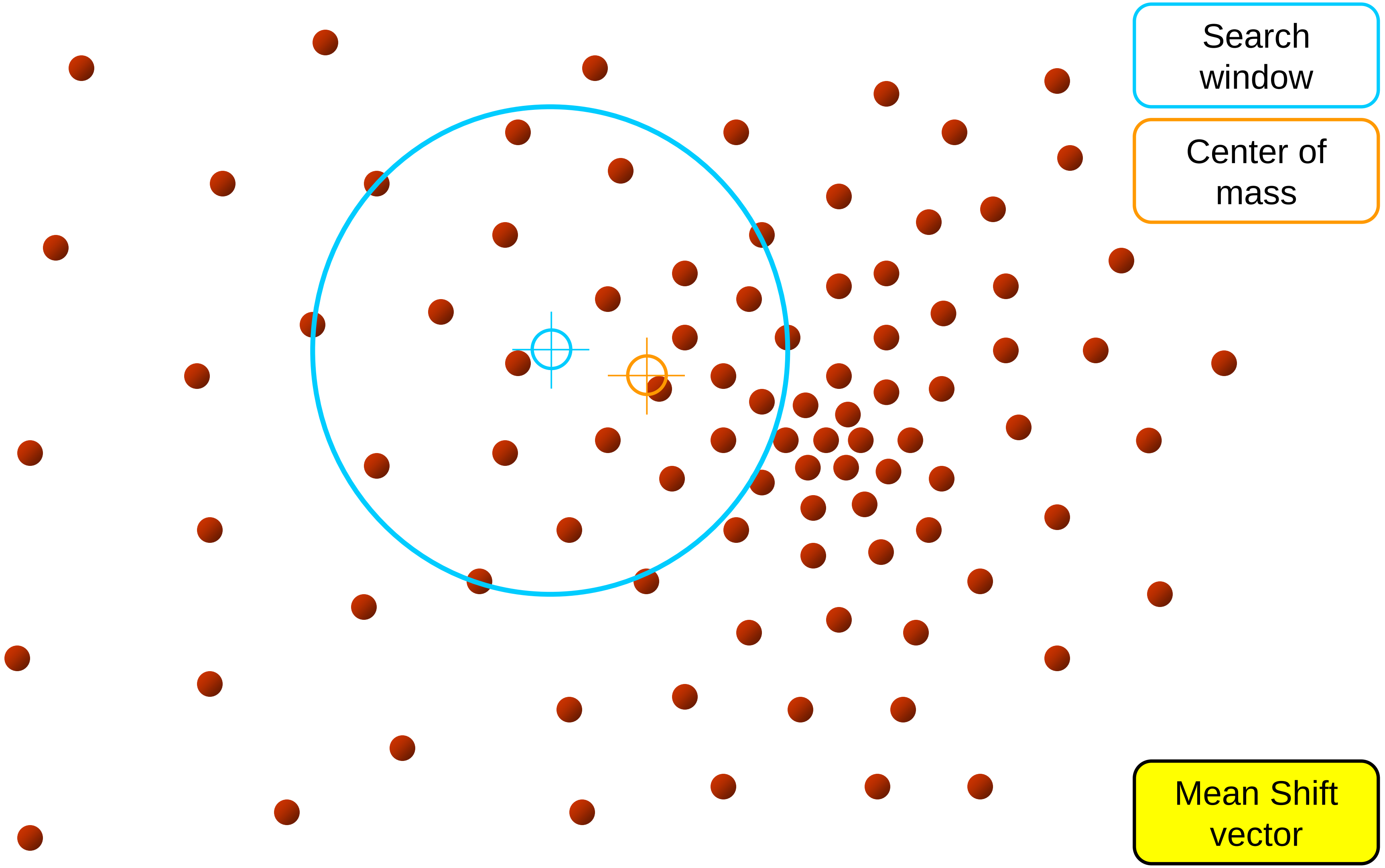
Source: D. Hoiem

Mean shift



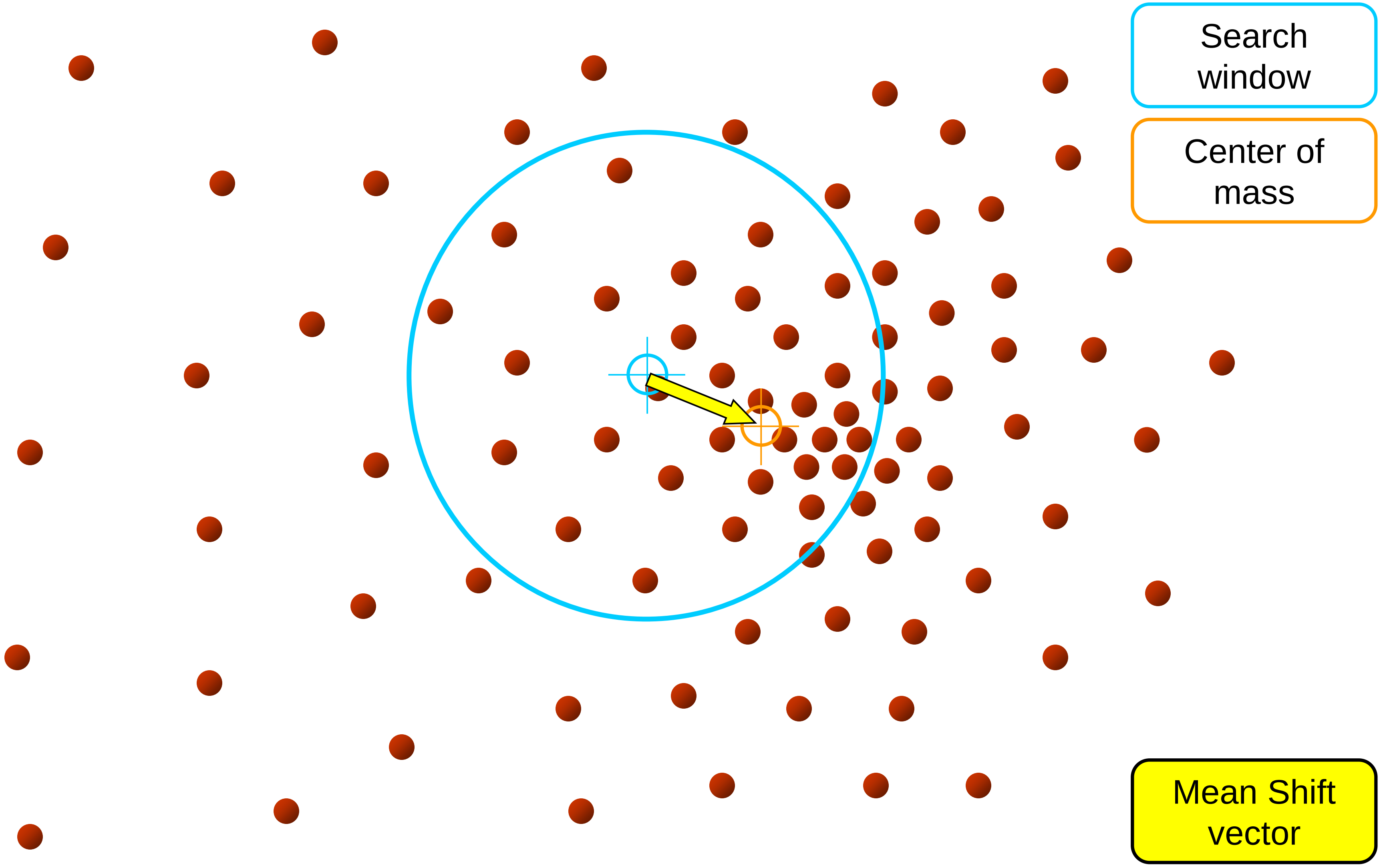
Slide by Y. Ukrainitz & B. Sarel

Mean shift



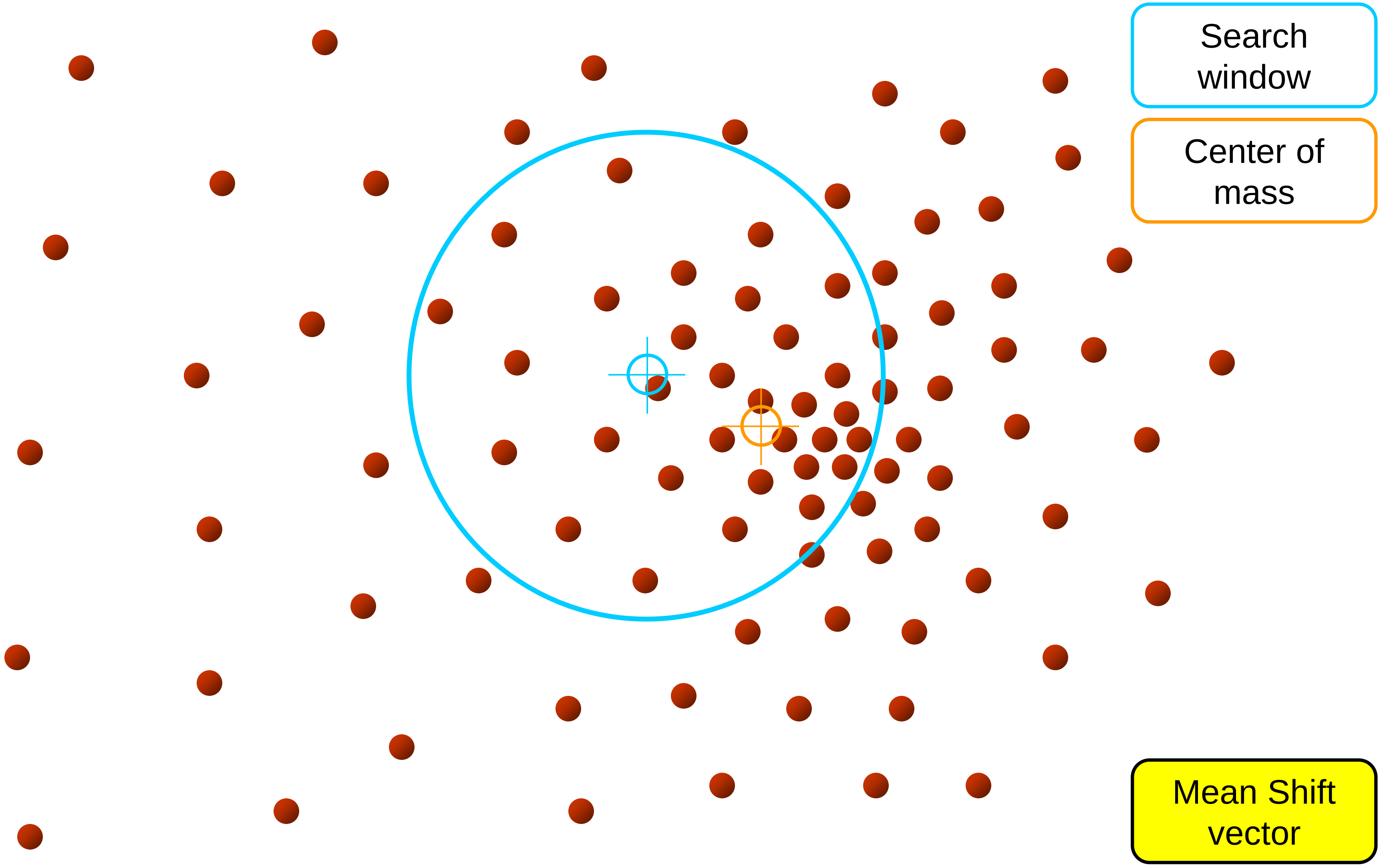
Slide by Y. Ukrainitz & B. Sarel

Mean shift



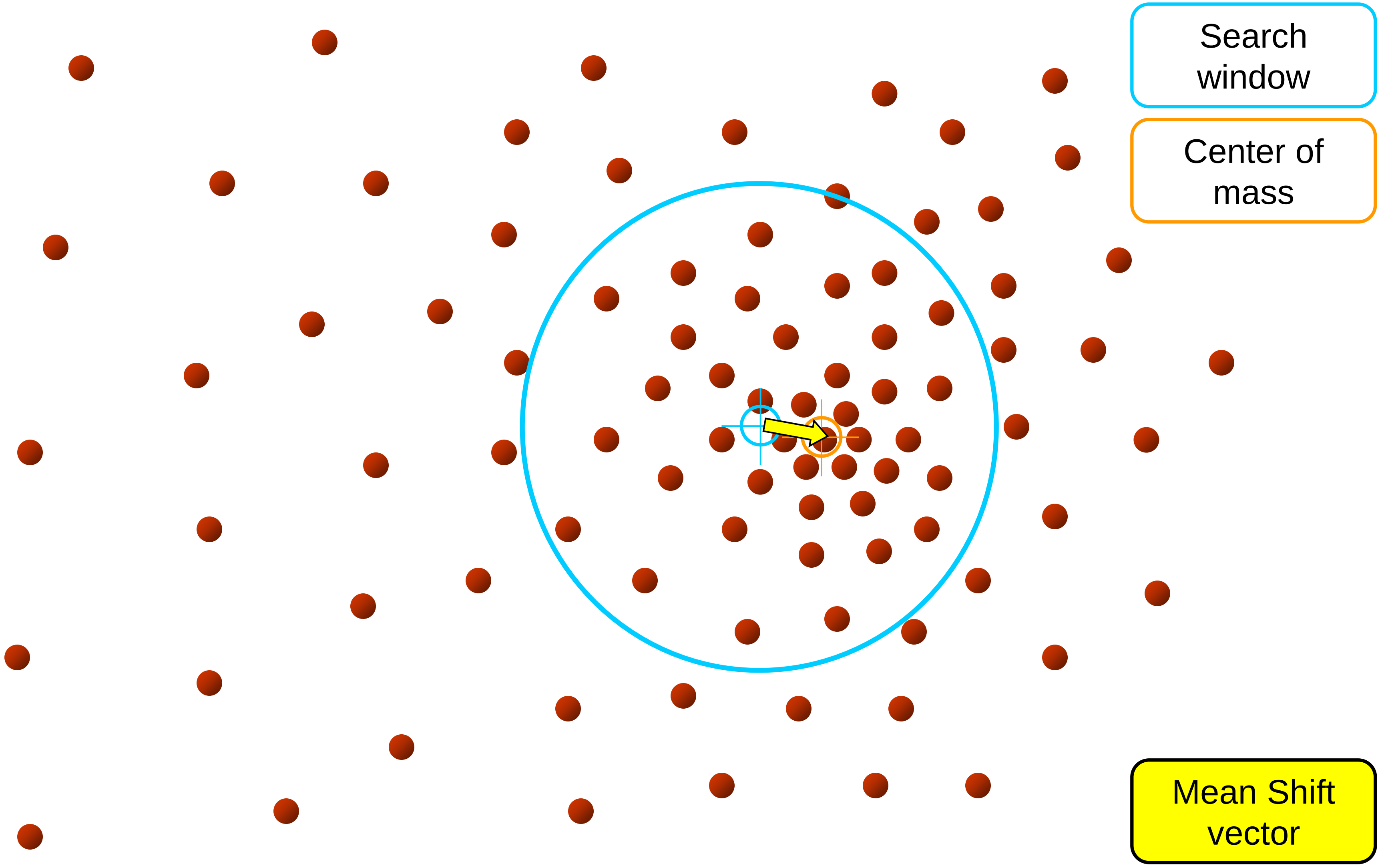
Slide by Y. Ukrainitz & B. Sarel

Mean shift



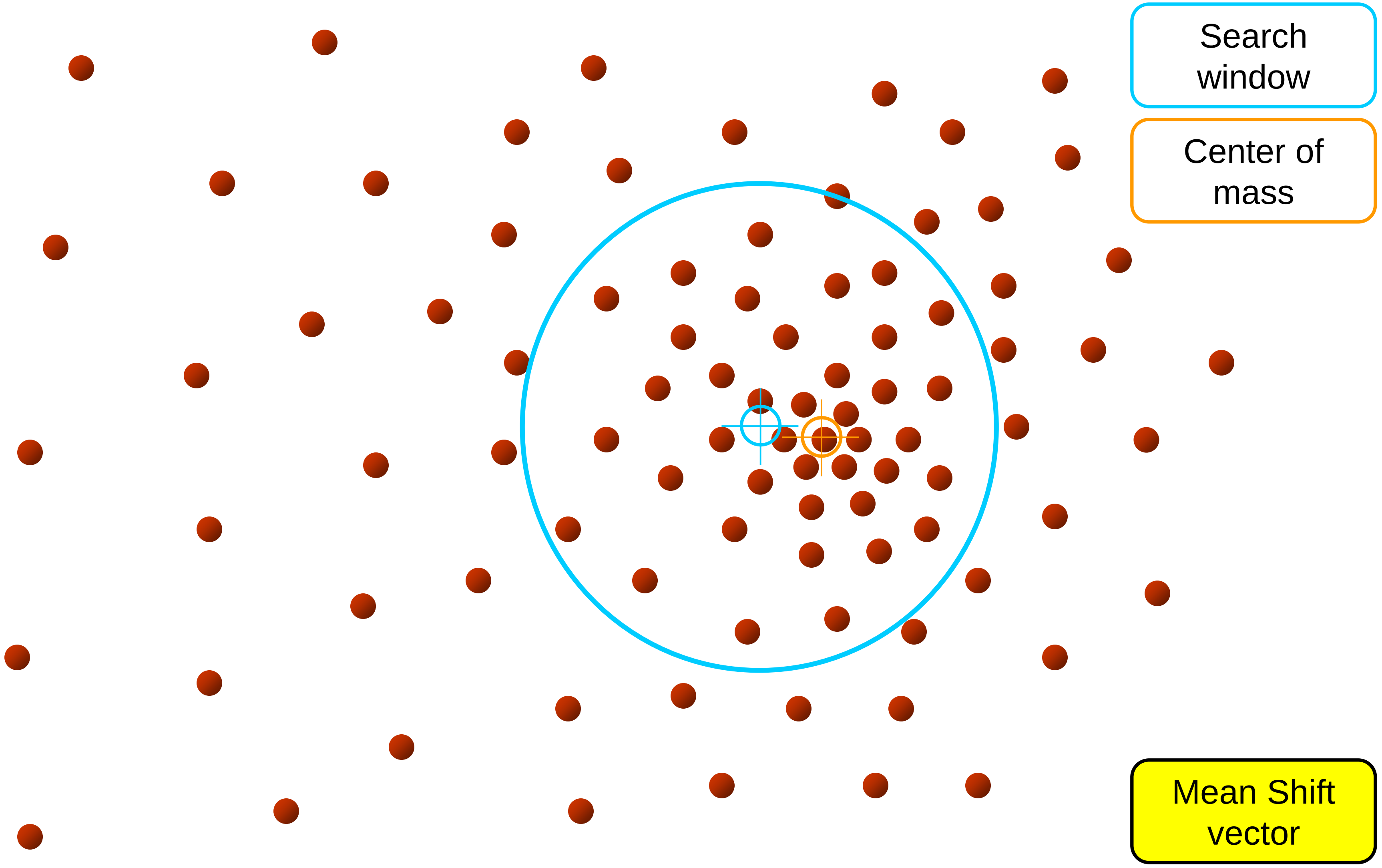
Slide by Y. Ukrainitz & B. Sarel

Mean shift



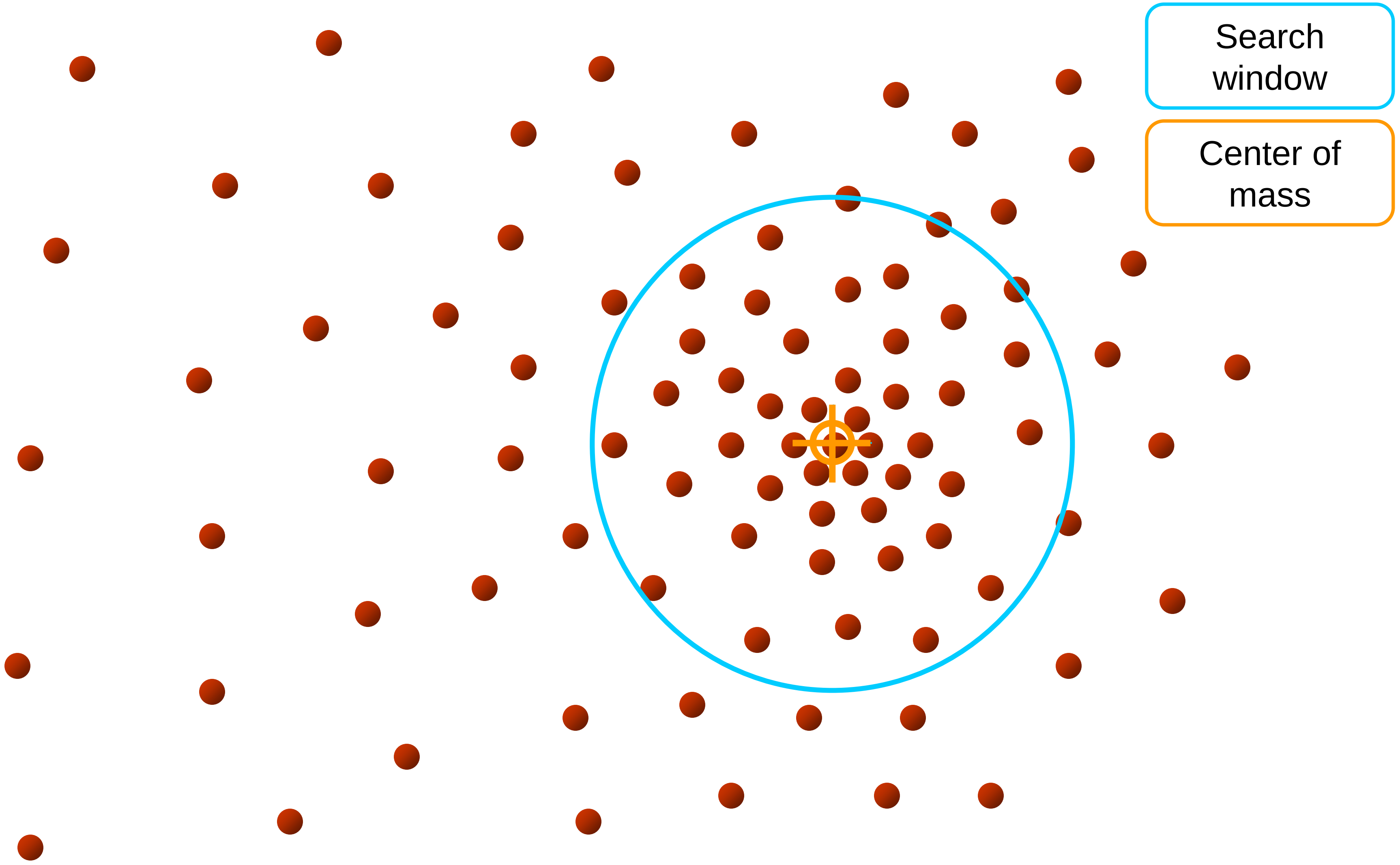
Slide by Y. Ukrainitz & B. Sarel

Mean shift



Slide by Y. Ukrainitz & B. Sarel

Mean shift



Slide by Y. Ukrainitz & B. Sarel

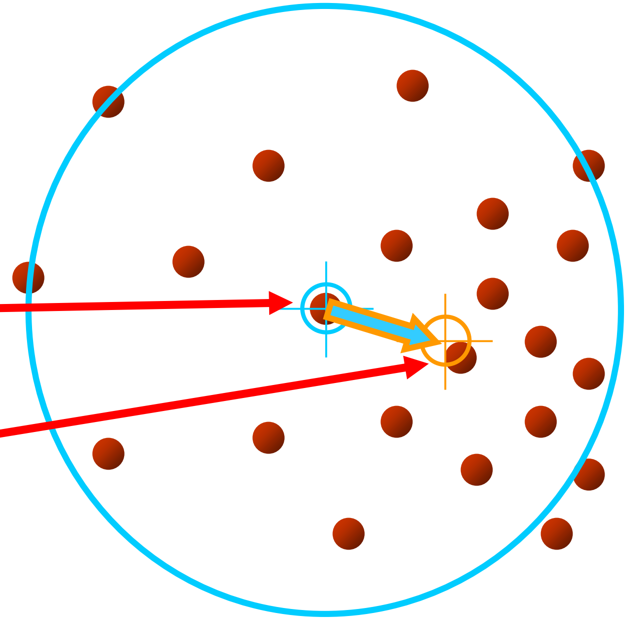
Computing the mean shift

