CS 2770: Computer Vision

Image Filtering and Texture

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Plan for this lecture

- Filters: math and properties
- Types of filters
  - Linear
    - Smoothing
    - Other
  - Non-linear
    - Median
- Texture representation with filters
- Anti-aliasing for image subsampling

How images are represented (Matlab)

- Color images represented as a matrix with multiple channels (=1 if grayscale)
- Suppose we have a NxM RGB image called "im"
  - im(1,1,1) = top-left pixel value in R-channel
  - im(y, x, b) = y pixels **down (rows)**, x pixels **to right (cols)** in bth channel
  - im(N, M, 3) = bottom-right pixel in B-channel
- imread(filename) returns a uint8 image (values 0 to 255)

column →

row ↓

R

0.92	0.93	0.94	0.97	0.62	0.37	0.85	0.97	0.93	0.92	0.99
0.95	0.89	0.82	0.89	0.56	0.31	0.75	0.92	0.81	0.95	0.91
0.89	0.72	0.51	0.55	0.51	0.42	0.57	0.41	0.49	0.91	0.92
0.96	0.95	0.88	0.94	0.56	0.46	0.91	0.87	0.90	0.97	0.95
0.71	0.81	0.81	0.87	0.57	0.37	0.80	0.88	0.89	0.79	0.85
0.49	0.62	0.60	0.58	0.50	0.60	0.58	0.50	0.61	0.45	0.33
0.86	0.84	0.74	0.58	0.51	0.39	0.73	0.92	0.91	0.49	0.74
0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93
0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93

G

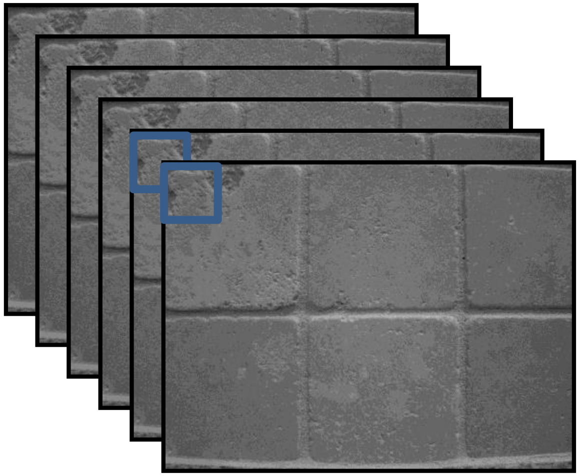
									0.92	0.99
									0.95	0.91
									0.91	0.92
									0.97	0.95
									0.79	0.85
									0.45	0.33
									0.49	0.74
									0.82	0.93
									0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93

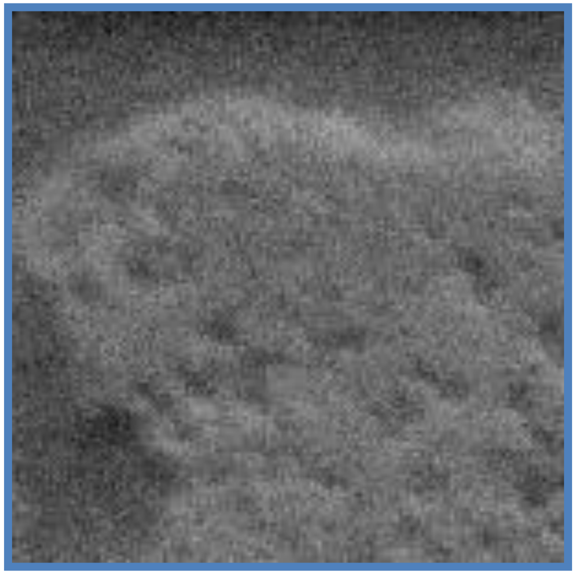
B

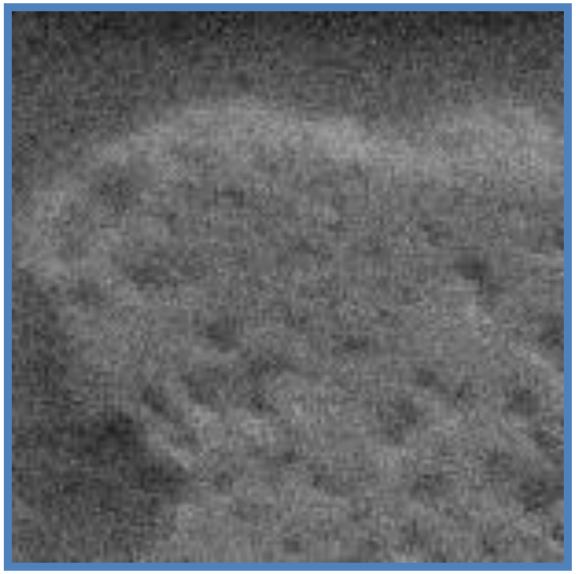
									0.92	0.99
									0.95	0.91
									0.91	0.92
									0.97	0.95
									0.79	0.85
									0.45	0.33
									0.49	0.74
									0.82	0.93
									0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93

Adapted from Derek Hoiem

Enter: Noise







- We talked about how the same object will look very different across images
- Even multiple images of the same static scene will not be identical
- How could we reduce the noise, i.e., give an estimate of the true intensities?
- What if there's only one image?

Common types of noise

- **Salt and pepper noise**: random occurrences of black and white pixels
- **Impulse noise:** random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution

Original

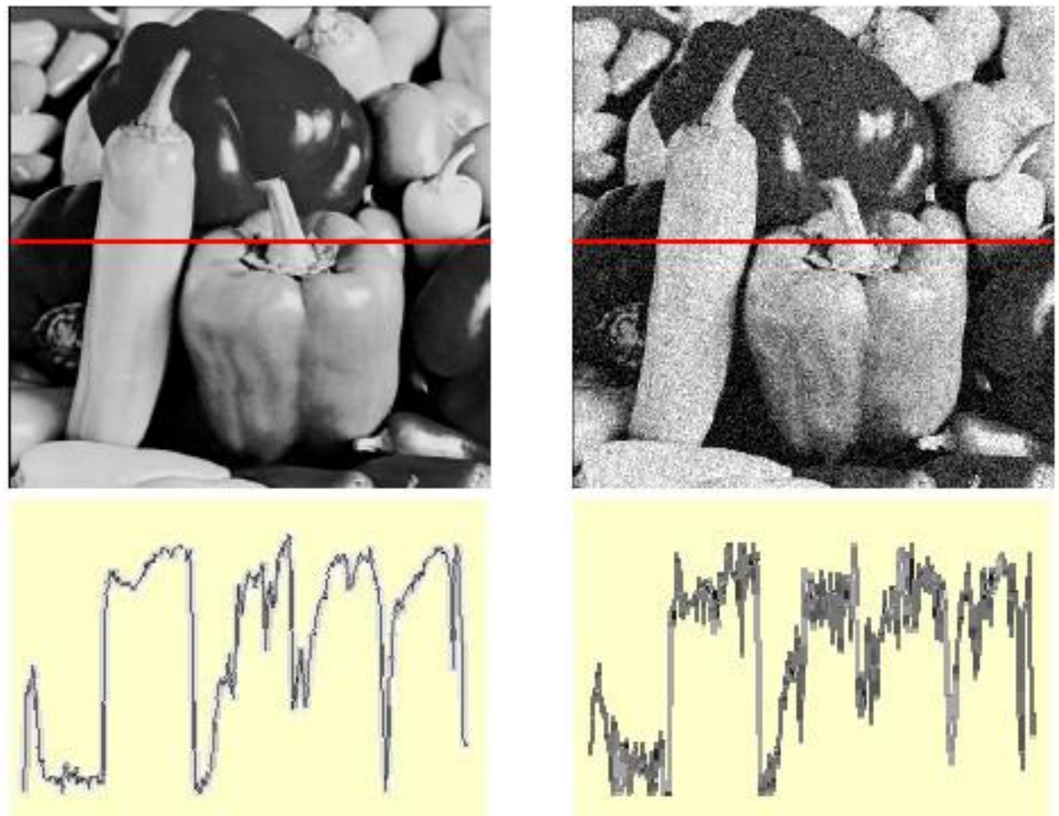
Salt and pepper noise

Impulse noise

Gaussian noise

Source: S. Seitz

Gaussian noise



$$f(x,y) = \overbrace{\bar{f}(x,y)}^{\text{Ideal Image}} + \overbrace{\eta(x,y)}^{\text{Noise process}}$$

Gaussian i.i.d. ("white") noise:
$\eta(x,y) \sim \mathcal{N}(\mu, \sigma)$

```
>> noise = randn(size(im)).*sigma;
>> output = im + noise;
```

What is impact of the sigma?

Fig: M. Hebert

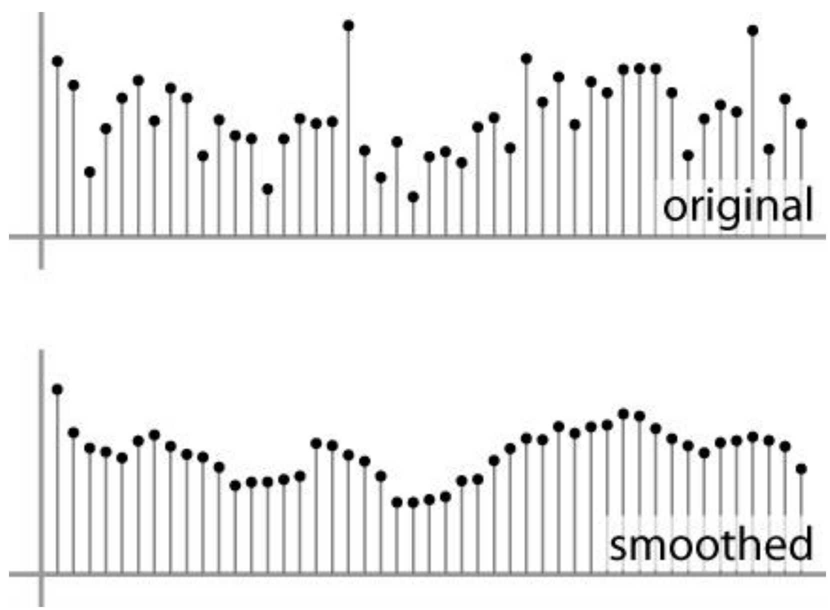
First attempt at a solution

- Let’s replace each pixel with an average of all the values in its neighborhood
- Assumptions:
  - Expect pixels to be like their neighbors
  - Expect noise processes to be independent from pixel to pixel

Kristen Grauman

First attempt at a solution

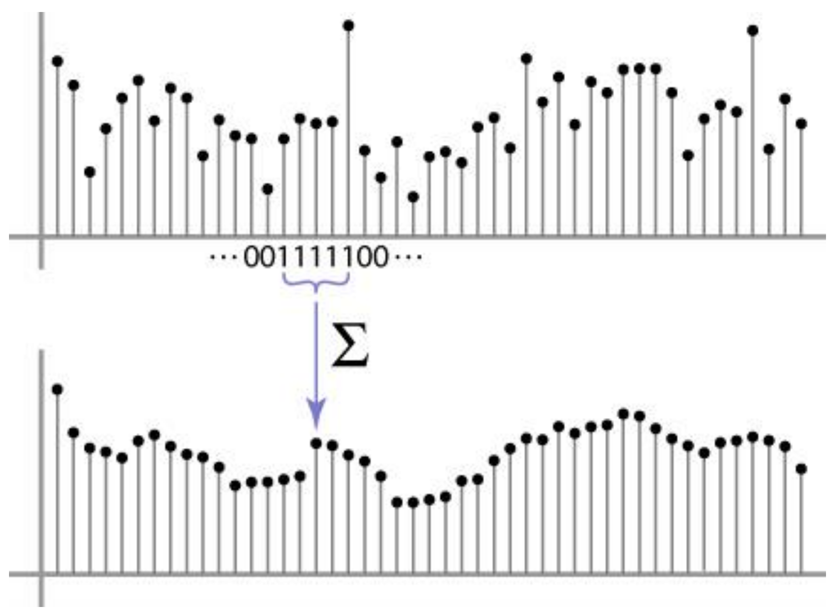
- Let’s replace each pixel with an average of all the values in its neighborhood
- Moving average in 1D:



Source: S. Marschner

Weighted Moving Average

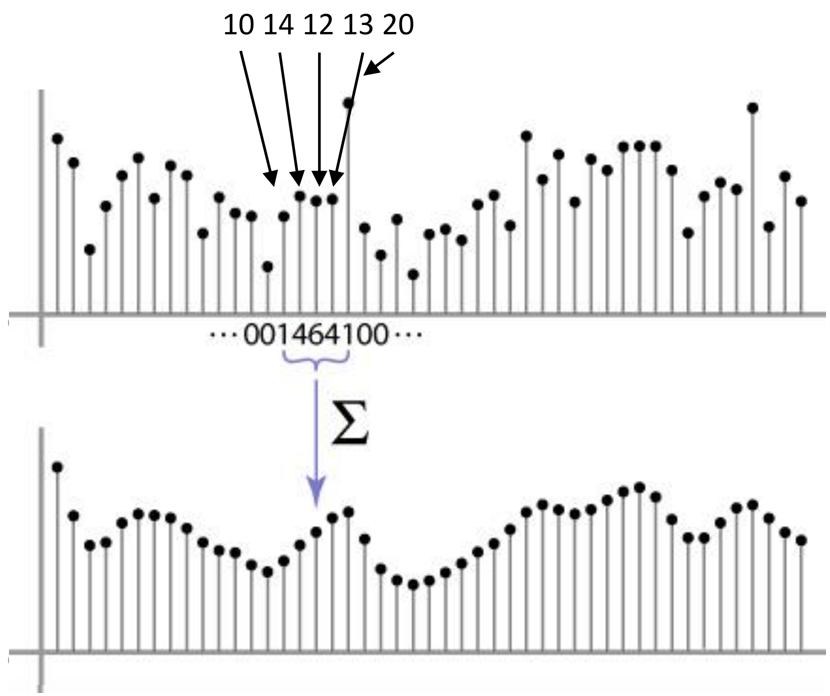
- Can add weights to our moving average
- *Weights* [1, 1, 1, 1, 1] / 5



Source: S. Marschner

Weighted Moving Average

- Non-uniform weights [1, 4, 6, 4, 1] / 16



Central pixel =
10*1 +
14*4 +
12*6 +
13*4 +
20*1

Adapted from S. Marschner

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0								

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10							

Source: S. Seitz

Moving Average In 2D

$F[x,y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x,y]$

	0	10	20						

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30					

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30	30				

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

Source: S. Seitz

Image filtering

- Compute a function of the local neighborhood at each pixel in the image
  - Function specified by a “filter” or mask saying how to combine values from neighbors.
  - Element-wise multiplication

- Uses of filtering:
  - Enhance an image (denoise, resize, etc)
  - Extract information (texture, edges, etc)
  - Detect patterns (template matching)

Adapted from Derek Hoiem

Correlation filtering

Say the averaging window size is 2k+1 x 2k+1:

$$G[i,j] = \underbrace{\frac{1}{(2k+1)^2}}_{\textit{Attribute uniform weight to each pixel}} \underbrace{\sum_{u=-k}^{k} \sum_{v=-k}^{k} F[i+u, j+v]}_{\textit{Loop over all pixels in neighborhood around image pixel F[i,j]}}$$

Now generalize to allow **different weights** depending on neighboring pixel's relative position:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} \underbrace{H[u,v]}_{\textit{Non-uniform weights}} F[i+u, j+v]$$

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Correlation filtering

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i+u, j+v]$$

This is called **cross-correlation**, denoted $G = H \otimes F$

Filtering an image: replace each pixel with a linear combination of its neighbors.

The filter "**kernel**" or "**mask**" $H[u,v]$ is the prescription for the weights in the linear combination.

Averaging filter

- What values belong in the kernel *H* for the moving average example?

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$\otimes$

$H[u, v]$

$$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

?

"box filter"

$G[x, y]$

	0	10	20	30	30				

$$G = H \otimes F$$

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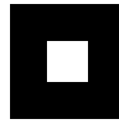
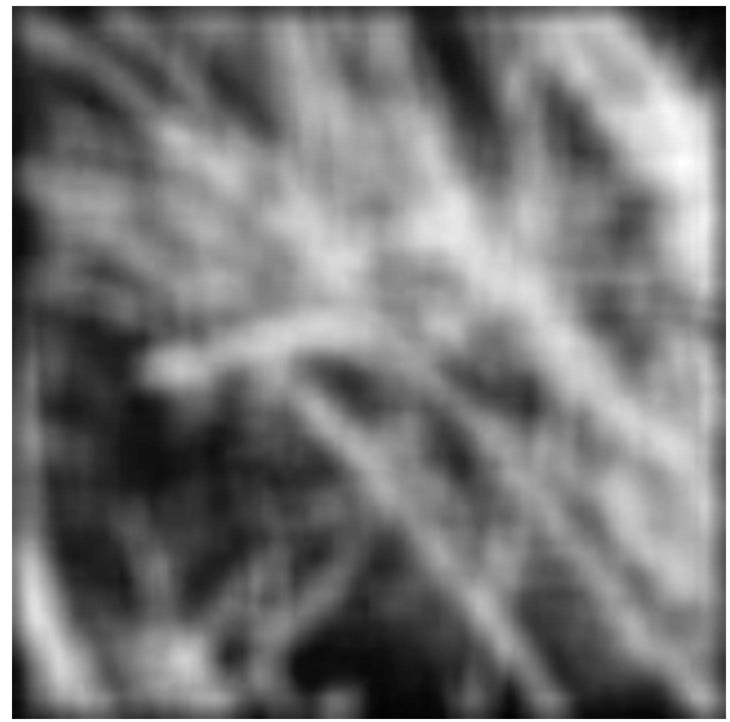
Smoothing by averaging



depicts box filter:
white = high value, black = low value

original



filtered

What if the filter size was 5 x 5 instead of 3 x 3?

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Gaussian filter

- What if we want nearest neighboring pixels to have the most influence on the output?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

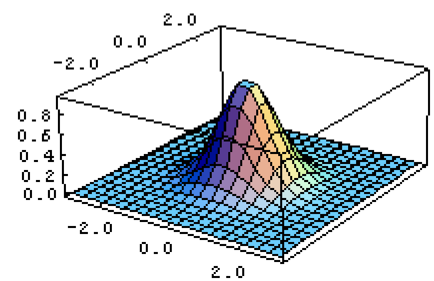
$\frac{1}{16}$

1	2	1
2	4	2
1	2	1

$H[u, v]$

This kernel is an approximation of a 2d Gaussian function:

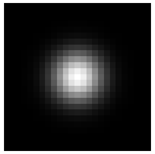
$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$

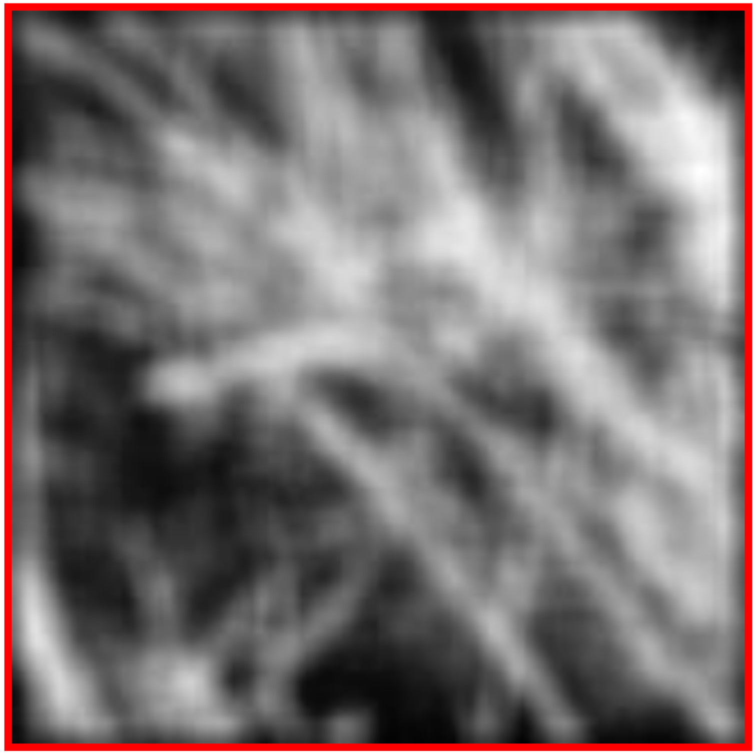


- Removes high-frequency components from the image ("low-pass filter").

Source: S. Seitz

Smoothing with a Gaussian





Vs box filter

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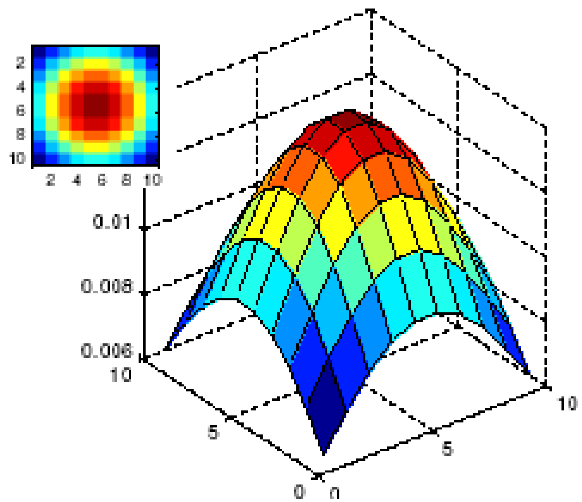
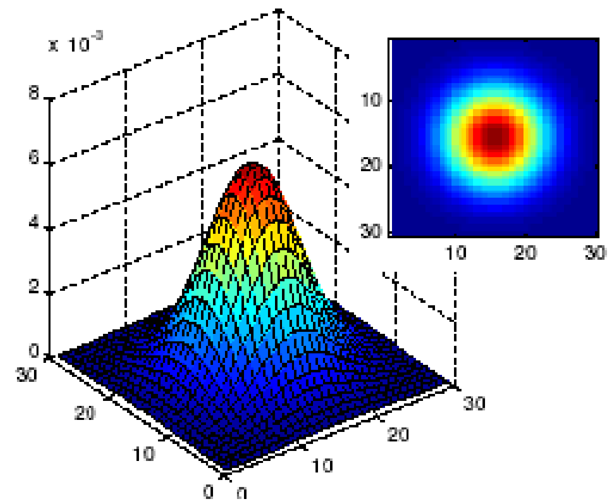
Gaussian filters

- What parameters matter here?
- **Size** of kernel or mask
  - Note, Gaussian function has infinite support, but discrete filters use finite kernels