<u>Simple Mean Shift procedure</u>:

- Compute mean shift vector

- Translate the Kernel window by *m(x)*

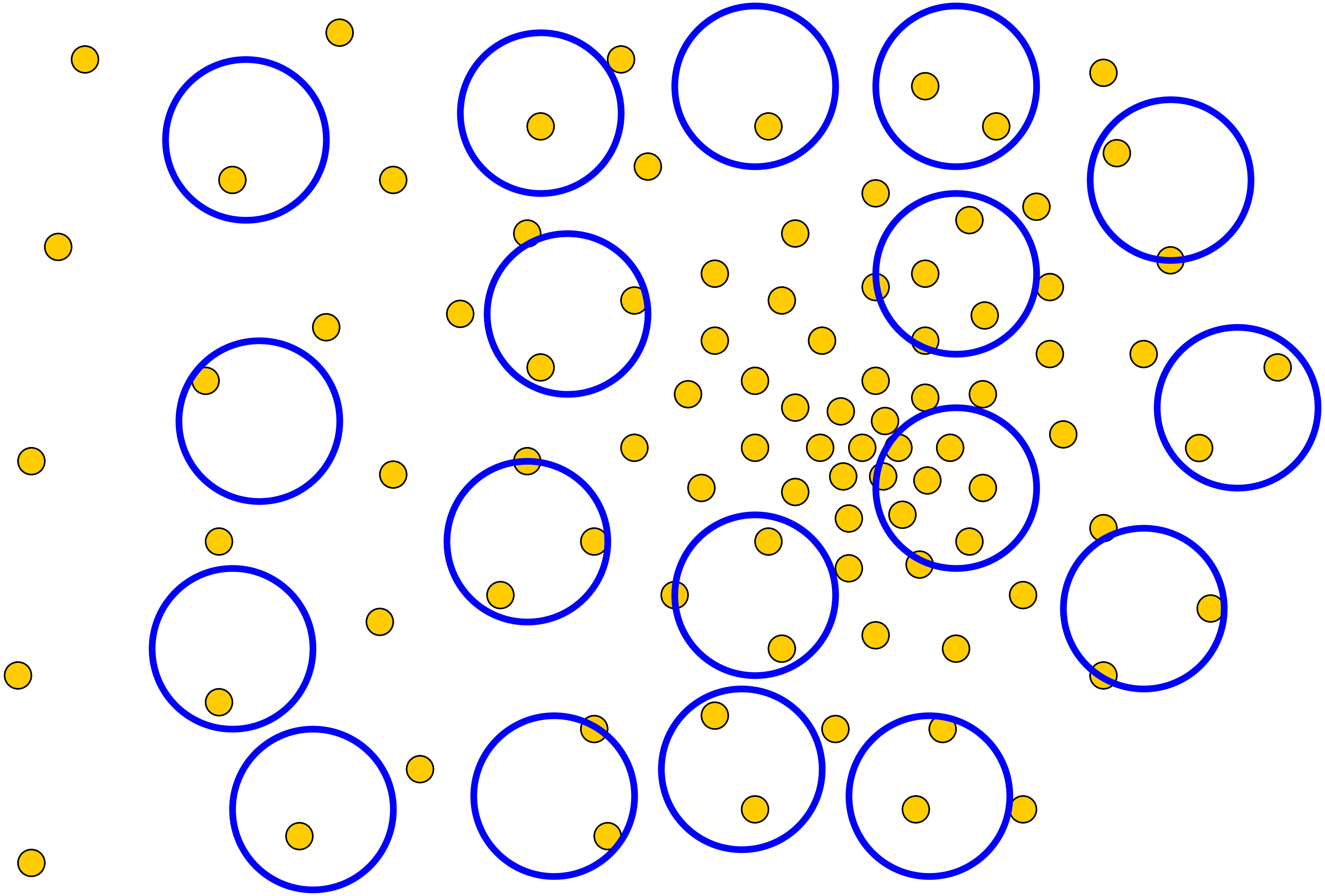
$$m(x) = \boxed{\frac{\sum_{i=1}^{n} x_i K(x_i - x)}{\sum_{i=1}^{n} K(x_i - x)}} - \boxed{x}$$



$$K(x_i - x) = e^{-\frac{\left\|x_i - x\right\|^2}{\sigma}}$$

Adapted from Y. Ukrainitz & B. Sarel

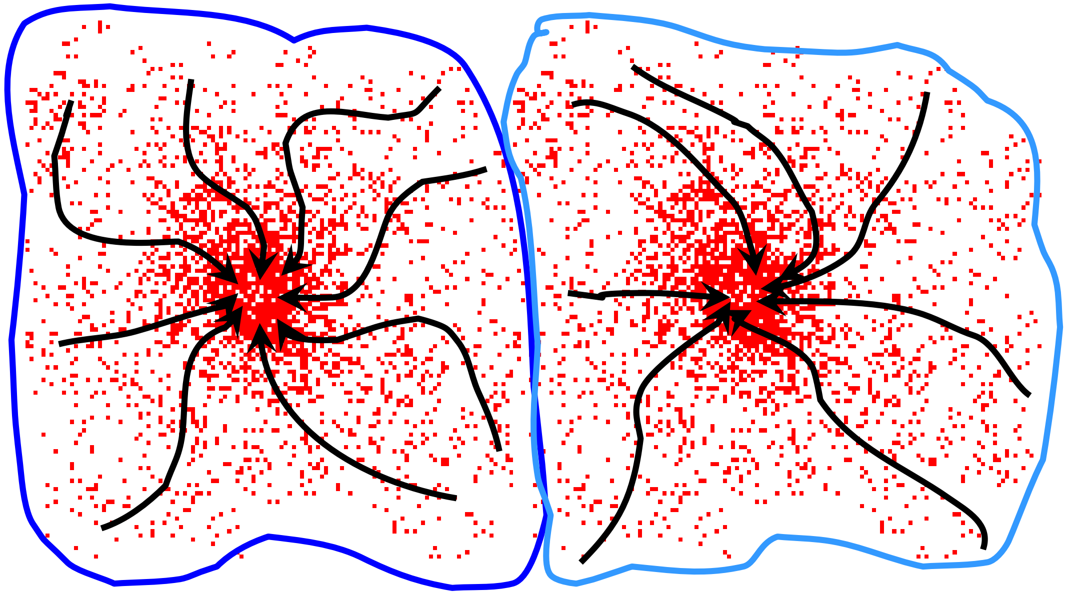
Points in same cluster converge



Source: D. Hoiem

Mean shift clustering

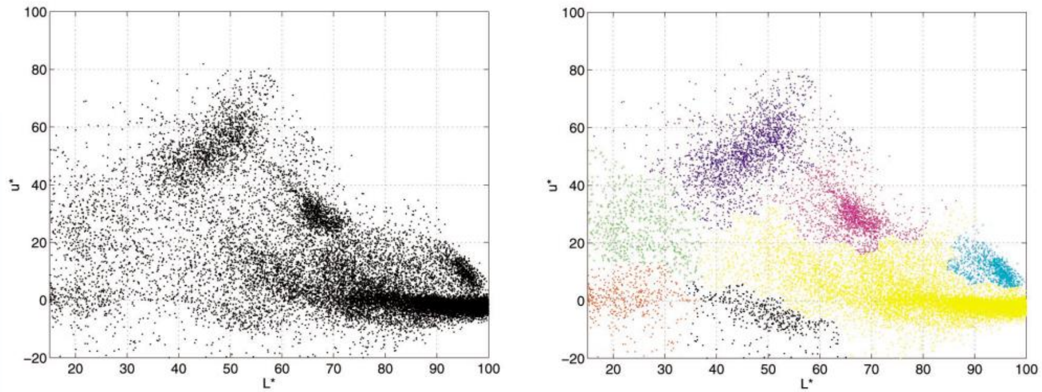
- Cluster: all data points in the attraction basin of a mode
- Attraction basin: the region for which all trajectories lead to the same mode



Slide by Y. Ukrainitz & B. Sarel

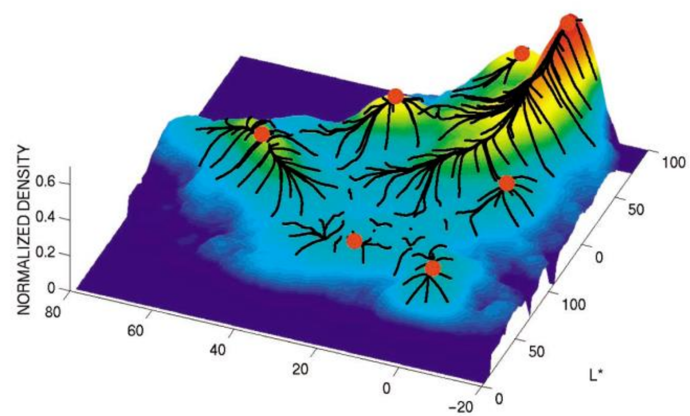
Mean shift clustering/segmentation

- Compute features for each point (color, texture, etc)
- Initialize windows at individual feature points
- Perform mean shift for each window until convergence
- Merge windows that end up near the same “peak” or mode



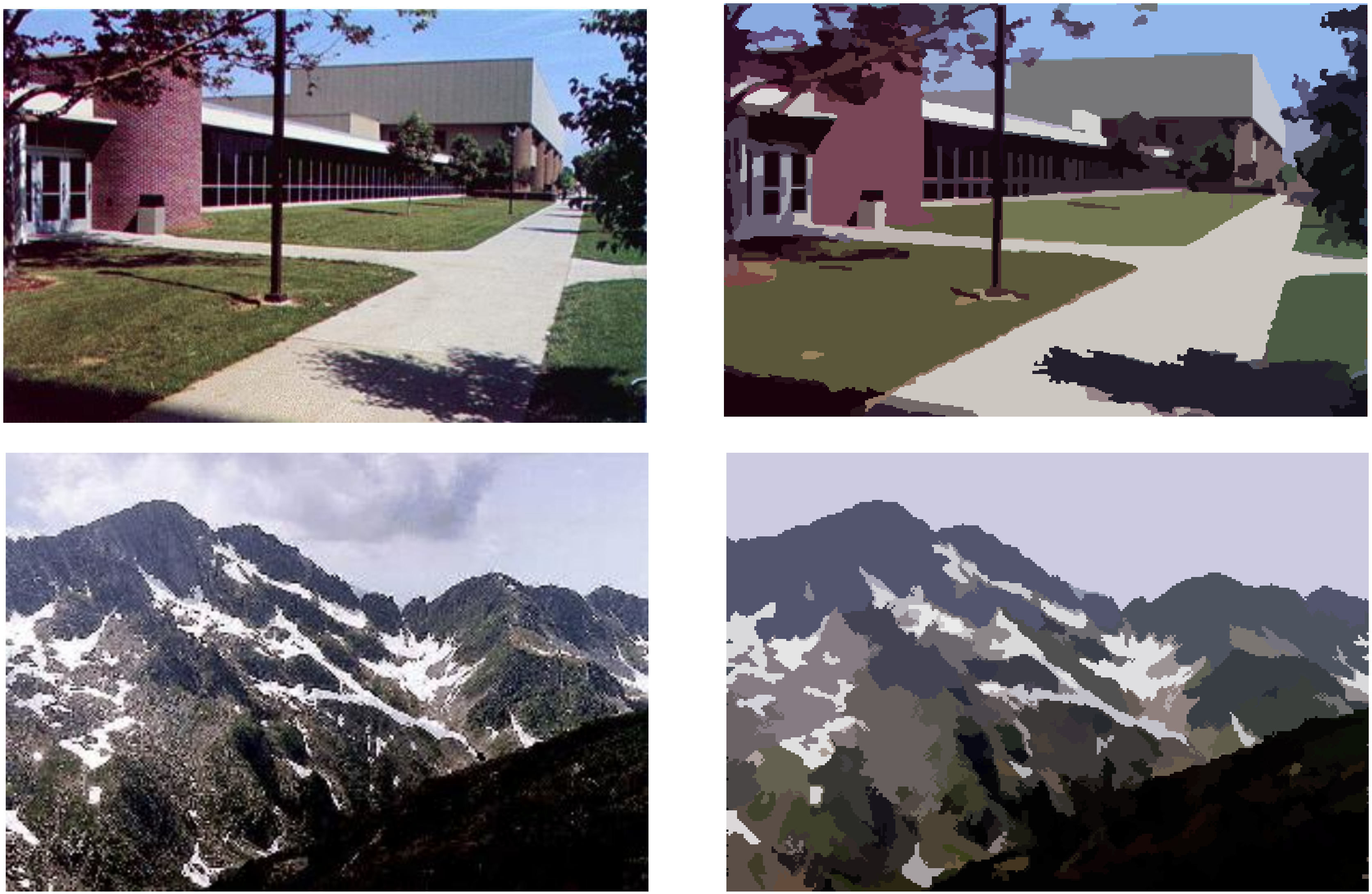
(a)

(b)



Source: D. Hoiem

Mean shift segmentation results



http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html

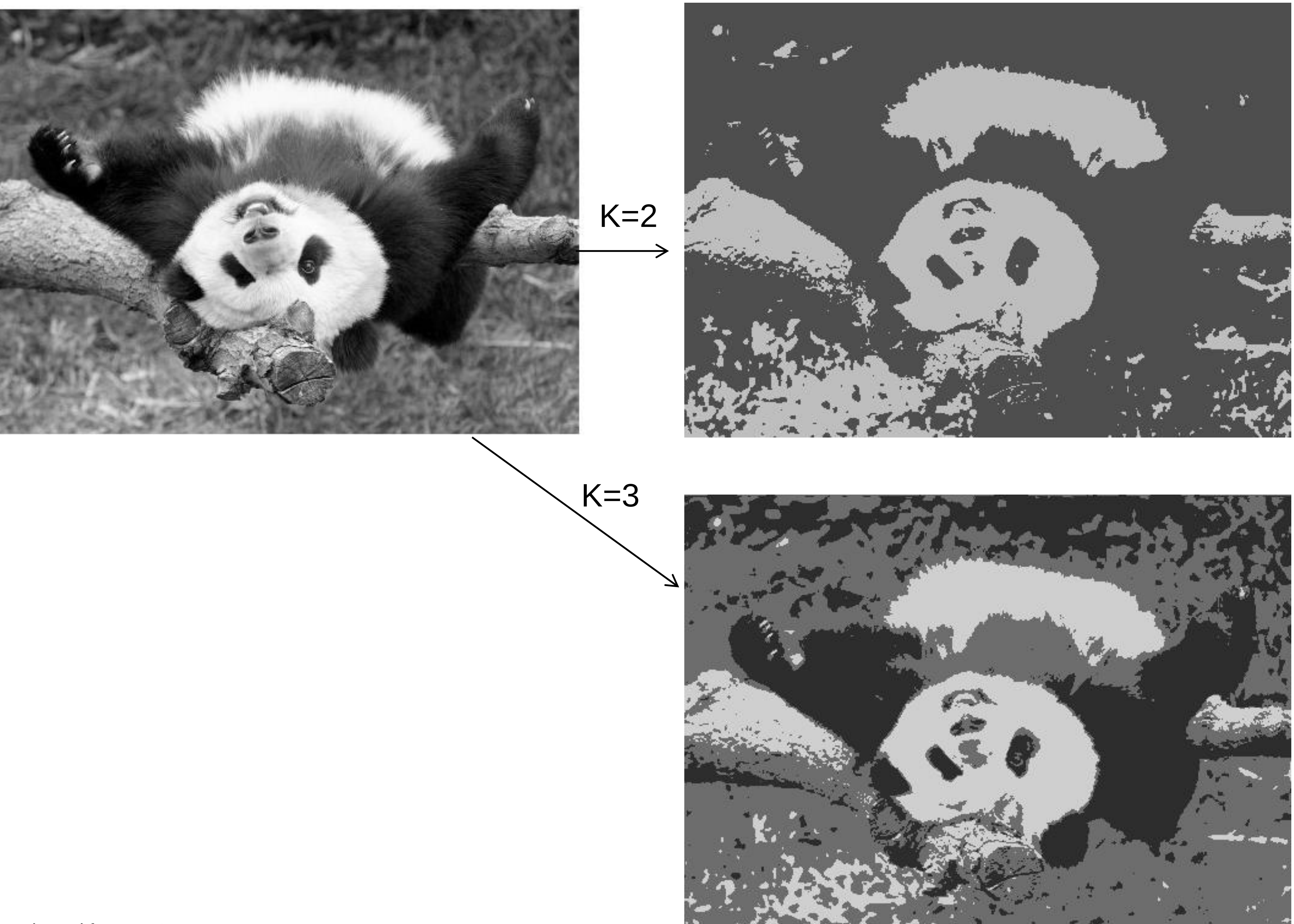
Segmentation as clustering

Depending on what we choose as the *feature space*, we can group pixels in different ways.

Grouping pixels based on **intensity** similarity

Feature space: intensity value (1-d)

Source: K. Grauman

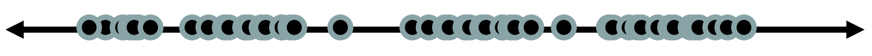


Adapted from K. Grauman

Segmentation as clustering

Depending on what we choose as the *feature space*, we can group pixels in different ways.

Grouping pixels based on **intensity** similarity



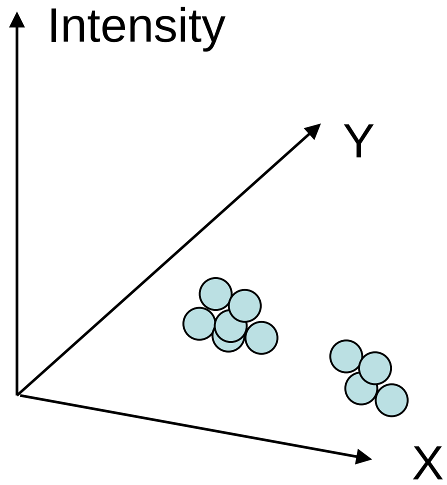
Clusters based on intensity similarity don’t have to be spatially coherent.

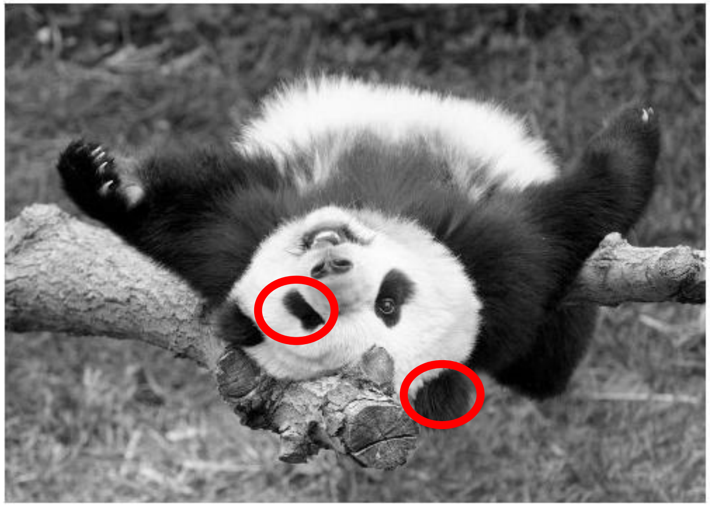
Source: K. Grauman

Segmentation as clustering

Depending on what we choose as the *feature space*, we can group pixels in different ways.

Grouping pixels based on **intensity+position** similarity





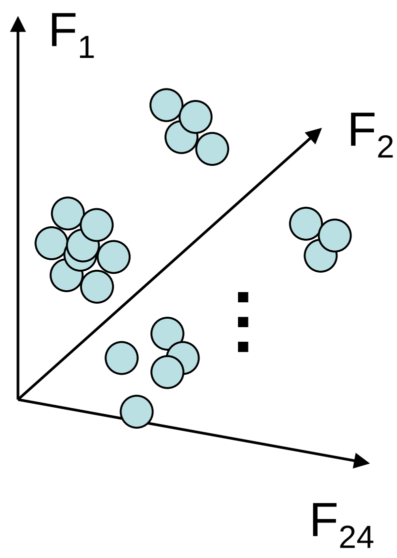
Both regions are black, but if we also include **position (x,y),** then we could group the two into distinct segments; way to encode both similarity & proximity.

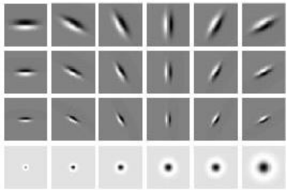
Source: K. Grauman

Segmentation as clustering

Depending on what we choose as the *feature space*, we can group pixels in different ways.

Grouping pixels based on **texture** similarity





Filter bank of 24 filters

Feature space: filter bank responses (e.g., 24-d)

Source: K. Grauman

Summary

- Edges: threshold gradient magnitude
- Lines: edge points vote for parameters of line, circle, etc. (works for general objects)
- Segments: use clustering (e.g. K-means) to group pixels by intensity, texture, etc.

Plan for this lecture

- Group pixels into:
  - Edges: Extract gradients and threshold
  - Lines: Find which edge points are collinear or belong to another shape
  - Segments: Find which pixels form a consistent region, e.g. via clustering
- Transform pixels:
  - Find relationships between multiple views of the same world point
  - *Both parts rely on finding geometric relationships between pixels*

Why multiple views?

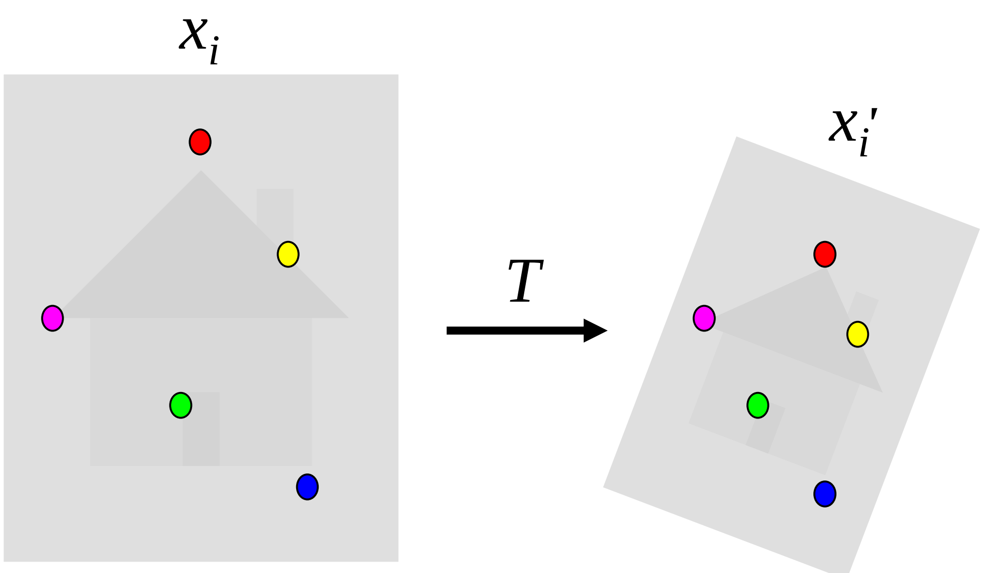
- Structure and depth are inherently ambiguous from single views.