σ = 5 with 10 x 10 kernel



σ = 5 with 30 x 30 kernel

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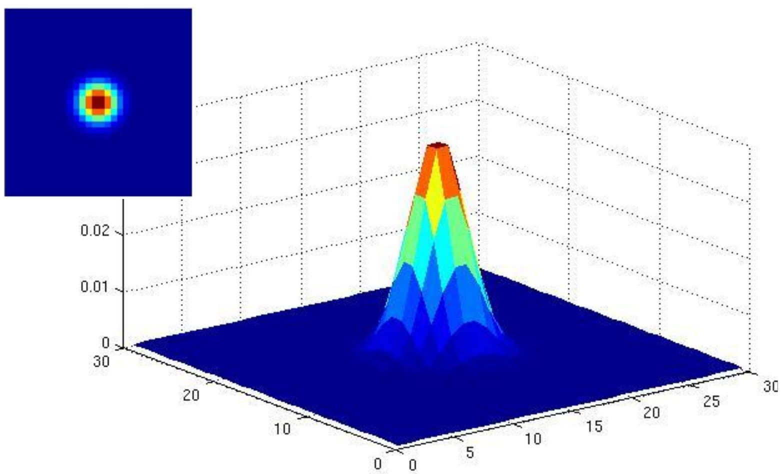
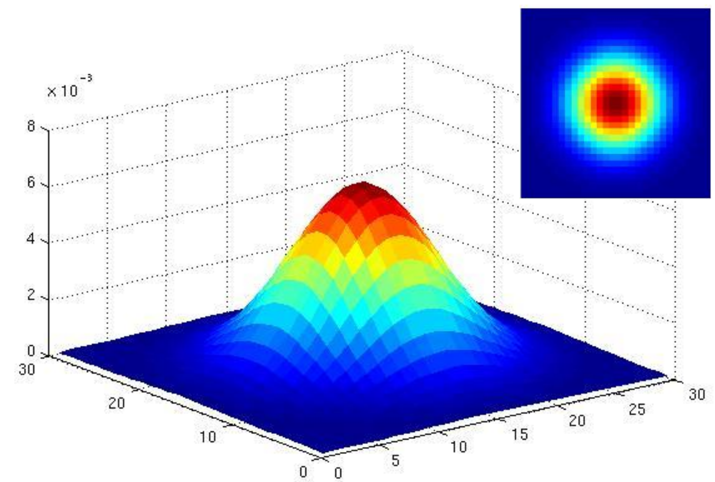
Gaussian filters

- What parameters matter here?
- **Variance** of Gaussian: determines extent of smoothing



σ = 2 with 30 x 30 kernel



σ = 5 with 30 x 30 kernel

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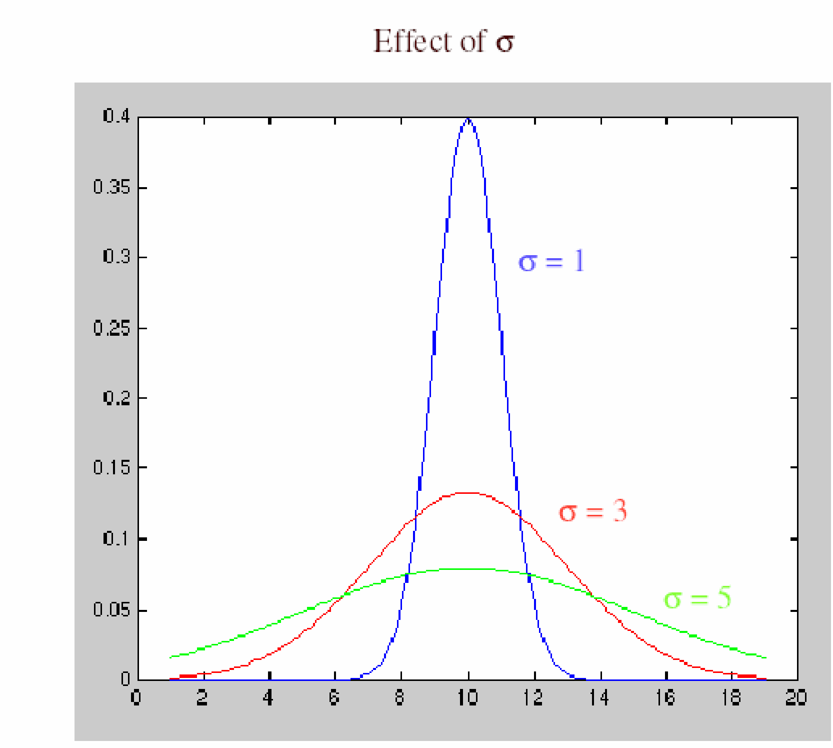
Gaussian filters

How big should the filter be?

- Values at edges should be near zero ← important!
- Rule of thumb for Gaussian: set filter half-width to about 3 σ

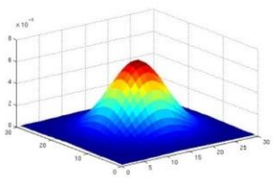


Source: Derek Hoiem

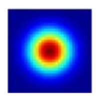
Gaussian filter in Matlab

```
>> hsize = 10;
>> sigma = 5;
>> h = fspecial(‘gaussian’ hsize, sigma);
```

```
>> mesh(h);
```



```
>> imagesc(h);
```



```
>> outim = imfilter(im, h); % correlation
>> imshow(outim);
```

outim

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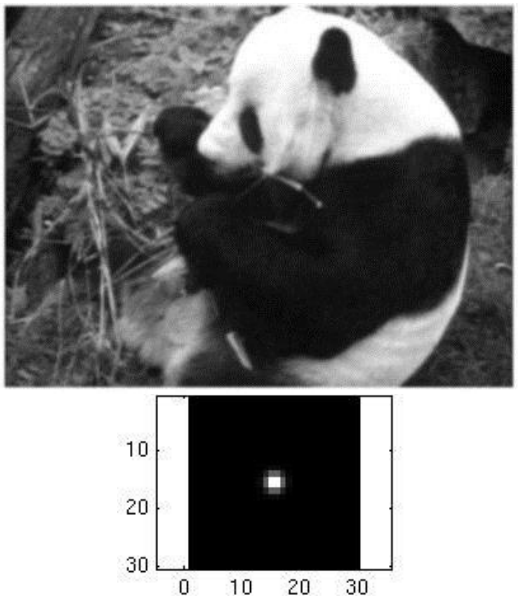
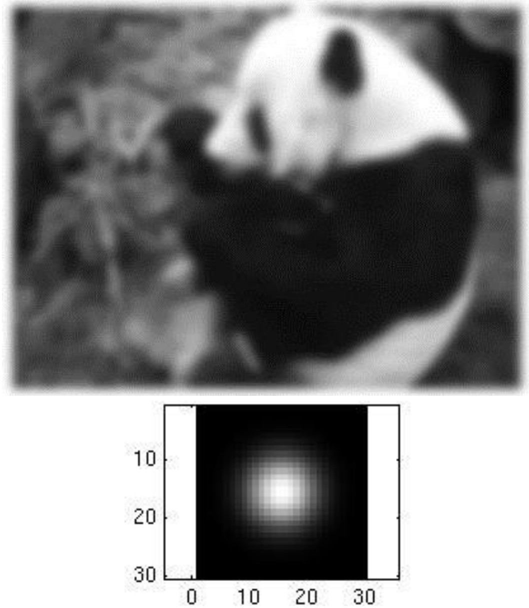
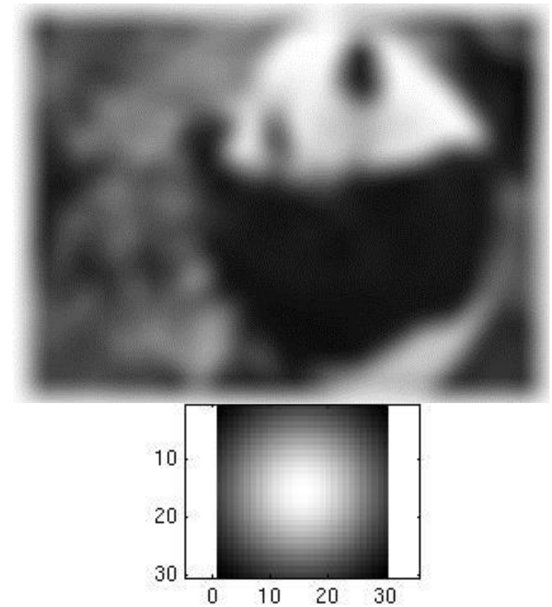
Smoothing with a Gaussian

Parameter σ is the “scale” / “width” / “spread” of the Gaussian kernel, and controls the amount of smoothing.





…




```
for sigma=1:3:10
   h = fspecial('gaussian‘, fsize, sigma);
   out = imfilter(im, h);
   imshow(out);
   pause;
end
```

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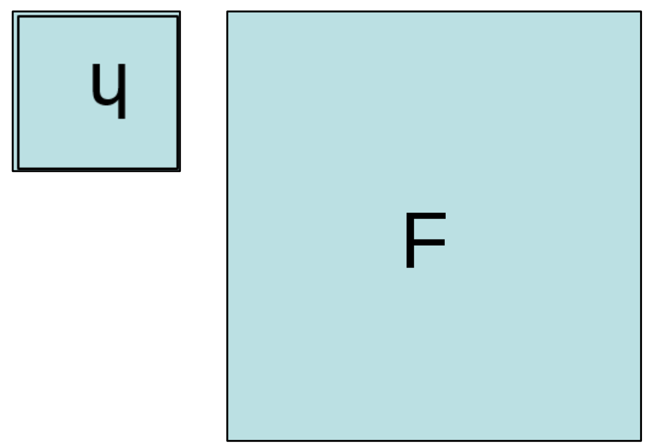
Convolution

- Convolution:
  - Flip the filter in both dimensions (bottom to top, right to left)
  - Then apply cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

$$G = H \star F$$

Notation for convolution operator



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Convolution vs. correlation

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u, j-v]$$

$$G = H \star F$$

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

$$G = H \otimes F$$

For a Gaussian or box filter, how will the outputs differ?

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Convolution vs. correlation

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u,j+v]$$

$$G = H \otimes F$$

u = -1, v = -1

F

5	2	5	4	4	
5	200	3	200	4	
1	5	5	4	4	(i, j)
5	5	1	1	2	
200	1	3	5	200	
1	200	200	200	1	

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

$$G = H \star F$$

H

.06	.12	.06	
.12	.25	.12	(0, 0)
.06	.12	.06	

Convolution vs. correlation

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

$$G = H \otimes F$$

u = -1, v = -1

v = 0

F

5	2	5	4	4	
5	200	3	200	4	
1	5	5	4	4	(i, j)
5	5	1	1	2	
200	1	3	5	200	
1	200	200	200	1	

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u, j-v]$$

$$G = H \star F$$

H

.06	.12	.06	
.12	.25	.12	(0, 0)
.06	.12	.06	

Convolution vs. correlation

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i+u, j+v]$$

$$G = H \otimes F$$

u = -1, v = -1

v = 0

v = +1

F

5	2	5	4	4
5	200	3	200	4
1	5	5	4	4
5	5	1	1	2
200	1	3	5	200
1	200	200	200	1

(i, j)

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i-u, j-v]$$

$$G = H \star F$$

H

.06	.12	.06
.12	.25	.12
.06	.12	.06

(0, 0)

Convolution vs. correlation

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

$$G = H \otimes F$$

u = -1, v = -1

v = 0

v = +1

u = 0, v = -1

F

5	2	5	4	4	
5	200	3	200	4	
1	5	5	4	4	(i, j)
5	5	1	1	2	
200	1	3	5	200	
1	200	200	200	1	

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u, j-v]$$

$$G = H \star F$$

H

.06	.12	.06	
.12	.25	.12	(0, 0)
.06	.12	.06	

Convolution vs. correlation

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i+u, j+v]$$

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u = -1, v = -1

F

5	2	5	4	4	
5	200	3	200	4	
1	5	5	4	4	(i, j)
5	5	1	1	2	
200	1	3	5	200	
1	200	200	200	1	

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i-u, j-v]$$

$$G = H \star F$$

H

.06	.12	.06	
.12	.25	.12	(0, 0)
.06	.12	.06	

Convolution vs. correlation

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i+u, j+v]$$

$$G = H \otimes F$$

u = -1, v = -1

v = 0

F

5	2	5	4	4	
5	200	3	200	4	
1	5	5	4	4	(i, j)
5	5	1	1	2	
200	1	3	5	200	
1	200	200	200	1	

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i-u, j-v]$$

$$G = H \star F$$

H

.06	.12	.06	
.12	.25	.12	(0, 0)
.06	.12	.06	

Convolution vs. correlation

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

$$G = H \otimes F$$

u = -1, v = -1

v = 0

v = +1

F

5	2	5	4	4	
5	200	3	200	4	
1	5	5	4	4	(i, j)
5	5	1	1	2	
200	1	3	5	200	
1	200	200	200	1	

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u, j-v]$$

$$G = H \star F$$

H

.06	.12	.06	
.12	.25	.12	(0, 0)
.06	.12	.06	

Convolution vs. correlation

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

$$G = H \otimes F$$

u = -1, v = -1

v = 0

v = +1

u = 0, v = -1

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u, j-v]$$

$$G = H \star F$$

F

5	2	5	4	4	
5	200	3	200	4	
1	5	5	4	4	(i, j)
5	5	1	1	2	
200	1	3	5	200	
1	200	200	200	1	

H

.06	.12	.06	
.12	.25	.12	(0, 0)
.06	.12	.06	

Properties of smoothing filters

- Smoothing
  - Values positive
  - Sum to 1 → overall intensity same as input
  - Amount of smoothing proportional to mask size
  - Remove “high-frequency” components; “low-pass” filter

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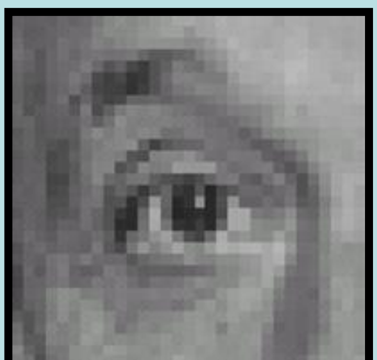
Predict the outputs using correlation filtering

$$* \begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix} = ?$$

$$* \begin{bmatrix}0&0&0\\0&0&1\\0&0&0\end{bmatrix} = ?$$



$$* \begin{bmatrix}0&0&0\\0&2&0\\0&0&0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} = ?$$

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Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

?