- Multiple views help us to perceive 3d shape and depth.

Kristen Grauman, images from Svetlana Lazebnik

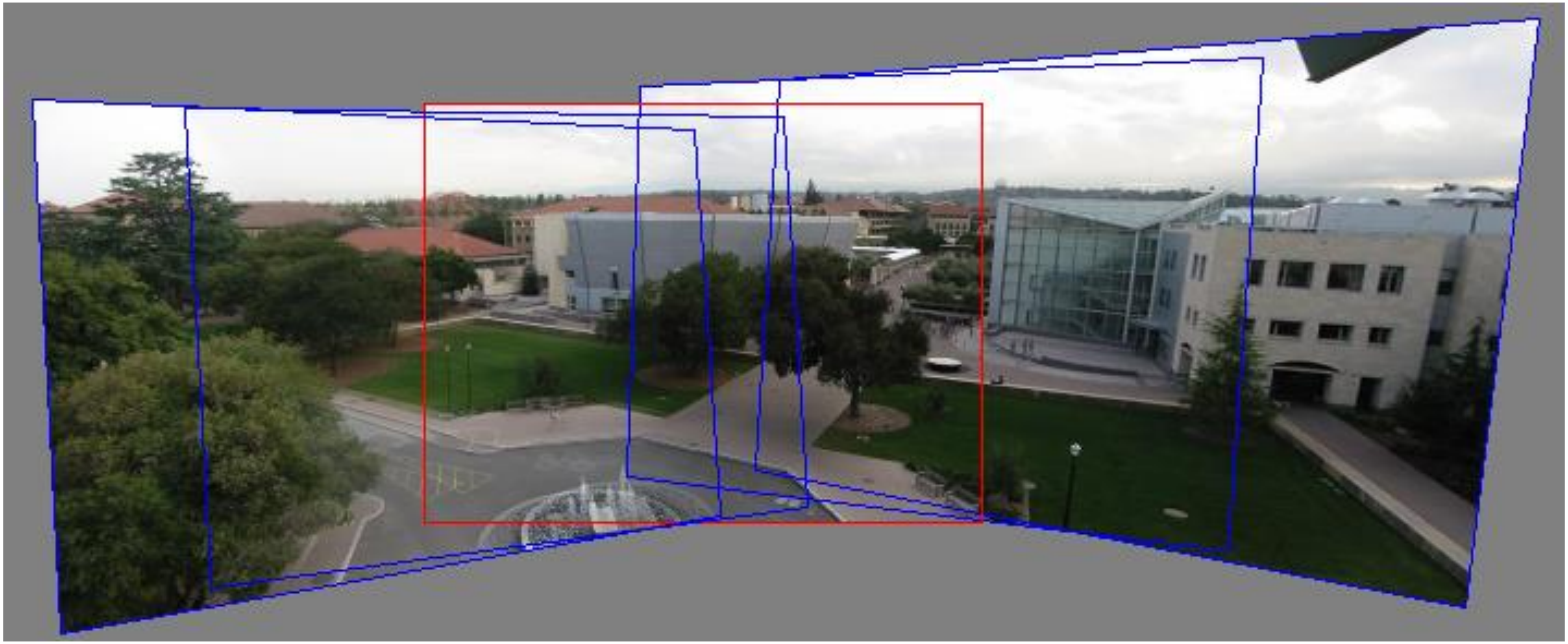
Alignment problem

- We previously discussed how to match features across images, of the same or different objects
- Now let's focus on the case of "two images of the same object"(e.g. x_i and x_i')
- What transformation relates x_i and x_i'?
- In *alignment*, we will **fit the parameters of some transformation** according to a set of matching feature pairs ("correspondences").



Adapted from Kristen Grauman and Derek Hoiem

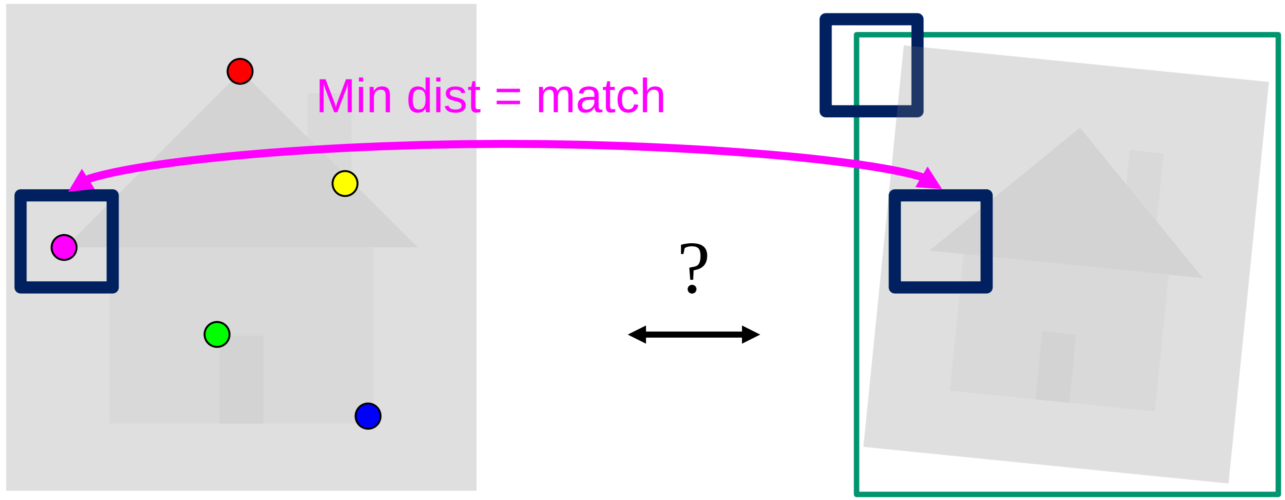
Motivation: Image mosaics



Kristen Grauman

Image from http://graphics.cs.cmu.edu/courses/15-463/2010_fall/

First, what are the correspondences?



- Compare content in **local** patches, find best matches.
  - Scan x_i' with template formed from a point in x_i, and compute e.g. Euclidean distance between SIFT features of the patches

Adapted from Kristen Grauman

Second, what are the transformations?

Examples of transformations:

translate

rotate

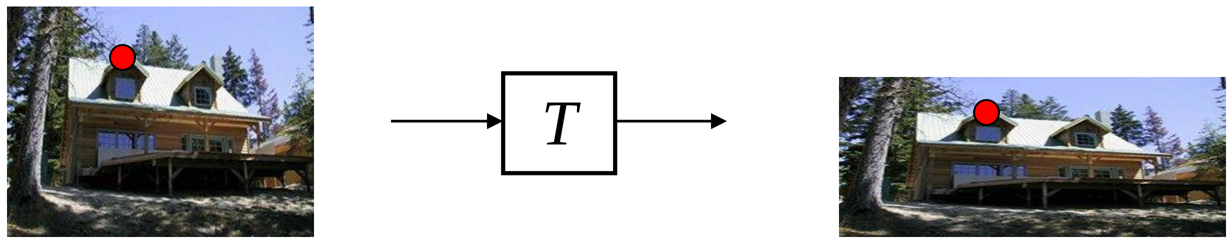
change aspect ratio

squish/shear

change perspective

Adapted from Alyosha Efros

Parametric (global) warping



$\mathbf{p} = (x,y)$ $\mathbf{p'} = (x',y')$

Transformation T is a coordinate-changing machine:

$$p' = T(p)$$

What does it mean that *T* is **global**?

- It is the same for any point p
- It can be described by just a few numbers (parameters)

Let's represent *T* as a matrix:

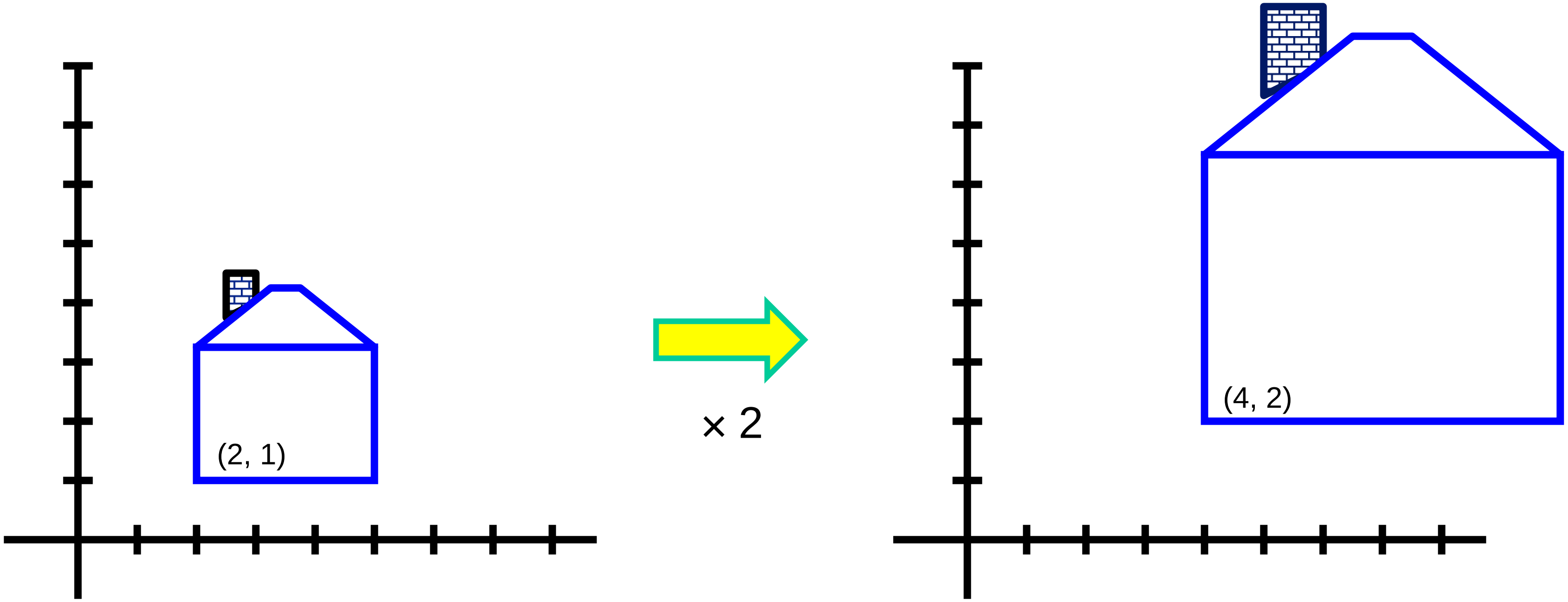
$$p' = \mathbf{M}p$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scaling

Scaling a coordinate means multiplying each of its components by a scalar

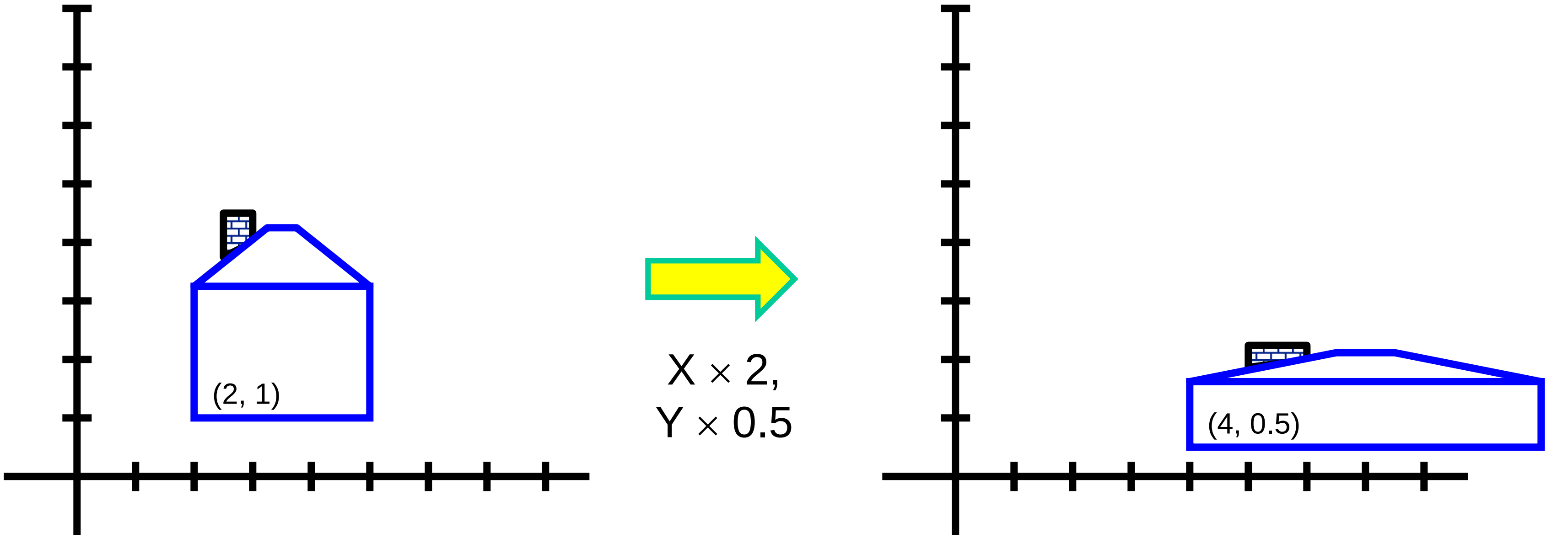
Uniform scaling means this scalar is the same for all components:



Adapted from Alyosha Efros

Scaling

Non-uniform scaling: different scalars per component



Adapted from Alyosha Efros

Scaling

Scaling operation:

$$x' = ax$$

$$y' = by$$

Or, in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}_{\textit{scaling matrix S}} \begin{bmatrix} x \\ y \end{bmatrix}$$

Adapted from Alyosha Efros

2D Linear transformations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Only linear 2D transformations can be represented with a 2x2 matrix.

Linear transformations are combinations of …

- Scale,
- Rotation,
- Shear, and
- Mirror

What transforms can we write w/ 2x2 matrix?

2D Scaling?

$$x' = s_x * x$$

$$y' = s_y * y$$

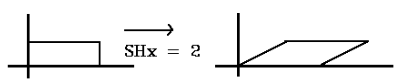
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Rotate around (0,0)? (see hidden slide)

$$x' = \cos\Theta * x - \sin\Theta * y$$
$$y' = \sin\Theta * x + \cos\Theta * y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Shear?



$$x' = x + sh_x * y$$

$$y' = sh_y * x + y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Modified from Alyosha Efros

Fig. from https://www.siggraph.org/education/materials/HyperGraph/modeling/mod_tran/2dshear.htm

What transforms can we write w/ 2x2 matrix?

2D Mirror about Y axis?

$$x' = -x$$
$$y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Mirror over (0,0)?

$$x' = -x$$
$$y' = -y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Translation?

$$\boldsymbol{x}' = \boldsymbol{x} + \boldsymbol{t}_x$$
$$\boldsymbol{y}' = \boldsymbol{y} + \boldsymbol{t}_y$$

CAN'T DO!

Homogeneous coordinates

To convert to homogeneous coordinates:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image
coordinates

Converting *from* homogeneous coordinates

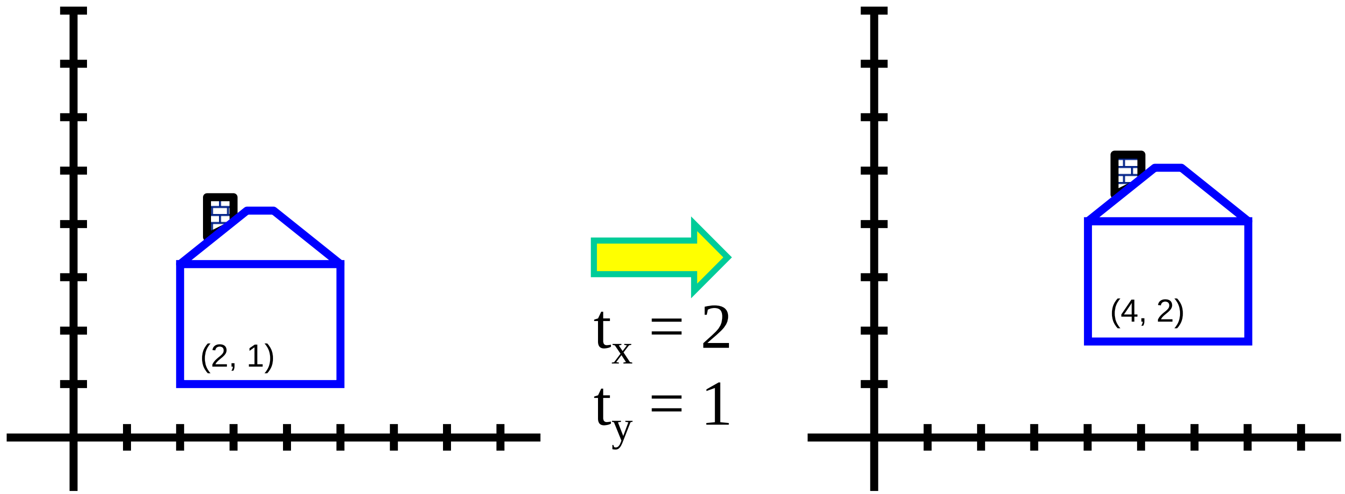
$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

Kristen Grauman

Translation

Homogeneous Coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$



Adapted from Alyosha Efros

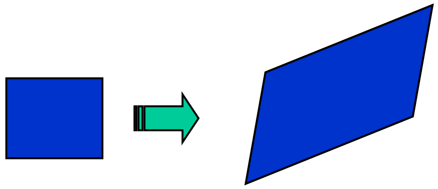
2D affine transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Affine transformations are combinations of …

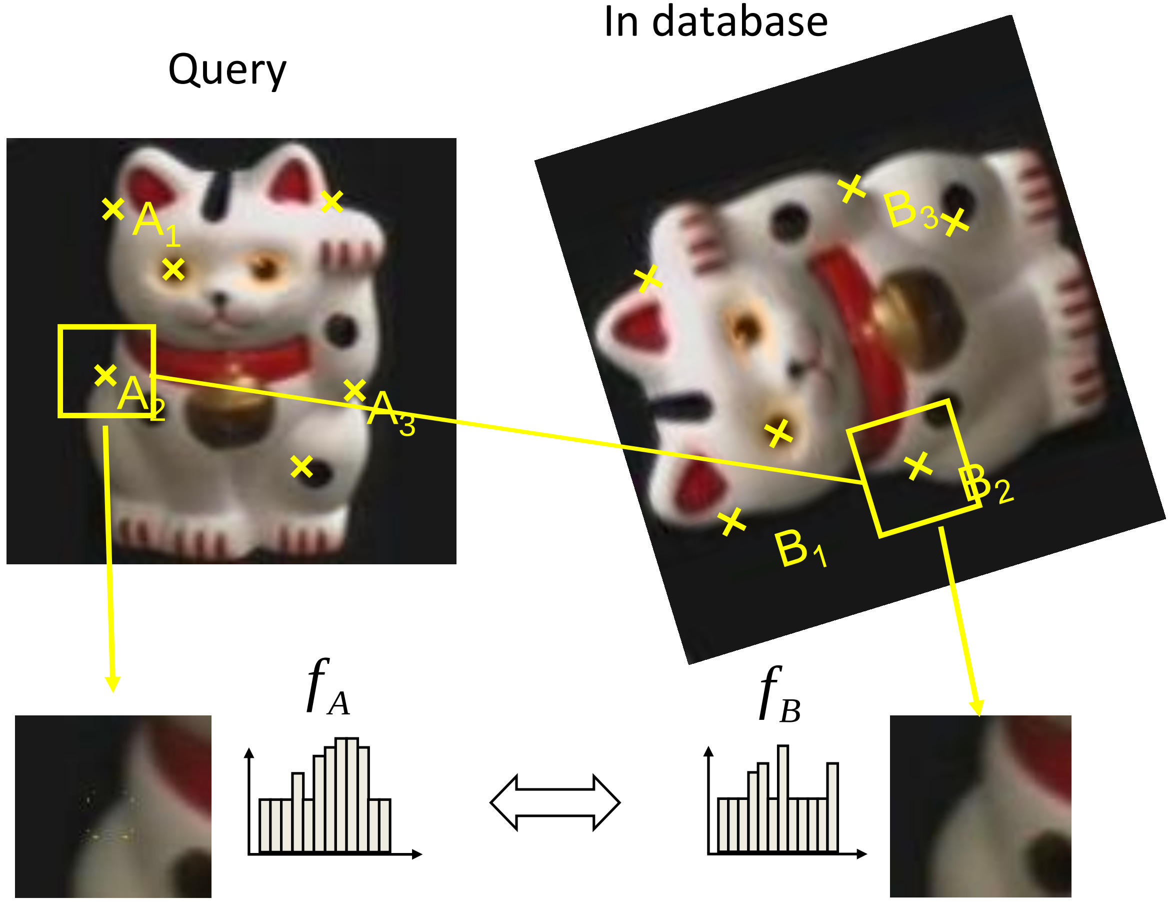
- Linear transformations, and
- Translations

Maps lines to lines, parallel lines remain parallel



Adapted from Alyosha Efros

Detour: Keypoint matching for search



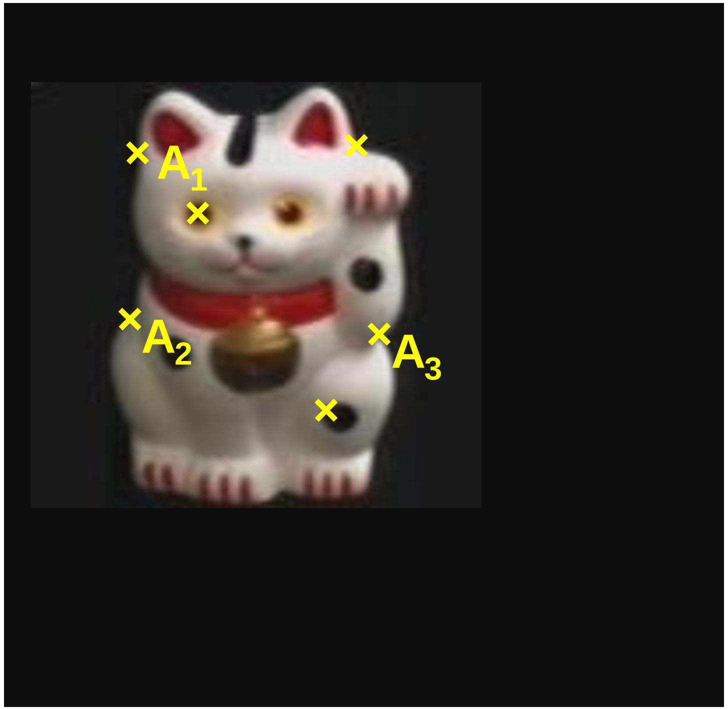
$$f_A \qquad \Longleftrightarrow \qquad f_B$$

$$d(f_A, f_B) < T$$

1. Find a set of distinctive key-points
2. Define a region around each keypoint (window)
3. Compute a local descriptor from the region
4. Match descriptors

Adapted from K. Grauman, B. Leibe

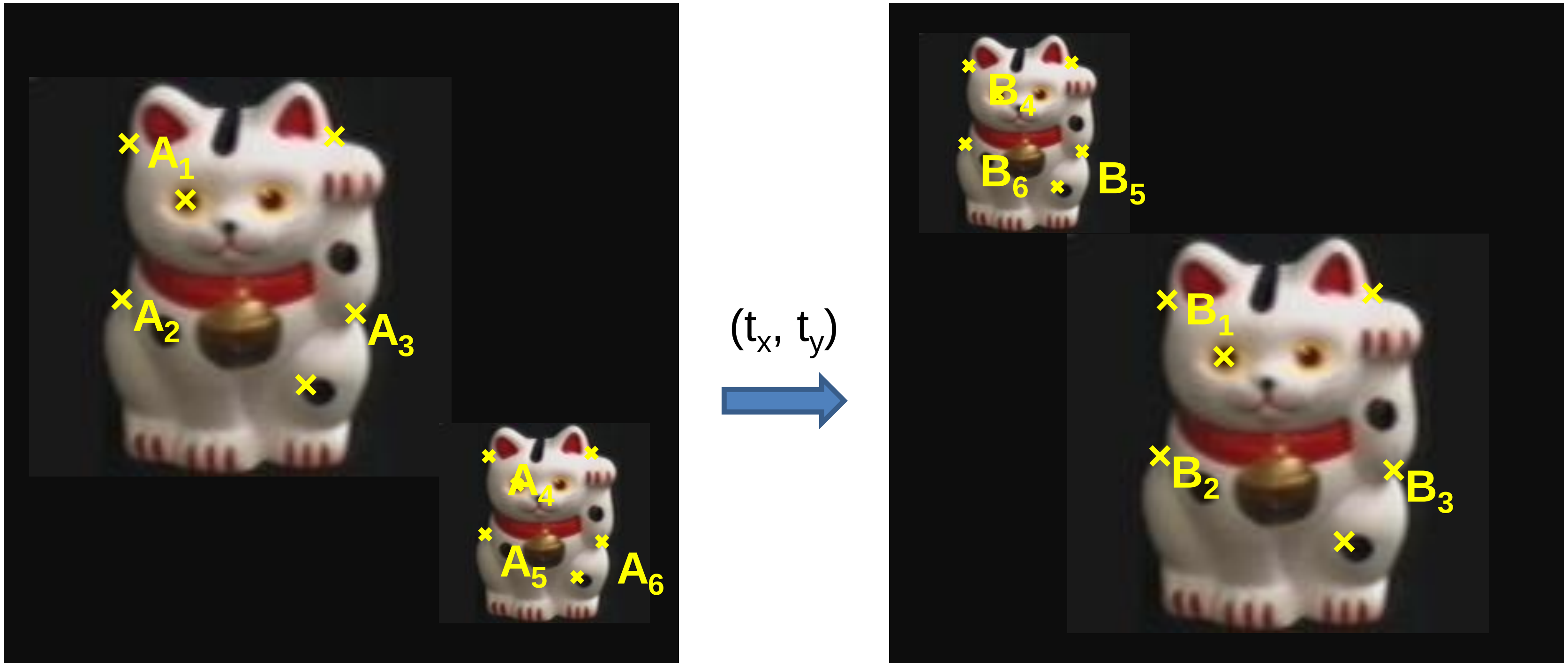
Detour: solving for translation with outliers



Given matched points in {A} and {B}, estimate the translation of the object

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Detour: solving for translation with outliers



Problem: outliers, multiple objects

Hough transform solution

1. Initialize a grid of parameter values
2. Each matched pair casts a vote for consistent values
3. Find the parameters with the most votes

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Adapted from Derek Hoiem

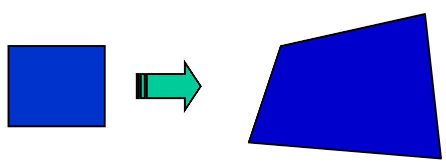
Projective transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Projective transformations:

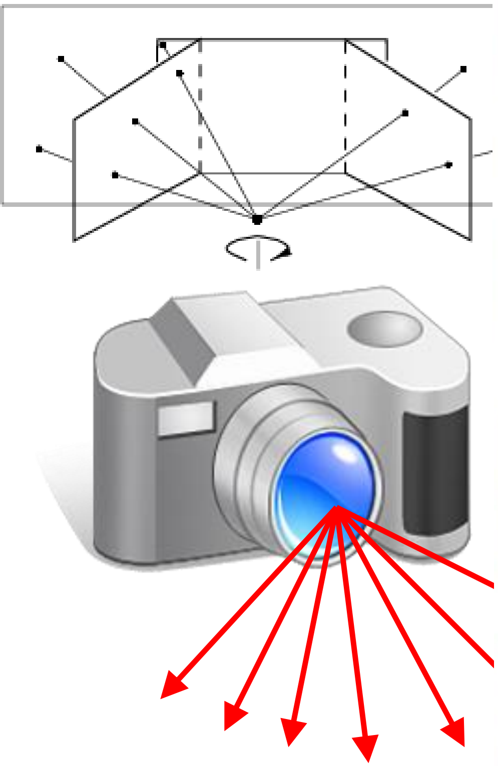
- Affine transformations, and
- Projective warps

Parallel lines do not necessarily remain parallel



Kristen Grauman

Image mosaics: Goals



. . .

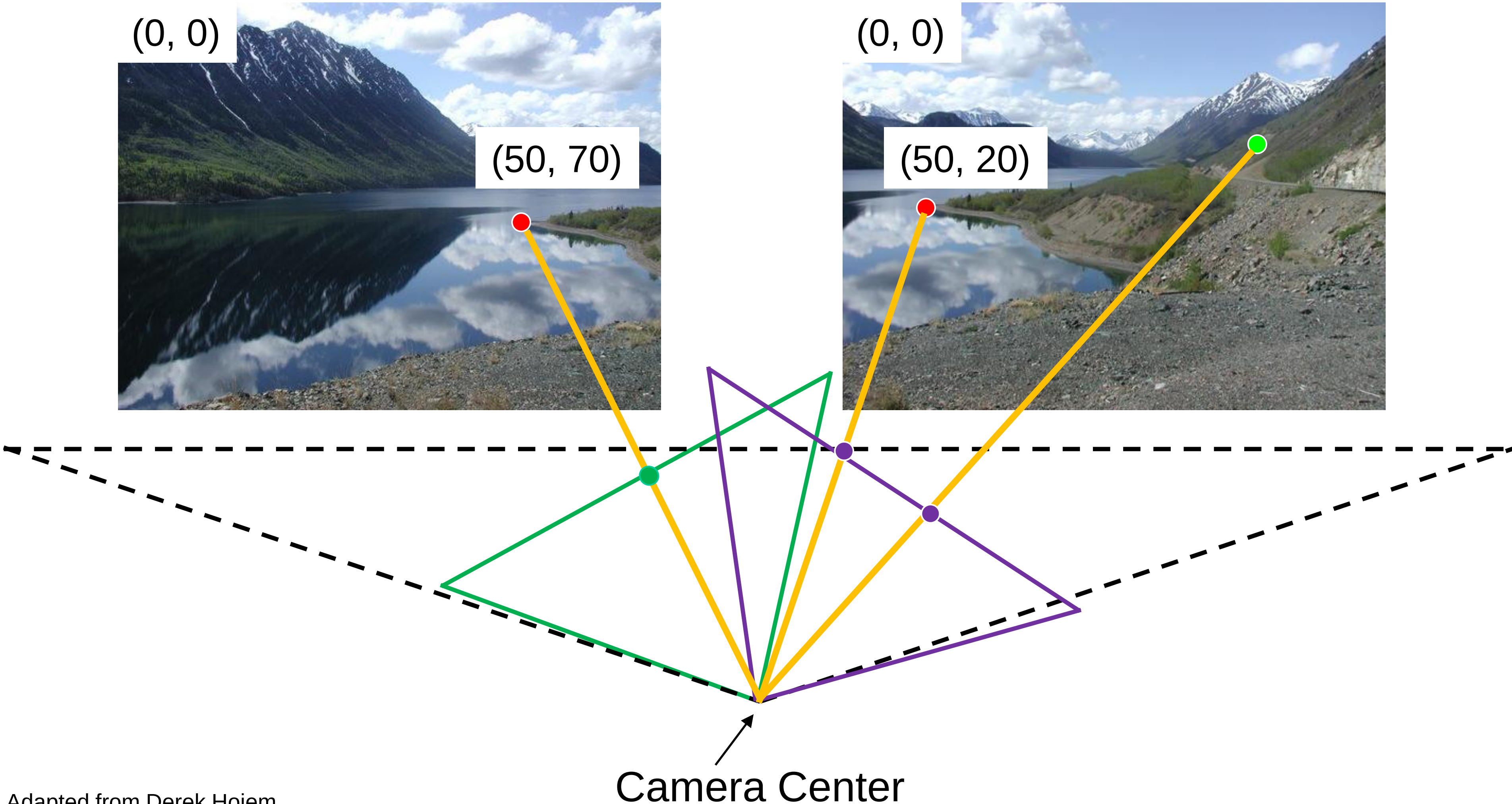
image from S. Seitz

Obtain a wider angle view by combining multiple images.