Source: D. Lowe

Practice with linear filters

0	0	0
0	1	0
0	0	0

Original

Filtered
(no change)

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

?

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

Shifted left by 1 pixel with correlation

Source: D. Lowe

Practice with linear filters

Original

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

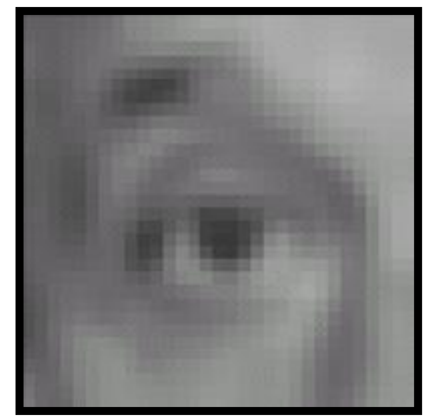
?

Source: D. Lowe

Practice with linear filters

Original

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$



Blur (with a box filter)

Source: D. Lowe

Practice with linear filters

Original

$$\begin{bmatrix}0&0&0\\0&2&0\\0&0&0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} \quad ?$$

Source: D. Lowe

Practice with linear filters

$$\begin{bmatrix}0 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

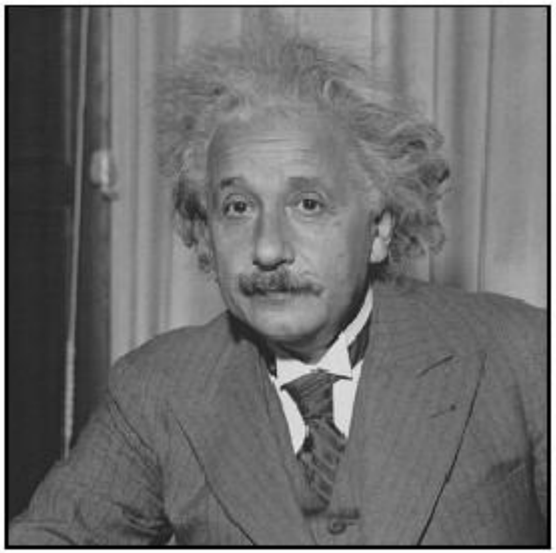
Original

Sharpening filter:

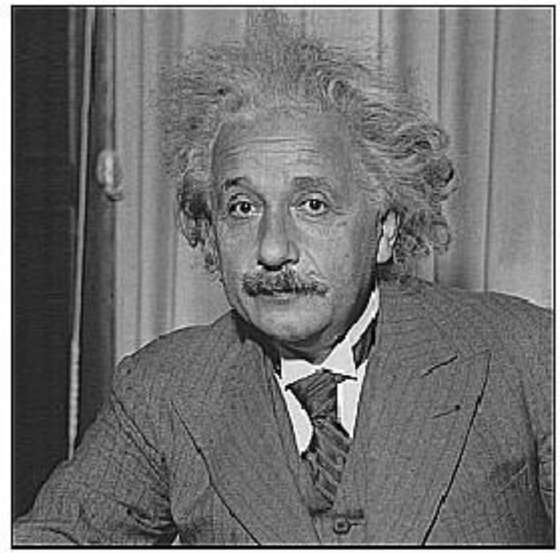
accentuates differences with local average

Source: D. Lowe

Sharpening



before



after

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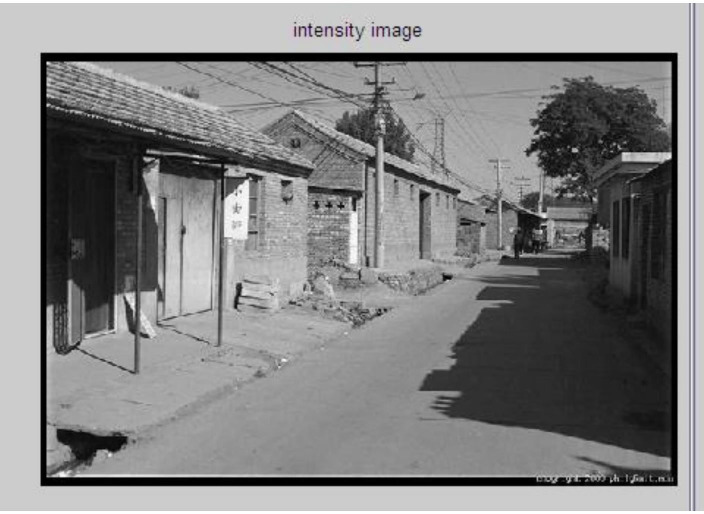
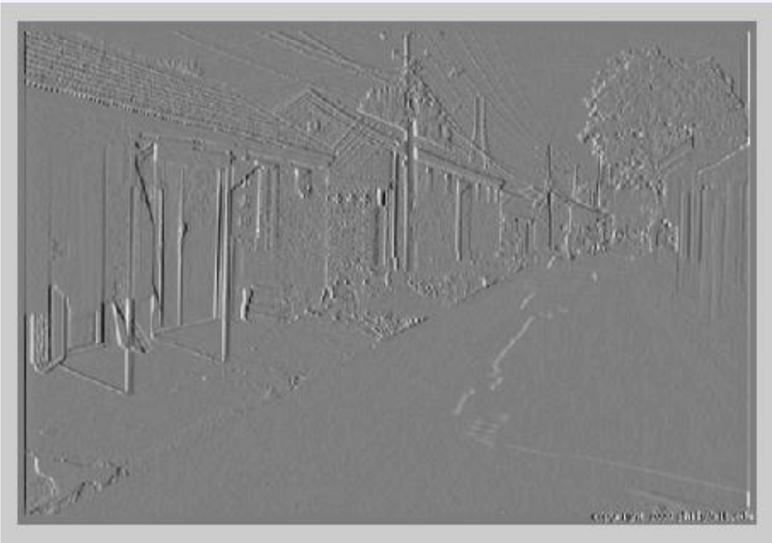
Filters for computing *gradients*

1	0	-1
2	0	-2
1	0	-1



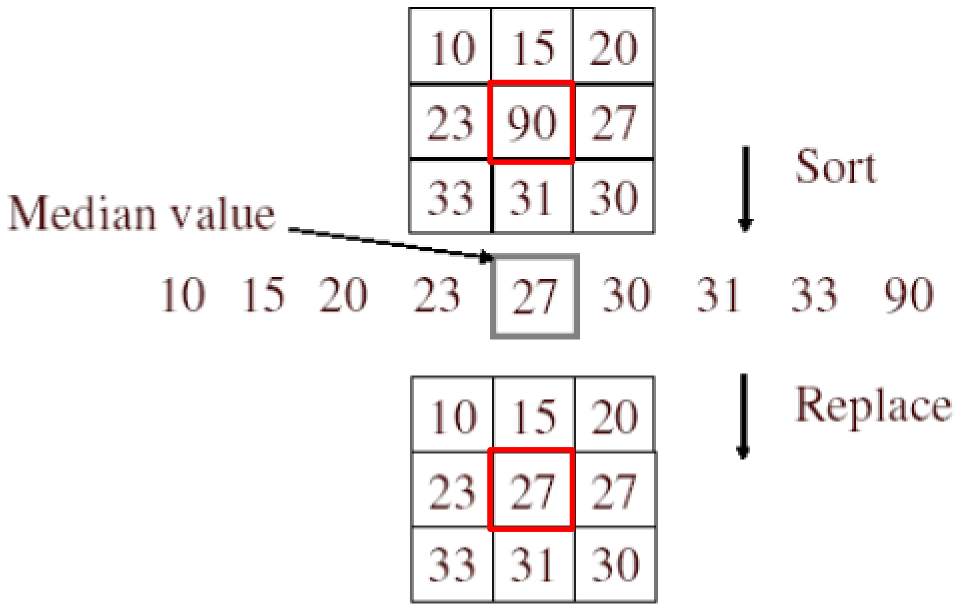
*

=



Slide credit: Derek Hoiem

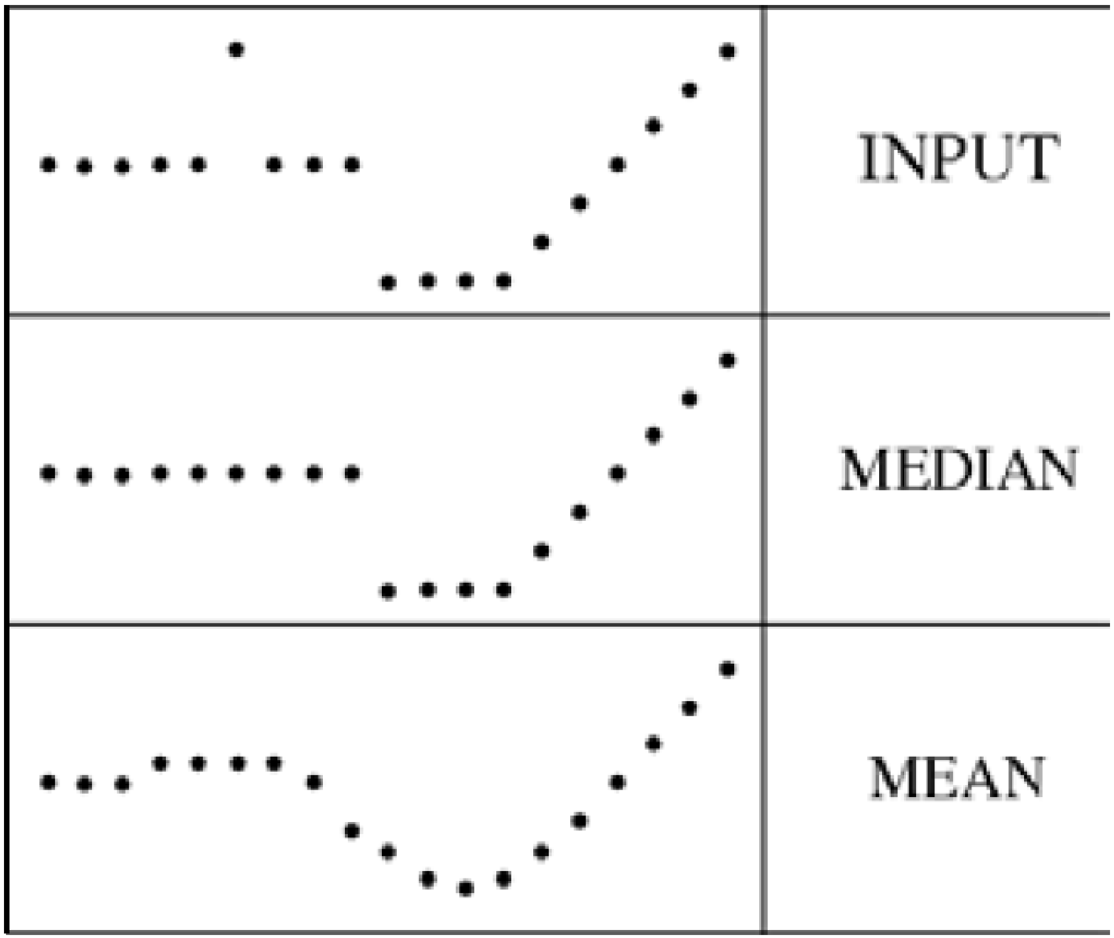
Median filter



- No new pixel values introduced
- Removes spikes: good for impulse, salt & pepper noise
- Non-linear filter

Median filter

- Median filter is edge preserving



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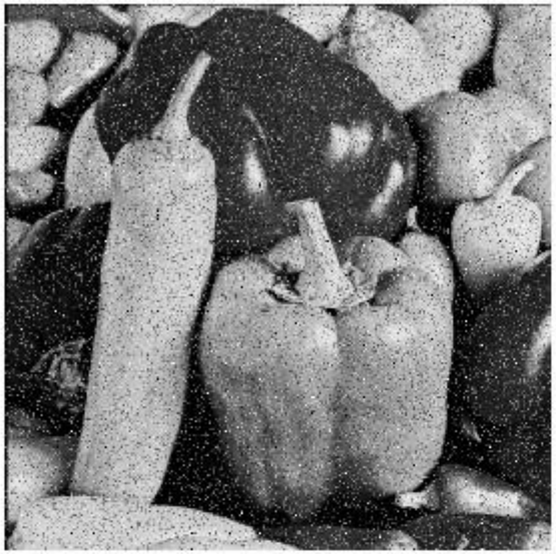
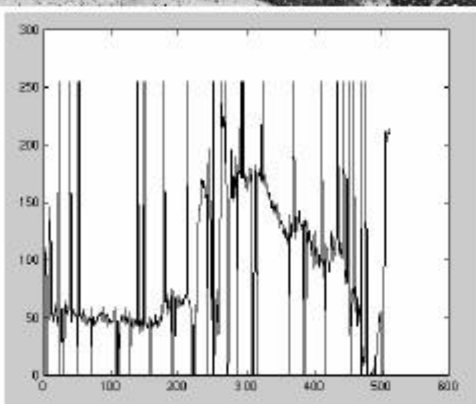
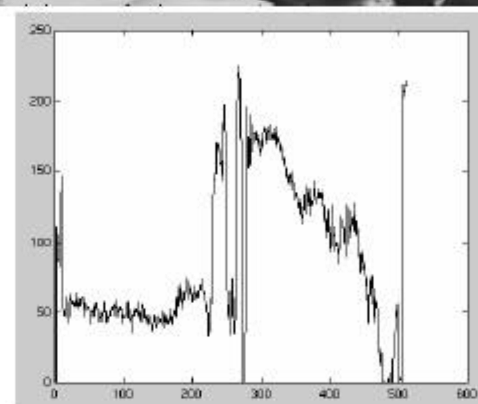
Median filter

Salt and pepper noise



Median filtered





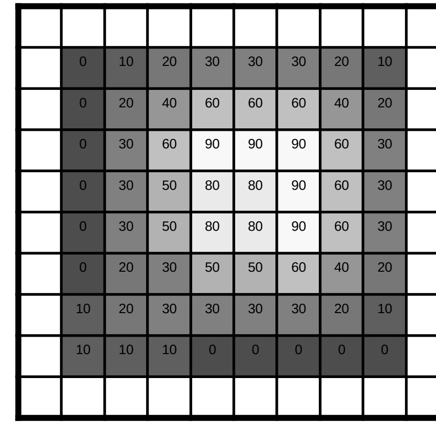
Plots of a row of the image

Matlab: output_im = medfilt2(im, [h w]);

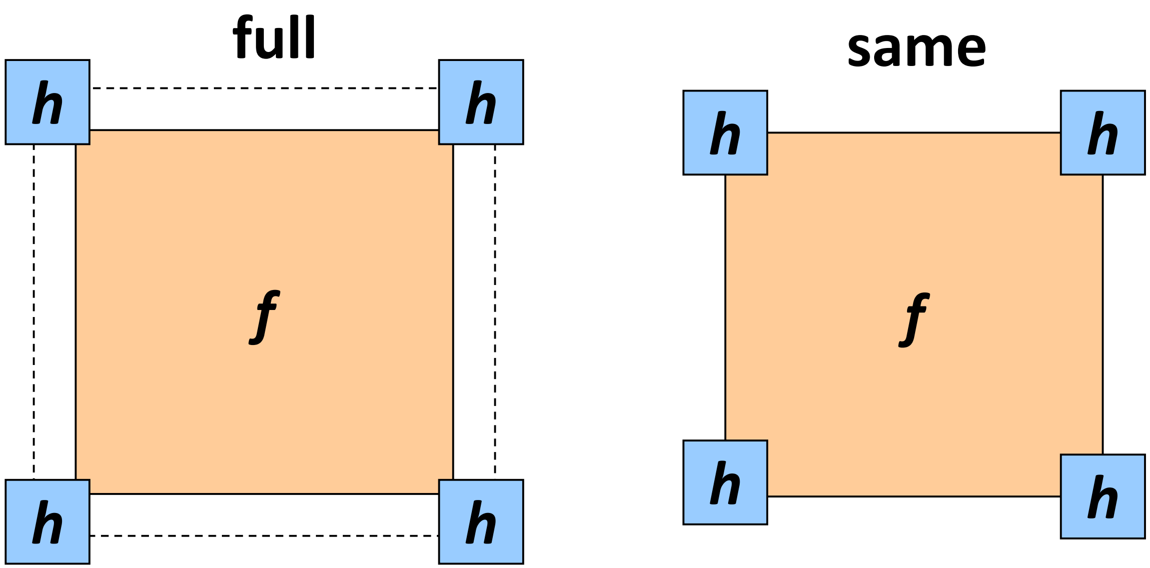
Source: M. Hebert

f = image
h = filter

Boundary issues



- What is the size of the output?
  - 'full': output size is larger than the size of ***f***
  - 'same': output size is same as ***f***

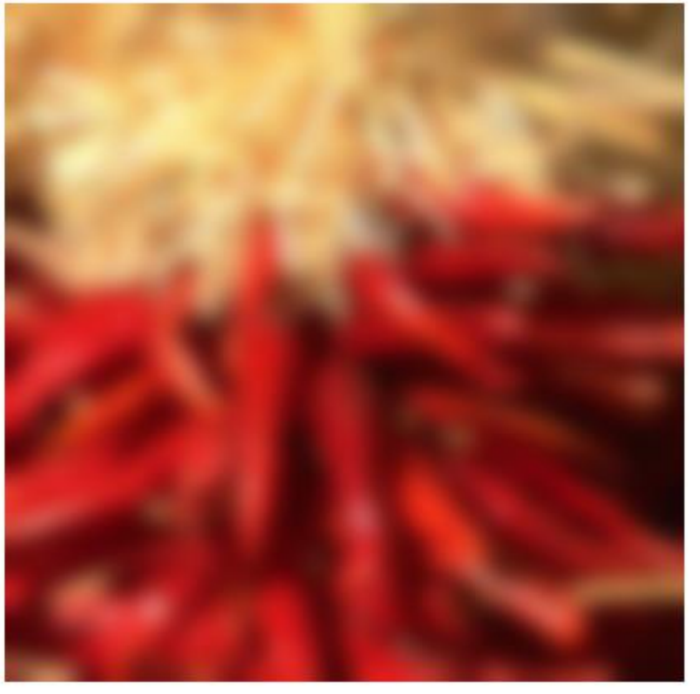


Adapted from S. Lazebnik

Boundary issues

- What about near the edge?
  - the filter window might fall off the edge of the image (in ‘same’ or ‘full’)
  - need to extrapolate
  - methods:

• clip filter (black)
• wrap around
• copy edge
• reflect across edge



Source: S. Marschner

Properties of convolution

- Commutative:

  $f * g = g * f$

- Associative

  $(f * g) * h = f * (g * h)$

- Distributes over addition

  $f * (g + h) = (f * g) + (f * h)$

- Scalars factor out

  $kf * g = f * kg = k(f * g)$

- Identity:

  unit impulse $e = [\ldots, 0, 0, 1, 0, 0, \ldots]$. $f * e = f$

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Separability

- In some cases, filter is separable, and we can factor into two steps:
  - Convolve all rows
  - Convolve all columns

Separability example

2D filtering
(center location only)

$$\underset{\text{filter}}{\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}} * \begin{bmatrix} 2 & 3 & 3 \\ 3 & 5 & 5 \\ 4 & 4 & 6 \end{bmatrix}$$

The filter factors
into an *outer* product
of 1D filters:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

Perform filtering
along rows:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 3 & 3 \\ 3 & 5 & 5 \\ 4 & 4 & 6 \end{bmatrix} = \begin{bmatrix} & 11 & \\ & 18 & \\ & 18 & \end{bmatrix}$$

Followed by filtering
along the remaining column:

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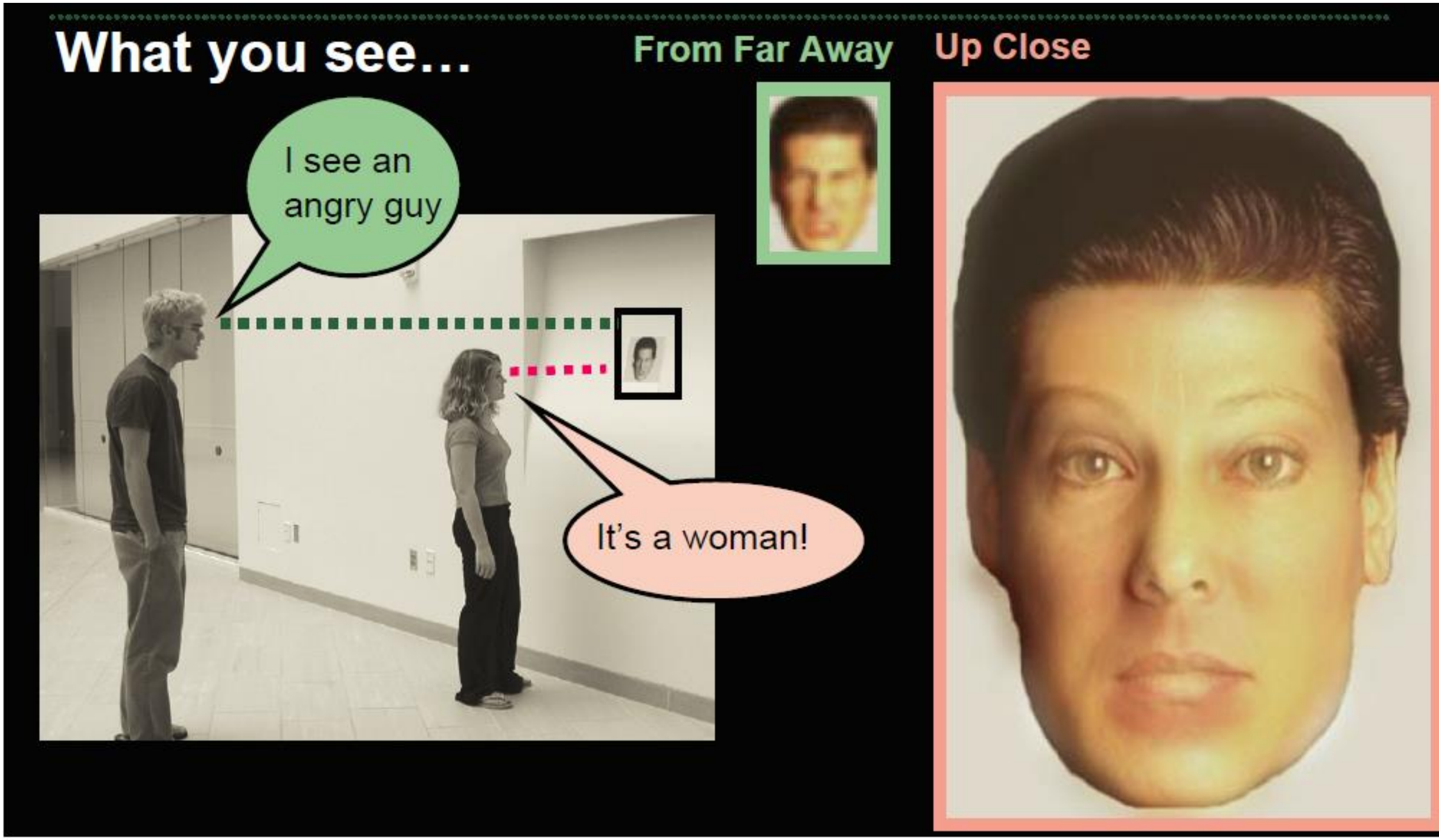
Application: Hybrid Images