Kristen Grauman

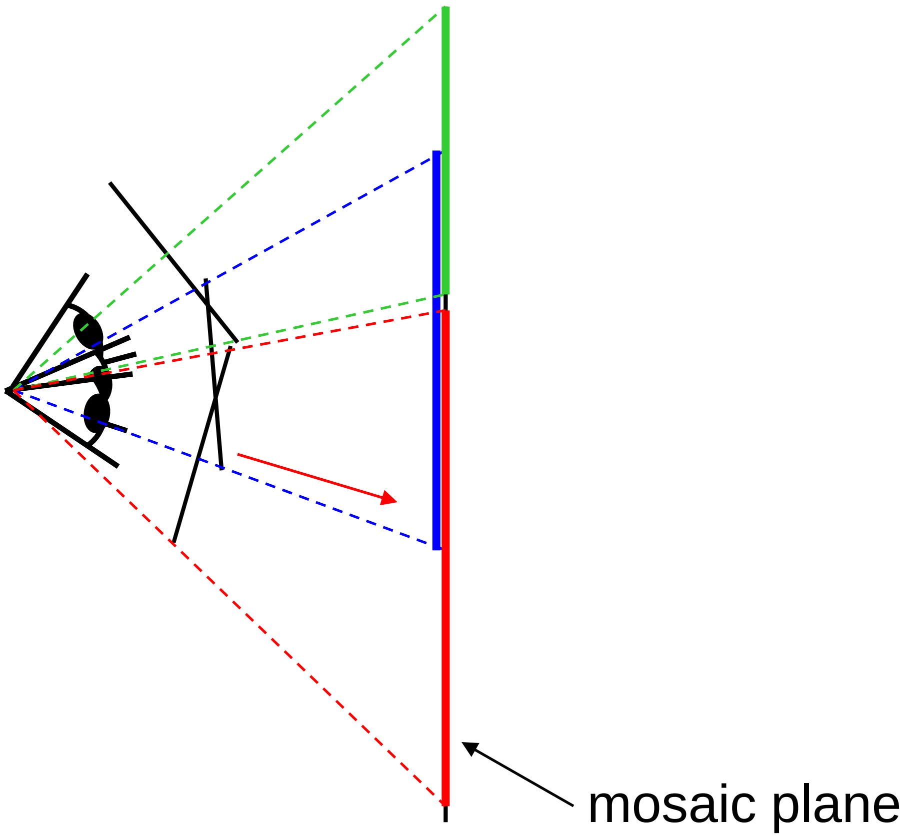
Image mosaics: Camera setup

Two images with camera rotation but no translation



Adapted from Derek Hoiem

Image mosaics: Many 2D views, one 3D object



The mosaic has a natural interpretation in 3D

- The images are reprojected onto a common plane
- The mosaic is formed on this plane
- Mosaic is a *synthetic wide-angle camera*

Steve Seitz

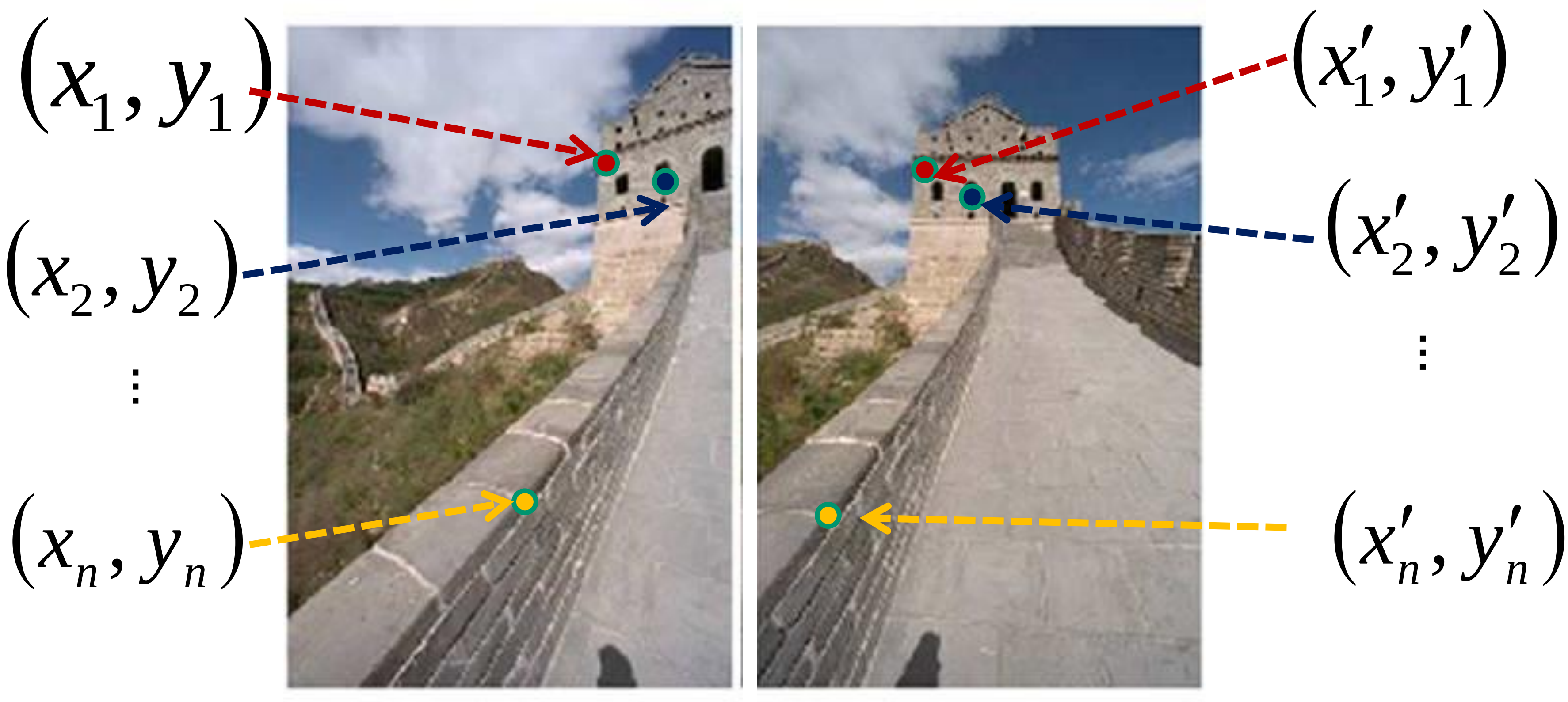
How to stitch together panorama (mosaic)?

Basic Procedure

- Take a sequence of images from the same position
  - Rotate the camera about its optical center
- **Compute the homography (transformation) between first and second image**
- **Transform the second image to overlap with the first (draw first image onto second canvas)**
- Blend the two together to create a mosaic
- (If there are more images, repeat)

Adapted from Steve Seitz

Computing the homography



To **compute** the homography given pairs of corresponding points in the images, we need to set up an equation where the parameters of **H** are the unknowns…

Kristen Grauman

Computing the homography

- Assume we have four matched points: How do we compute homography **H**?

p'=Hp

$$\mathbf{p}' = \begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} \quad \mathbf{H} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \quad \mathbf{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad \mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix}$$

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Can set scale factor $h_9 = 1$.
So, there are 8 unknowns.
Need at least 8 eqs, but the more the better…

A

$$\begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix} \mathbf{h} = \mathbf{0}$$

DEMO

Adapted from Derek Hoiem, Kristen Grauman

Derivation: http://www.cse.psu.edu/~rtc12/CSE486/lecture16.pdf

How to stitch together panorama (mosaic)?

Basic Procedure

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- (If there are more images, repeat)

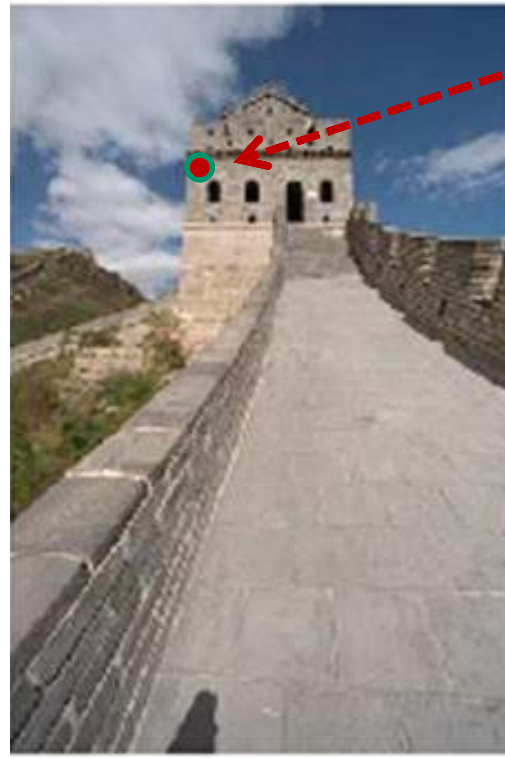
Adapted from Steve Seitz

Transforming the second image

$$(x, y)$$

Image 1

Image 2 canvas



$$\left({wx'}/{w}, {wy'}/{w} \right) = (x', y')$$

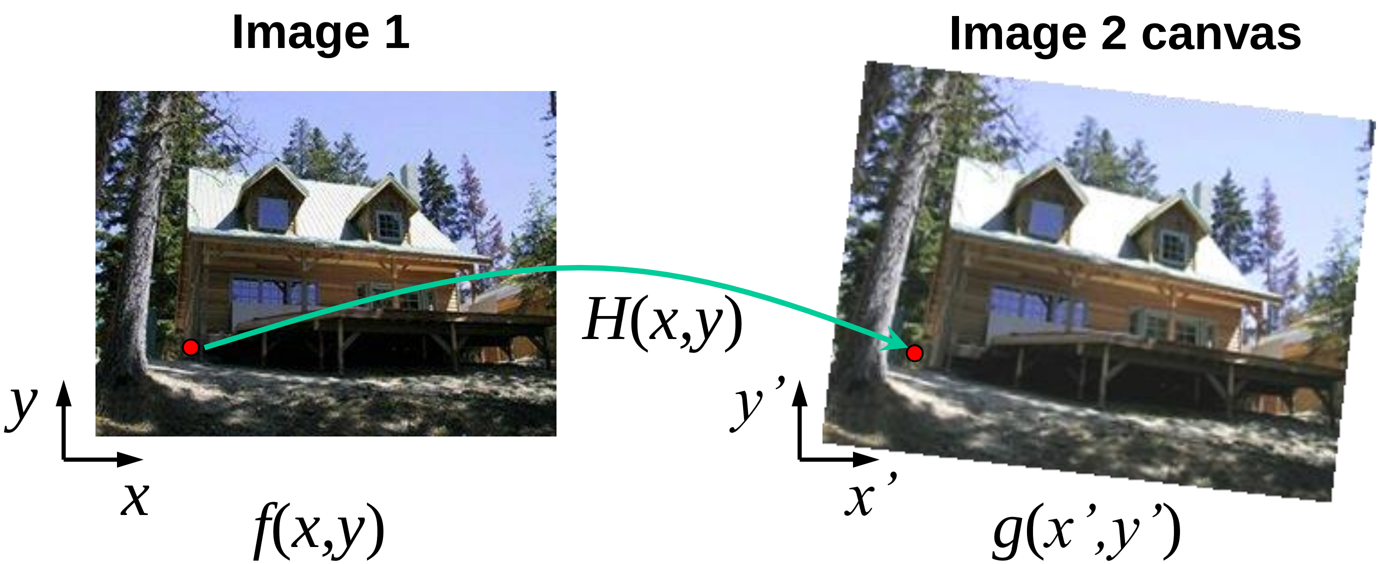
To **apply** a given homography **H**

- Compute **p' = Hp** (regular matrix multiply)
- Convert **p'** from homogeneous to image coordinates

$$\begin{array}{ccc} \begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} & = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} & \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \\ \mathbf{p'} & \mathbf{H} & \mathbf{p} \end{array}$$

Modified from Kristen Grauman

Transforming the second image



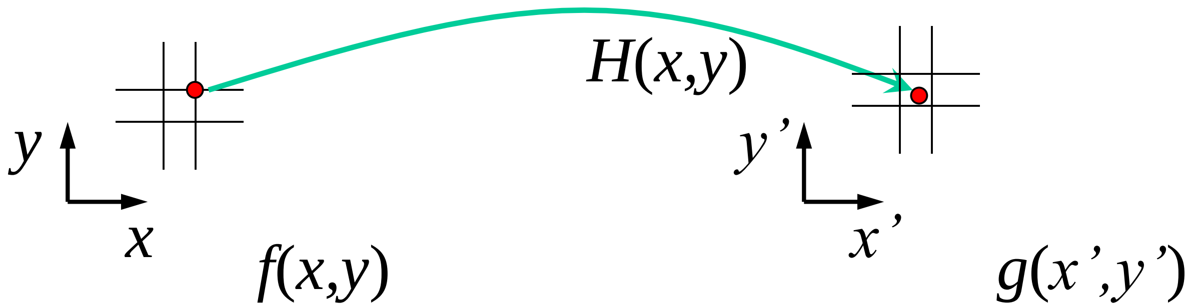
Forward warping:

Send each pixel $f(x,y)$ to its corresponding location

$(x',y') = H(x,y)$ in the right image

Modified from Alyosha Efros

Transforming the second image



Forward warping:

Send each pixel $f(x,y)$ to its corresponding location

$(x',y') = H(x,y)$ in the right image

Q: what if pixel lands “between” two pixels?