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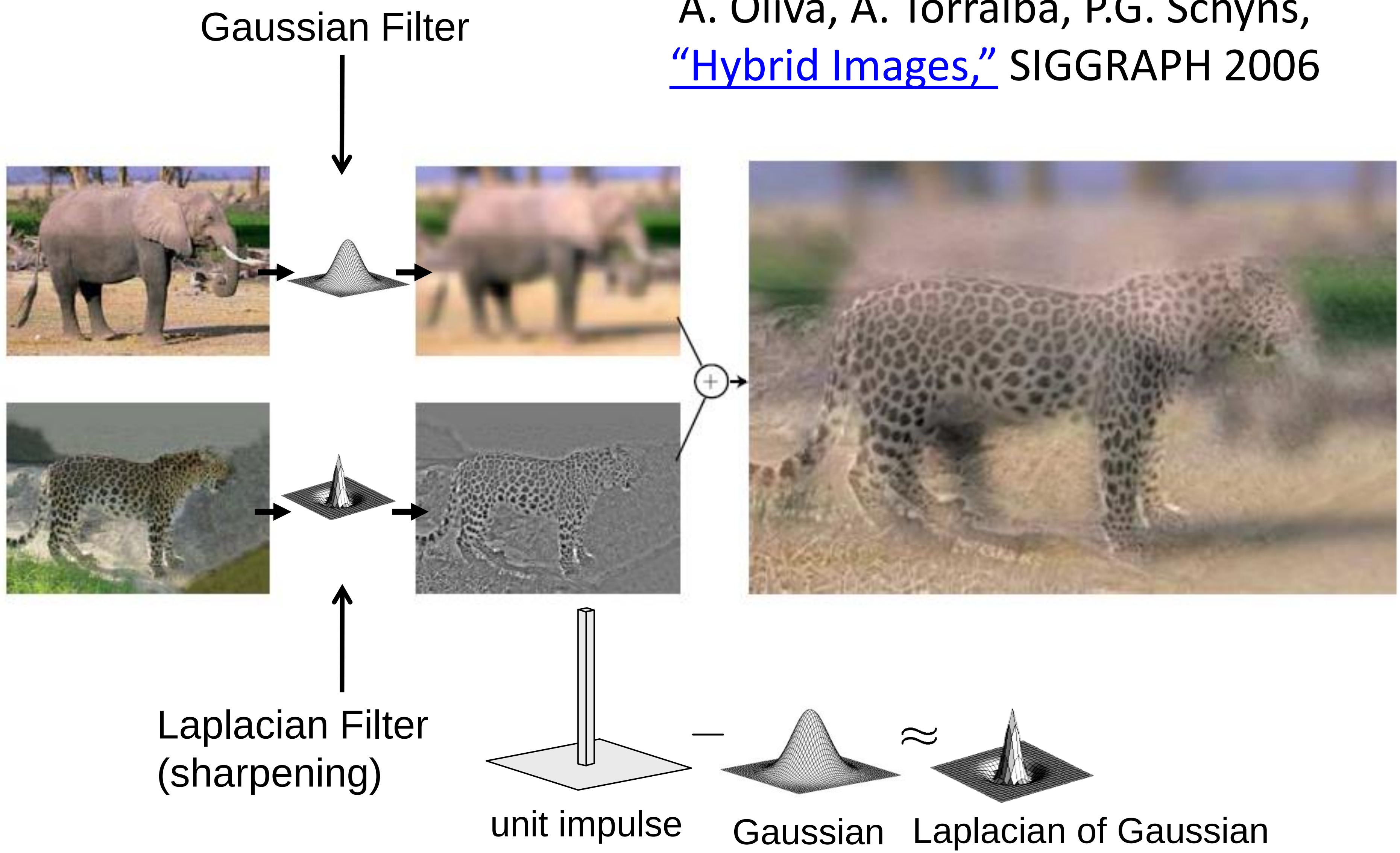
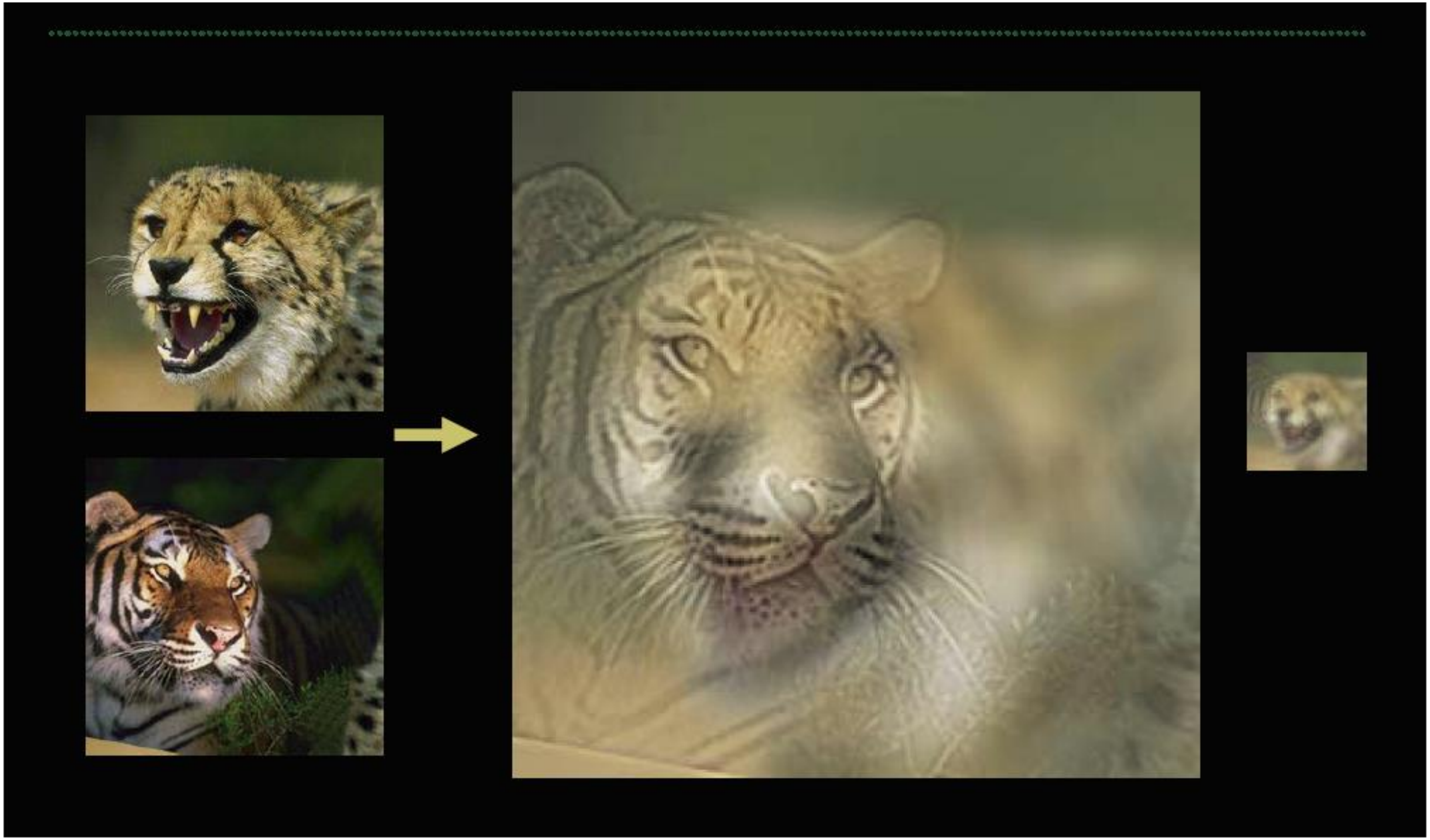
Aude Oliva & Antonio Torralba & Philippe G Schyns, SIGGRAPH 2006

Application: Hybrid Images

A. Oliva, A. Torralba, P.G. Schyns, [“Hybrid Images,”]() SIGGRAPH 2006



Application: Hybrid Images



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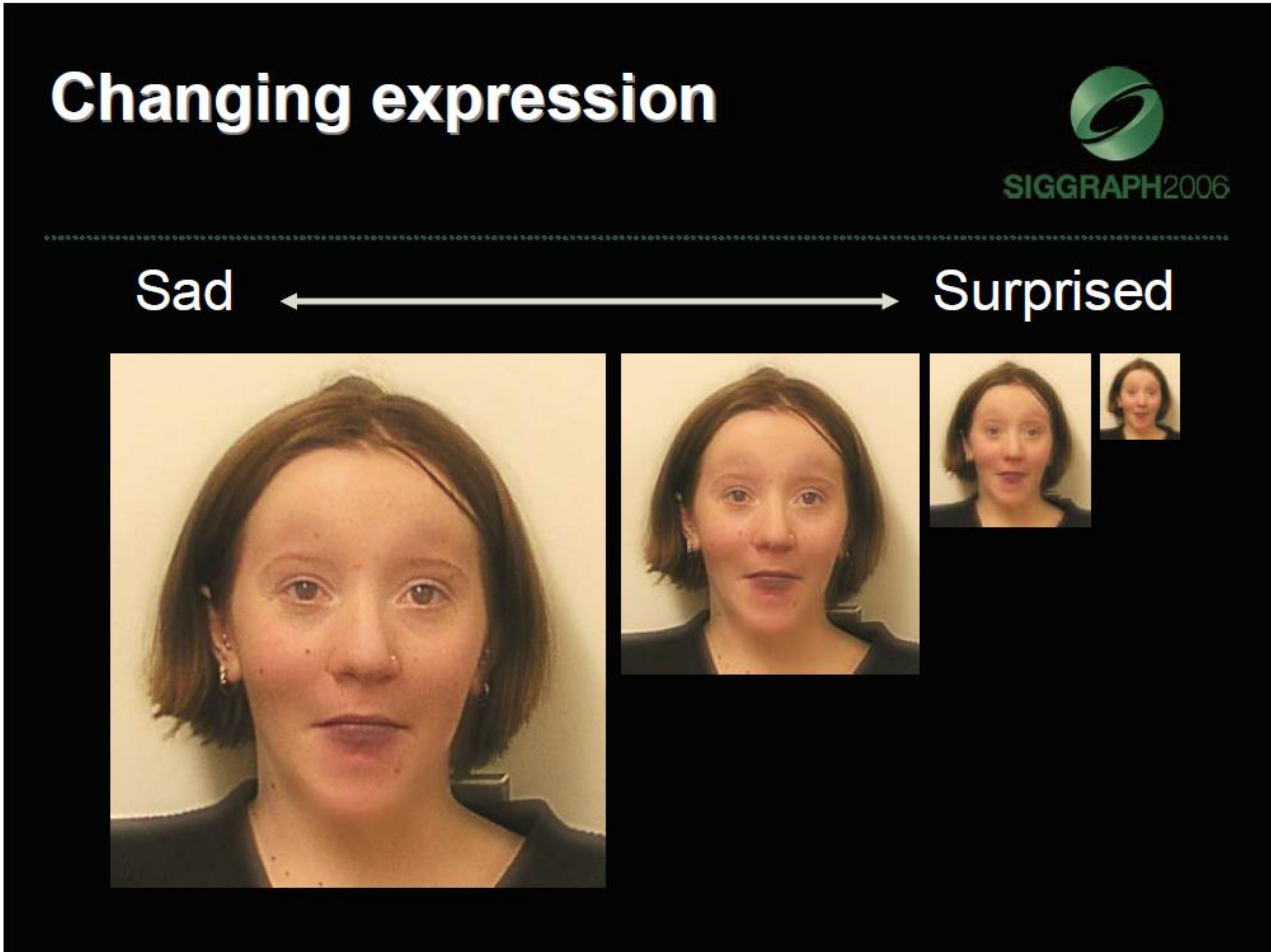
Aude Oliva & Antonio Torralba & Philippe G Schyns, SIGGRAPH 2006

Application: Hybrid Images



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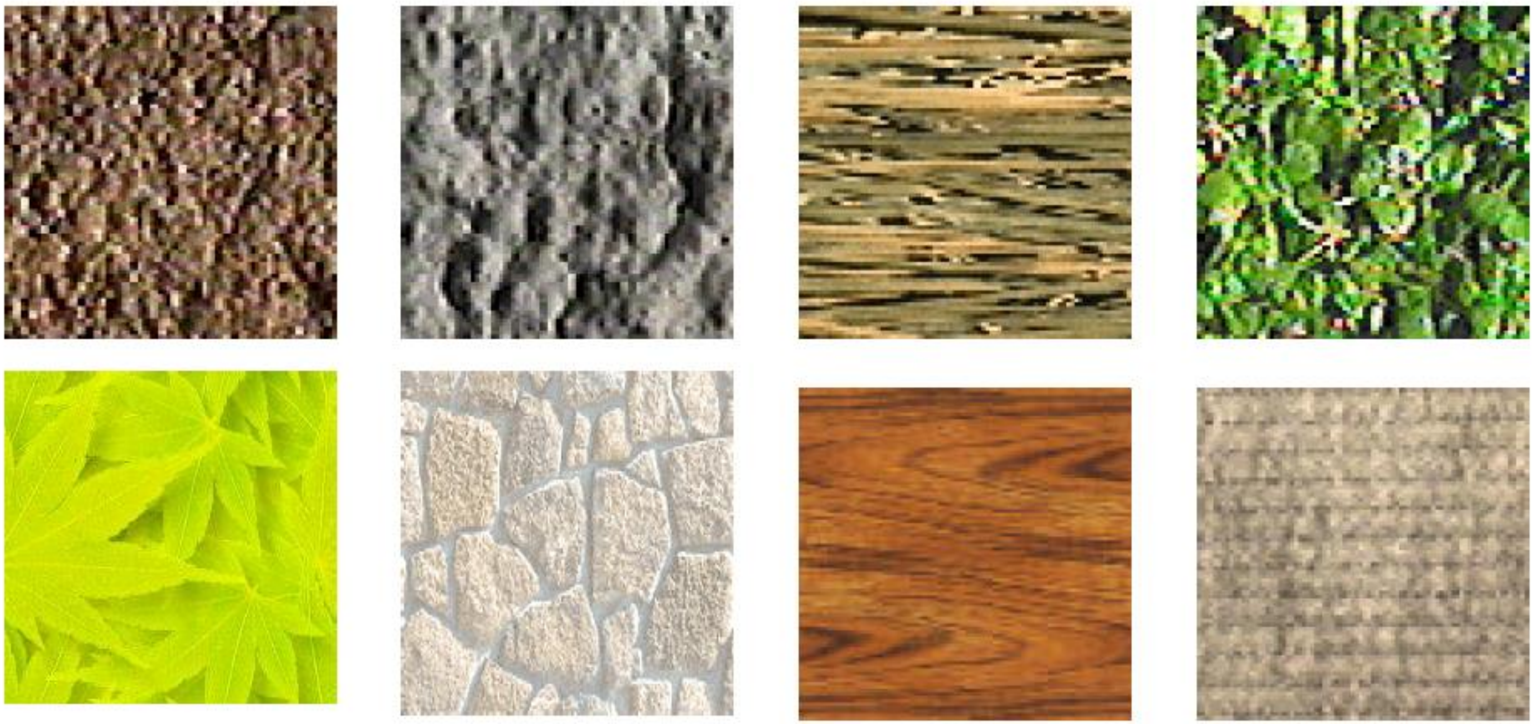
Aude Oliva & Antonio Torralba & Philippe G Schyns, SIGGRAPH 2006

Plan for this lecture

- Filters: math and properties
- Types of filters
  - Linear
    - Smoothing
    - Other
  - Non-linear
    - Median
- Texture representation with filters
- Anti-aliasing for image subsampling

Texture



Due to:

Patterns, marks, etches, blobs, holes, relief, etc.

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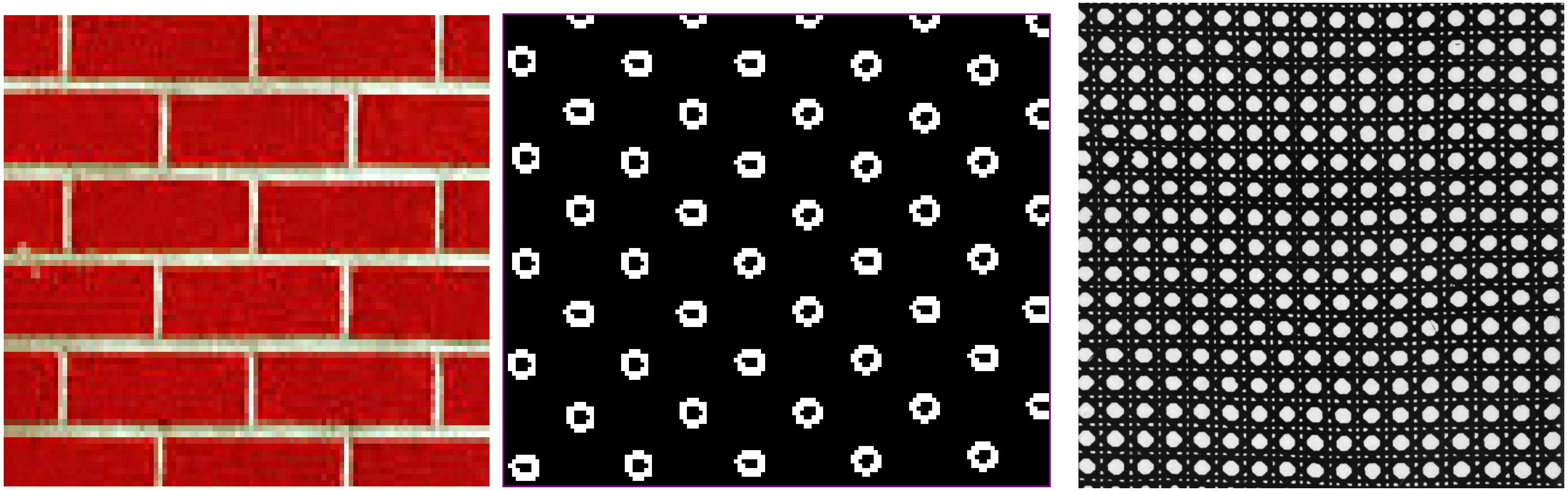
Includes: more regular patterns



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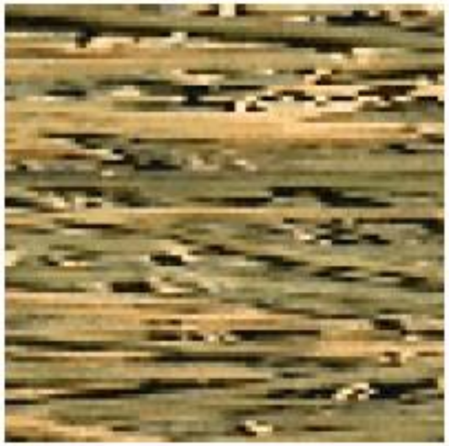
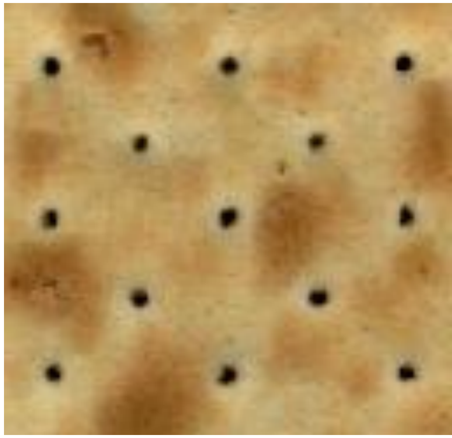
Includes: more random patterns



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http://animals.nationalgeographic.com/

Why analyze texture?

- Important for how we perceive objects
- Can be an important appearance cue that allows us to distinguish objects, especially if shape is similar across objects

Adapted from Kristen Grauman

Texture representation

- Textures are made up of repeated local patterns, so:
  - Find the patterns
    - Use filters that look like patterns (spots, bars, raw patches…)
    - Consider magnitude of response
  - Describe their statistics within each local window
    - E.g. mean, standard deviation

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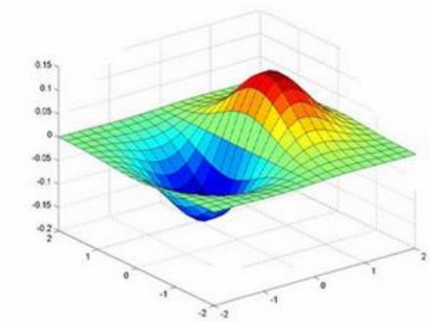
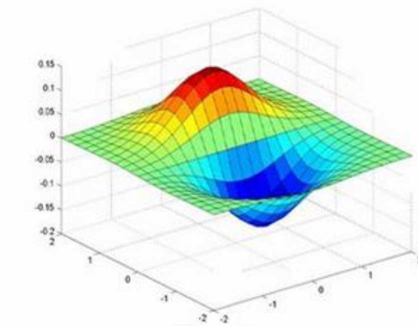
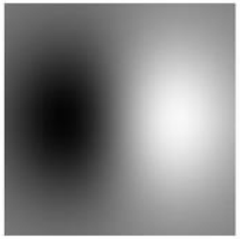
Derivative of Gaussian filter