A: distribute color among neighboring pixels (x’,y’)

Next: Stereo vision

- Homography: Same camera center, but camera rotates
- Stereo vision: Camera center is not the same (we have multiple cameras)

- Epipolar geometry
  - Relates cameras from two positions/cameras

- Stereo depth estimation
  - Recover depth from disparities between two images

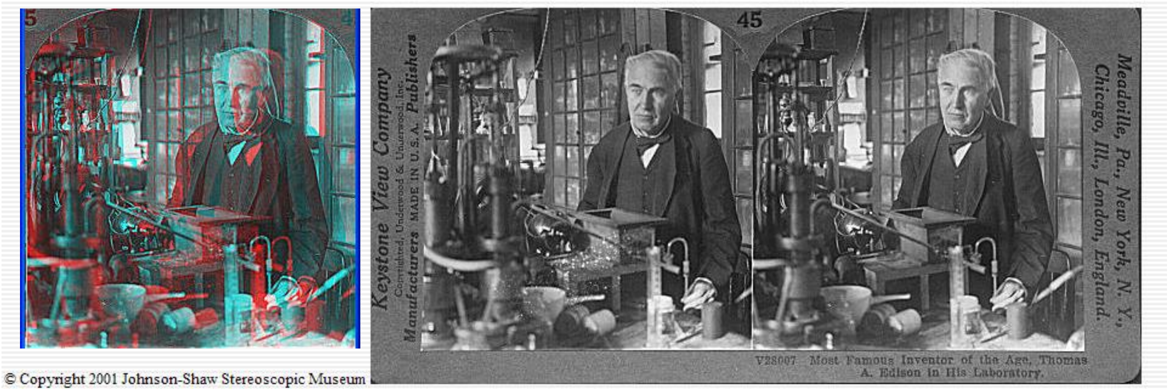
Adapted from Derek Hoiem

Stereo photography and stereo viewers

Take two pictures of the same subject from two slightly different viewpoints and display so that each eye sees only one of the images.


Invented by Sir Charles Wheatstone, 1838


Image from fisher-price.com



Kristen Grauman

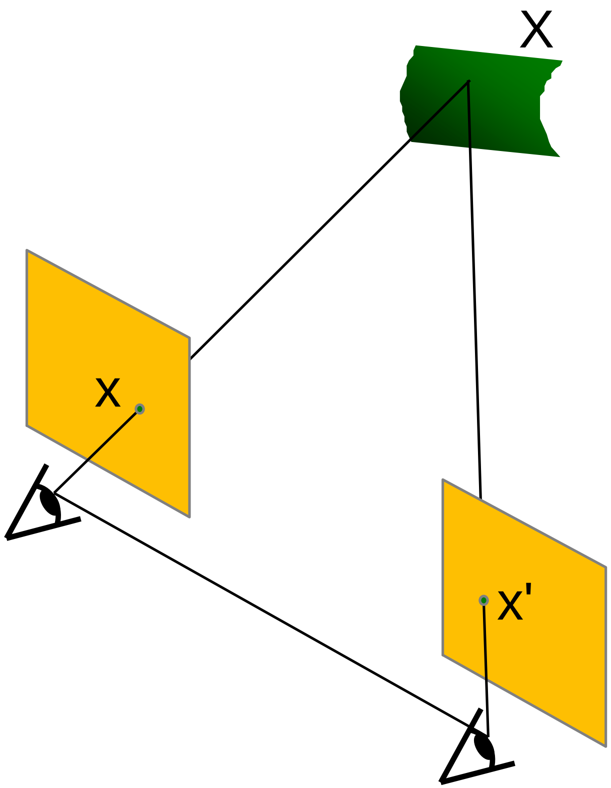
Depth from stereo for computers

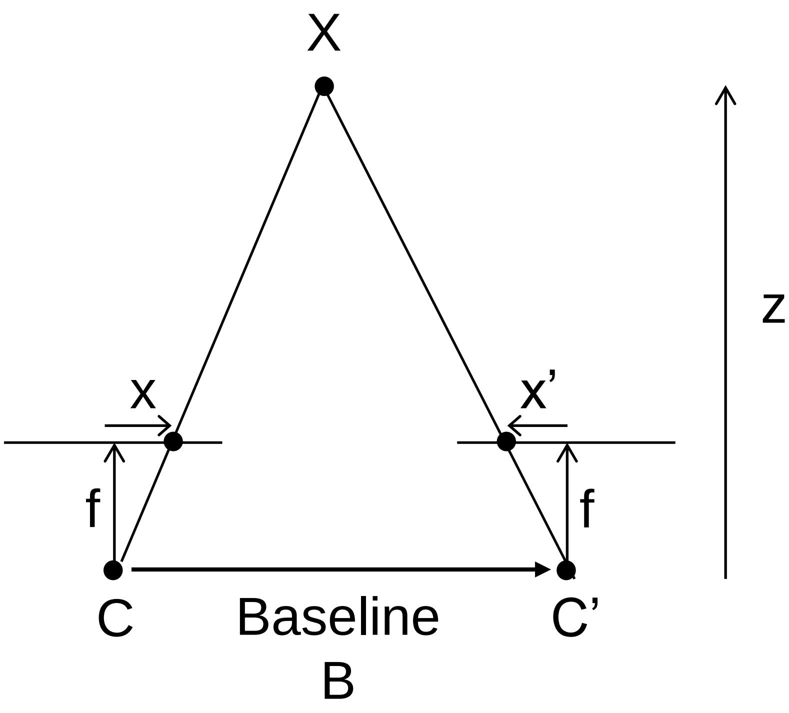
Two cameras, simultaneous views

Single moving camera and static scene

Depth from stereo

- Goal: recover depth by finding image coordinate x’ that corresponds to x, then measuring discrepancy between x and x’

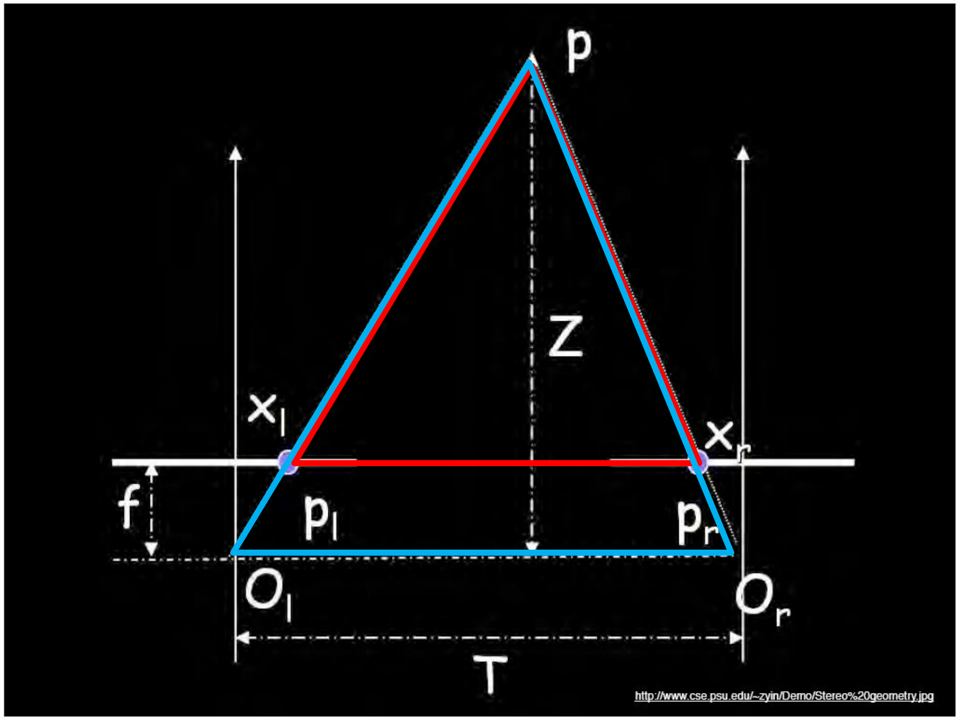




Derek Hoiem

Geometry for a simple stereo system

- Assume parallel optical axes, known camera parameters (i.e., calibrated cameras). **What is expression for Z?**



Similar triangles (p_l, P, p_r) and (O_l, P, O_r):

$$\frac{T + x_l - x_r}{Z - f} = \frac{T}{Z}$$

depth

$$Z = f \frac{T}{x_r - x_l}$$

disparity

Depth is inversely proportional to disparity.

Adapted from Kristen Grauman

Depth from disparity

- We have two images from different cameras.
- First, find **corresponding points** in two images
  - How to do this efficiently?
- Second, **estimate relative depth** from correspondences

image I(x,y)

Disparity map D(x,y)

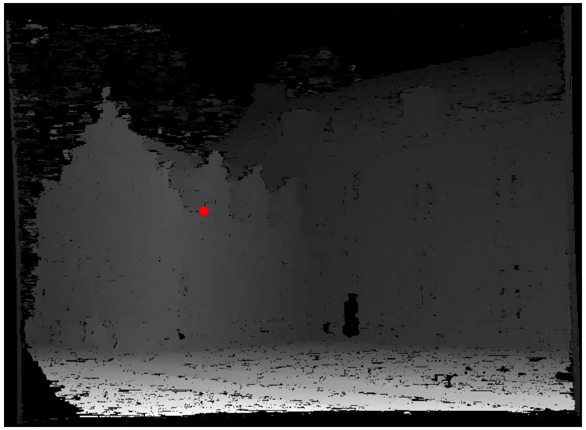
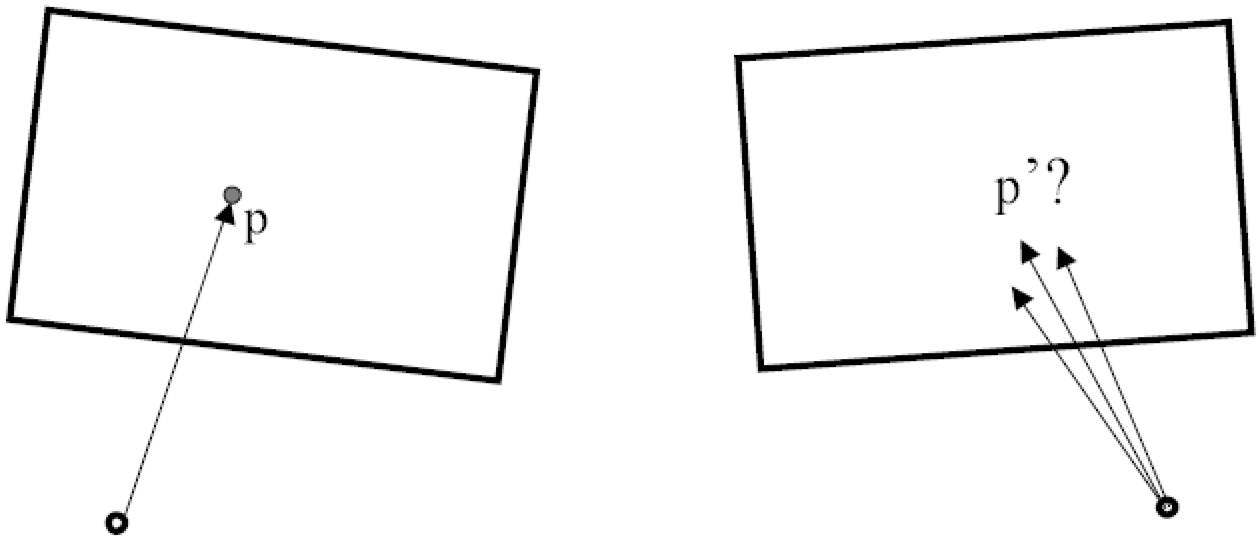


image I´(x´,y´)

Kristen Grauman

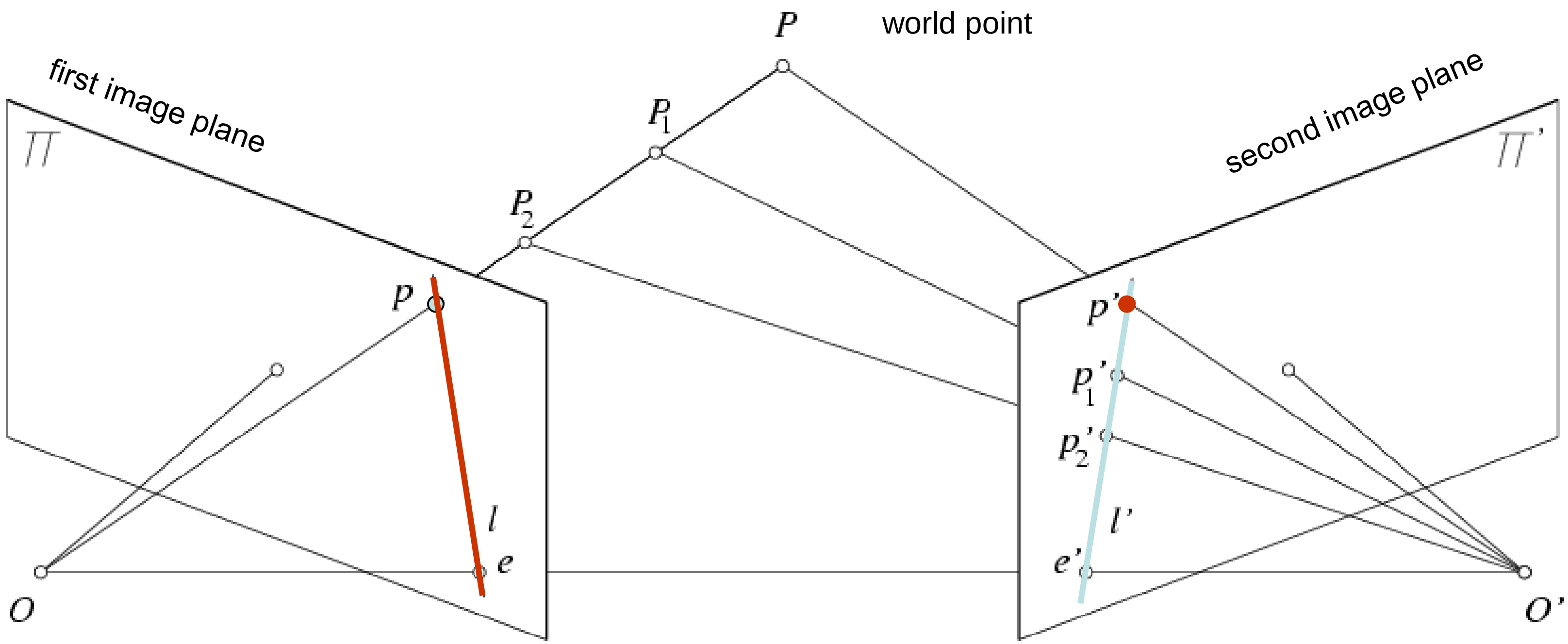
Stereo correspondence constraints



- Given p in left image, where can corresponding point p’ be?

Kristen Grauman

Epipolar constraint

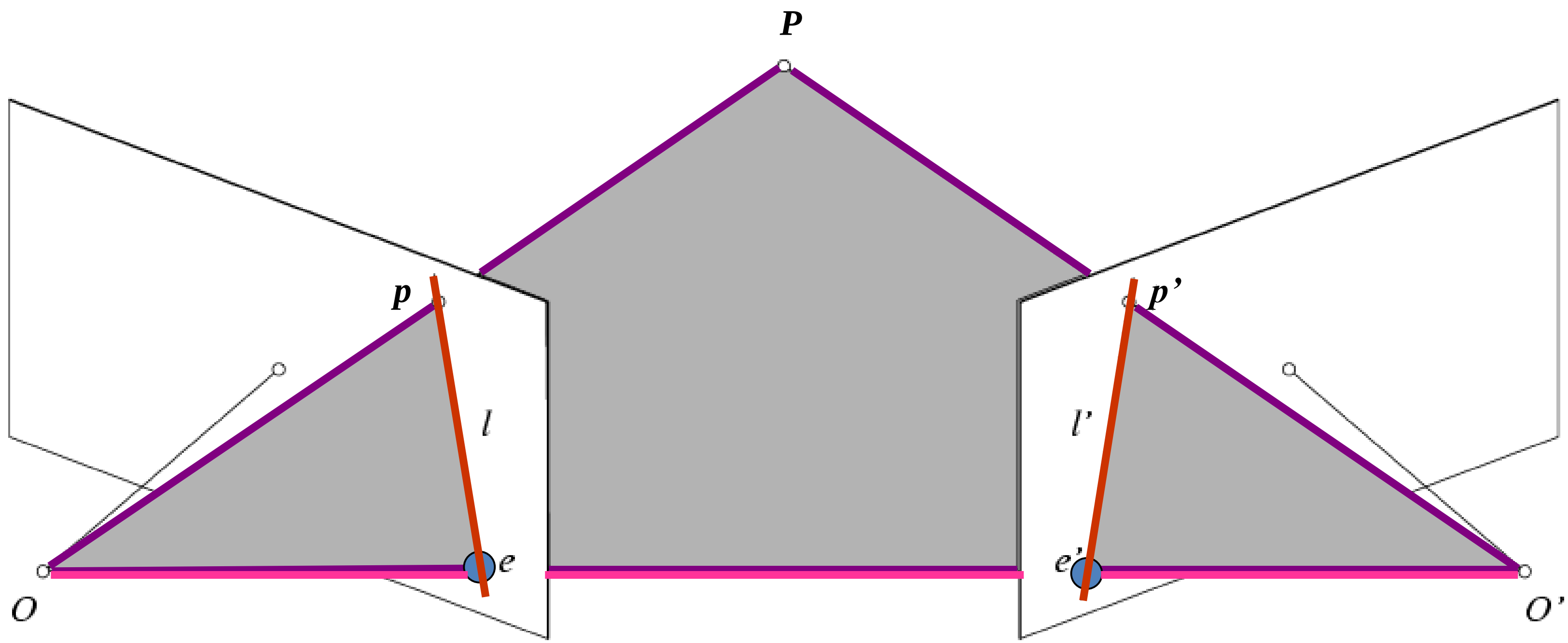


Geometry of two views constrains where the corresponding pixel for some image point in the first view must occur in the second view.

- It must be on the line where (1) the plane connecting the world point and optical centers, and (2) the image plane, intersect.
- Potential matches for *p* have to lie on the corresponding line *l'*.
- Potential matches for *p'* have to lie on the corresponding line *l*.

Adapted from Kristen Grauman, Derek Hoiem

Epipolar geometry: notation



- **Baseline** – line connecting the two camera centers
- **Epipoles**

= intersections of baseline with image planes

= projections of the other camera center

- **Epipolar Plane** – plane containing baseline
- **Epipolar Lines** - intersections of epipolar plane with image planes (always come in corresponding pairs)

Adapted from Derek Hoiem

Epipolar constraint

The epipolar constraint is useful because it reduces the correspondence problem to a 1D search along an epipolar line.

See hidden slides for details.

Kristen Grauman, image from Andrew Zisserman

Rigs related by:
Rotation: 3x3 matrix **R**
Translation: 3x1 vector **T**.

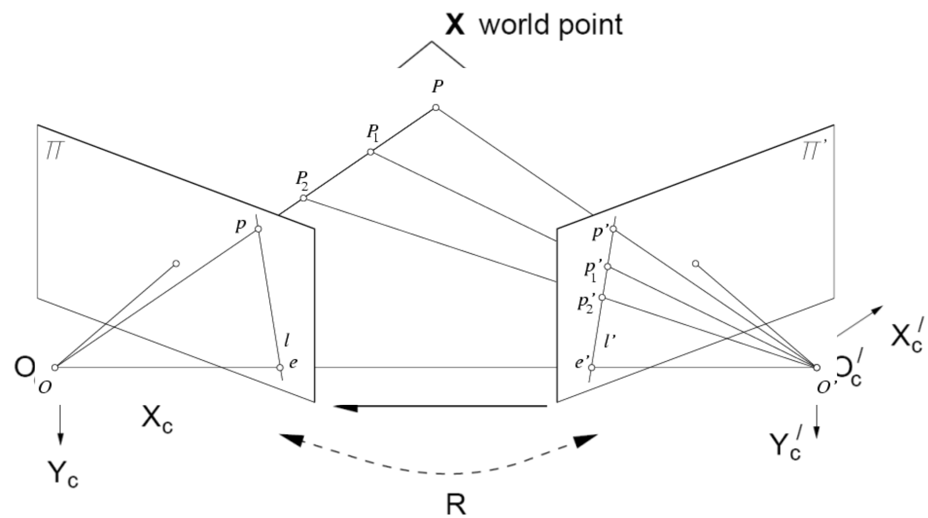
Essential matrix

$$\boxed{\mathbf{X}' \cdot \left(\mathbf{T} \times \mathbf{RX}\right) = 0}$$

$$\mathbf{X}' \cdot \left([\mathrm{T}_x]\mathbf{RX}\right) = 0$$

Let $\mathbf{E} = [\mathrm{T}_x]\mathbf{R}$

$$\mathbf{X}' \cdot \mathbf{EX} = \mathbf{X}'^T \mathbf{EX} = 0$$



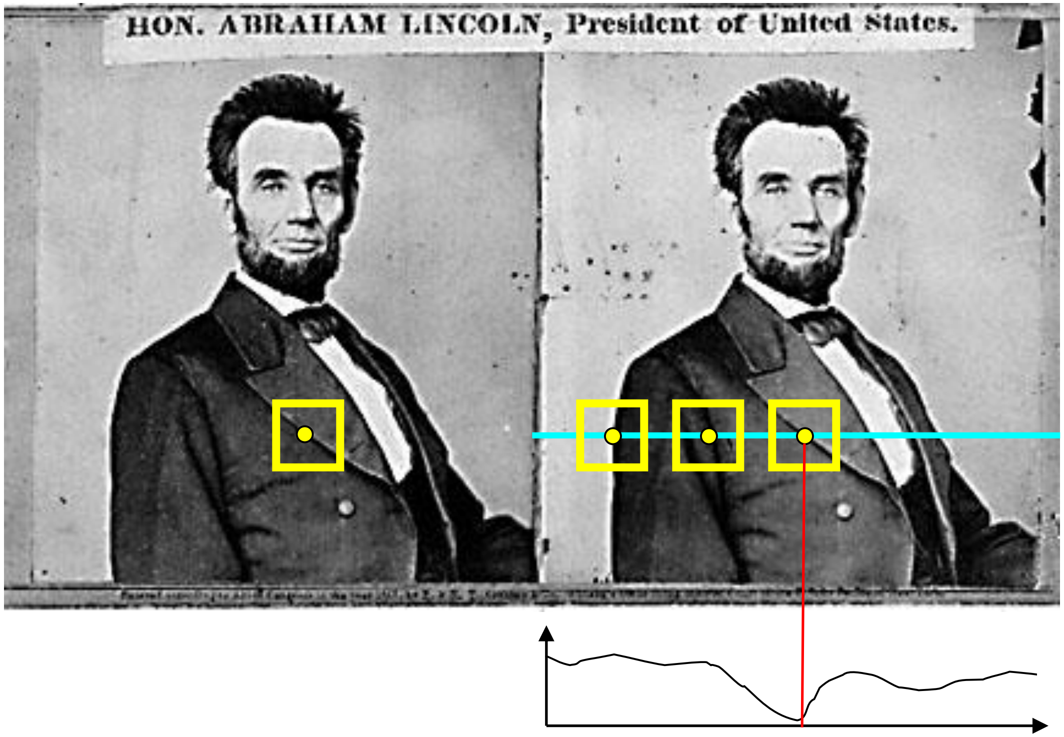
E is called the **essential matrix**, and it relates corresponding image points between both cameras, given the **rotation** and **translation**.

Before we said: If we observe a point in one image, its position in other image is constrained to lie on line defined by above. It turns out that:

- $E^T x$ is the epipolar line l' through x' in the second image, corresponding to x.
- Ex' is the epipolar line l through x in the first image, corresponding to x'.

Adapted from Kristen Grauman, Derek Hoiem

Basic stereo matching algorithm

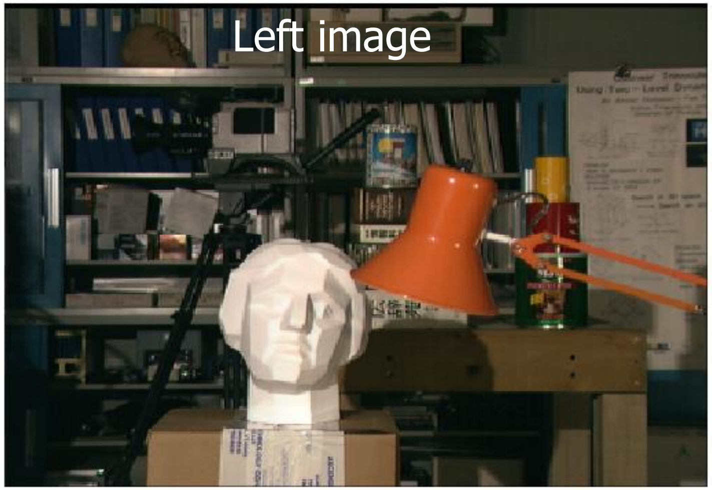


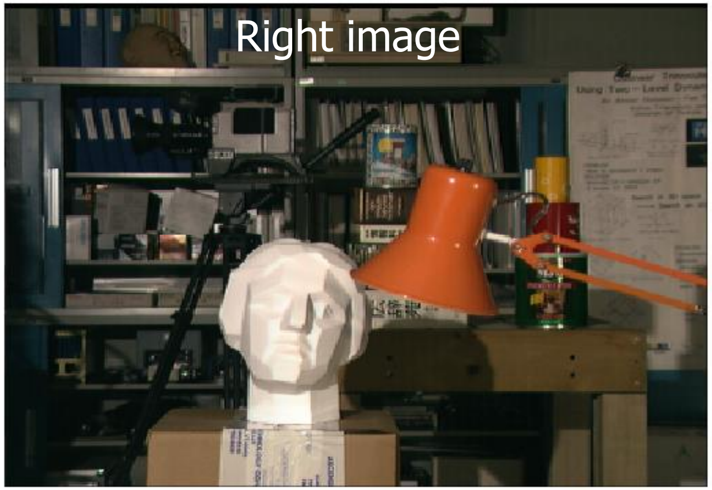
- For each pixel in the first image
  - Find corresponding epipolar scanline in the right image
  - Search along epipolar line and pick the best match x’ (e.g. smallest Euclidean distance between SIFT in patch)
  - Compute disparity x-x’ and set depth(x) = f*T/(x-x’)

Adapted from Derek Hoiem

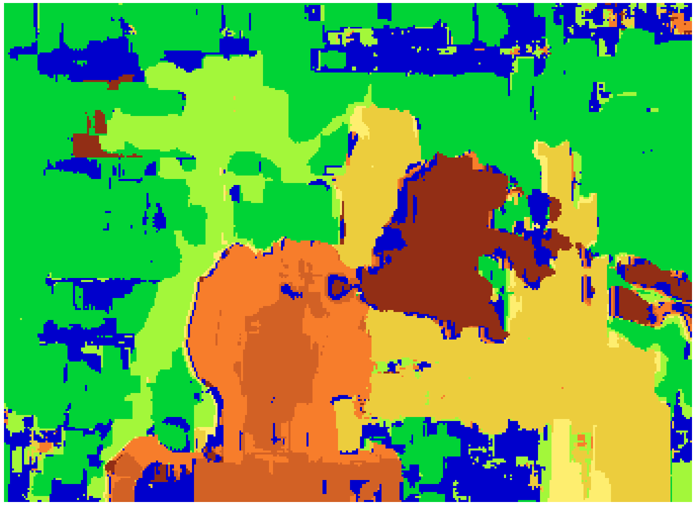
Results with window search

Data





Predicted depth



Ground truth

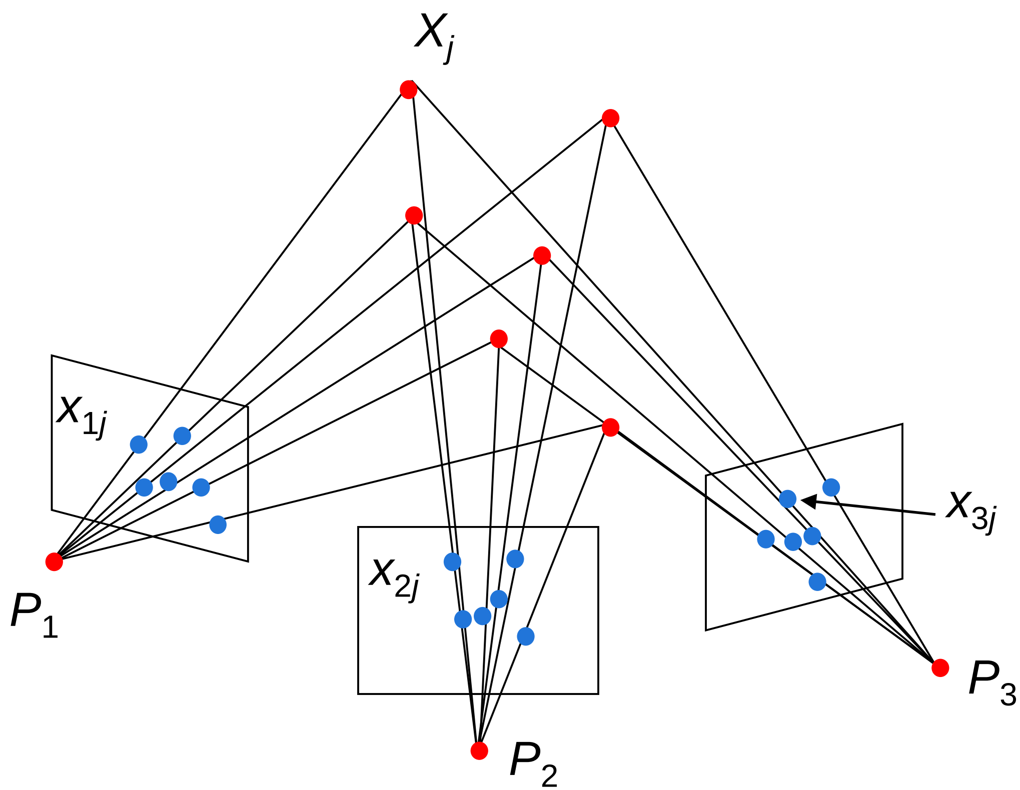
Derek Hoiem

Projective structure from motion

- Given: *m* images of *n* fixed 3D points

$$\mathbf{x}_{ij} = \mathbf{P}_i \mathbf{X}_j , \qquad i = 1,\dots, m, \quad j = 1, \dots , n$$

- Problem: estimate *m* projection matrices $\mathbf{P}_i$ and *n* 3D points $\mathbf{X}_j$ from the *mn* corresponding 2D points $\mathbf{x}_{ij}$



Svetlana Lazebnik

Photo tourism

Noah Snavely, Steven M. Seitz, Richard Szeliski, "Photo tourism: Exploring photo collections in 3D," SIGGRAPH 2006

http://phototour.cs.washington.edu/

3D from multiple images

Sameer Agarwala, Noah Snavely, Ian Simon, Steven M. Seitz, Richard Szeliski, "Building Rome in a Day," ICCV 2009

Summary of multiple views

- Write **2d transformations** as matrix-vector multiplication
- **Fitting transformations**: Solve for unknown parameters given corresponding points from two views – linear, affine, projective (homography)
- **Mosaics**: Uses homography and image warping to merge views taken from same center of projection
- **Stereo depth estimation:** Find corresponding points along epipolar scanline, then measure disparity (as inverse to depth)
- **Epipolar geometry:** Matching point in second image is on a line passing through its epipole; makes search for correspondences quicker

Adapted from Kristen Grauman and Derek Hoiem