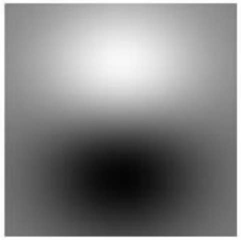


x-direction



y-direction

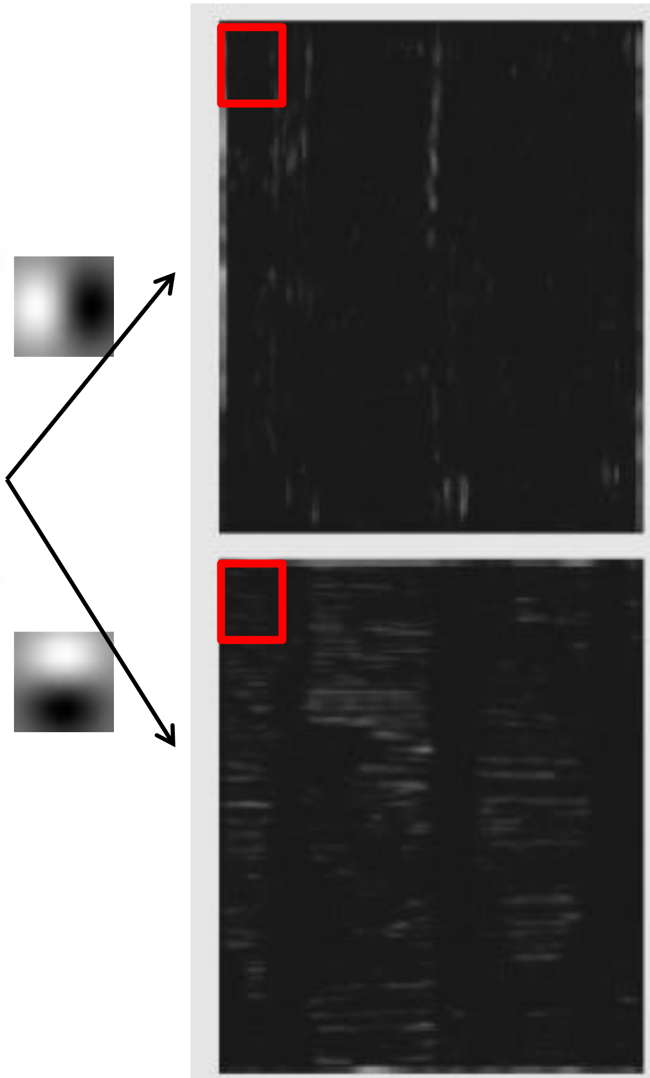




Figures from Noah Snavely

Texture representation: example

original image



derivative filter responses, squared

	mean d/dx value	**mean d/dy value**
Win. #1	4	10

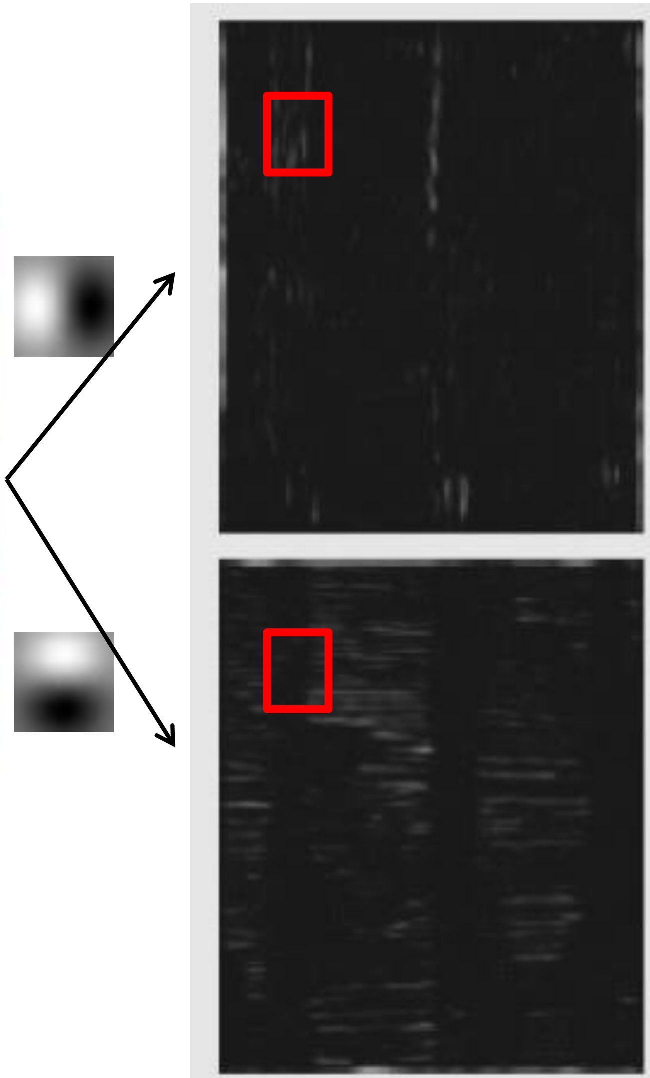
⋮

statistics to summarize patterns in small windows

Kristen Grauman

Texture representation: example

original image



derivative filter responses, squared

	mean d/dx value	**mean d/dy value**
Win. #1	4	10
Win.#2	18	7

⋮

statistics to summarize patterns in small windows

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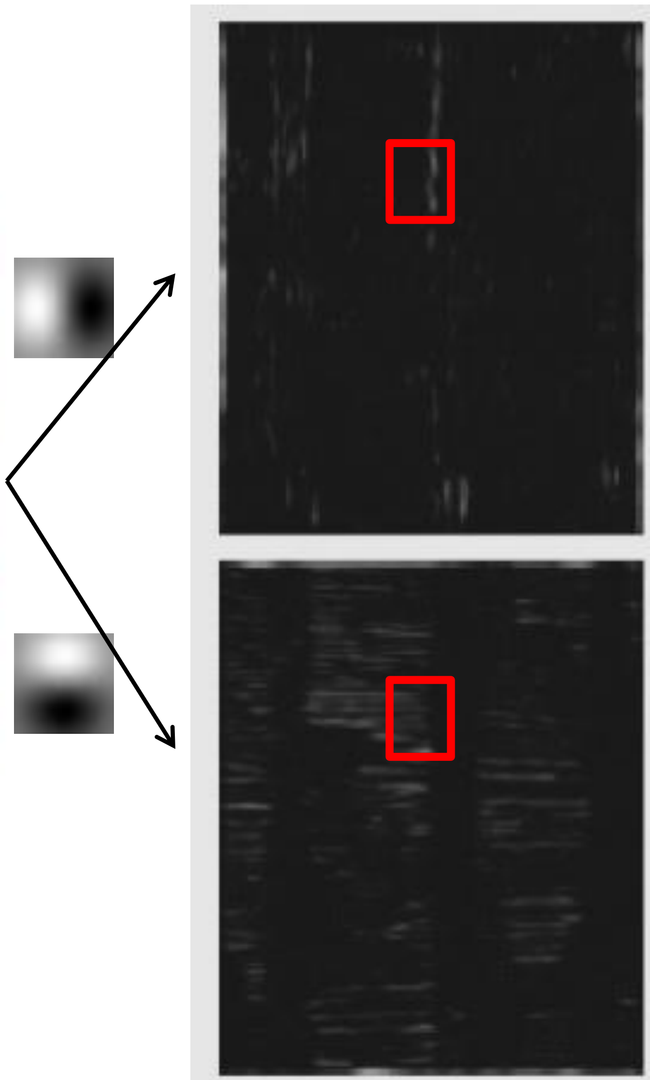
Texture representation: example

original image



derivative filter responses, squared

	mean d/dx value	mean d/dy value
Win. #1	4	10
Win.#2	18	7
⋮		
Win.#9	20	20

⋮

statistics to summarize patterns in small windows

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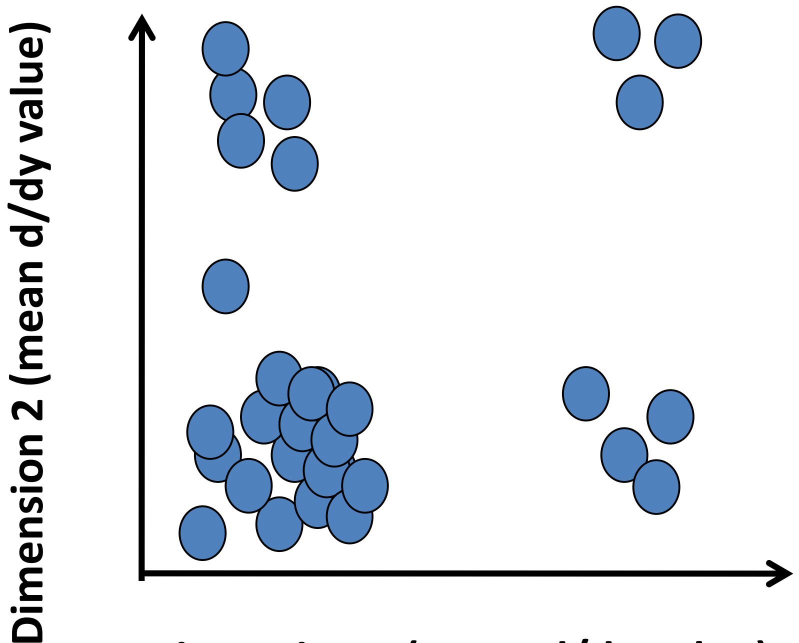
Texture representation: example



	mean d/dx value	mean d/dy value
Win. #1	4	10
Win.#2	18	7
⋮		
Win.#9	20	20

⋮

statistics to summarize patterns in small windows

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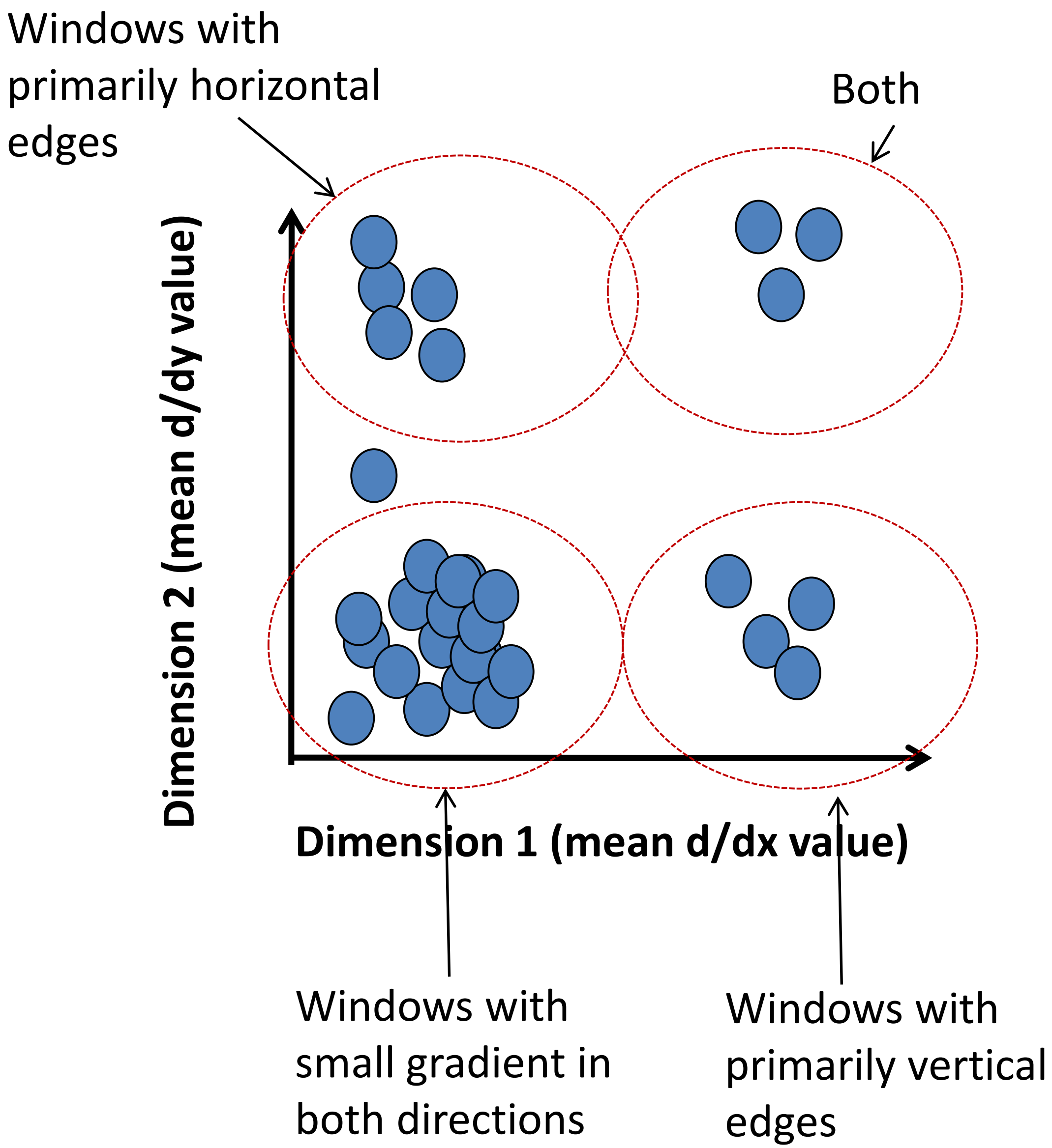
Texture representation: example



	mean d/dx value	mean d/dy value
Win. #1	4	10
Win.#2	18	7
⋮		
Win.#9	20	20

⋮

statistics to summarize patterns in small windows

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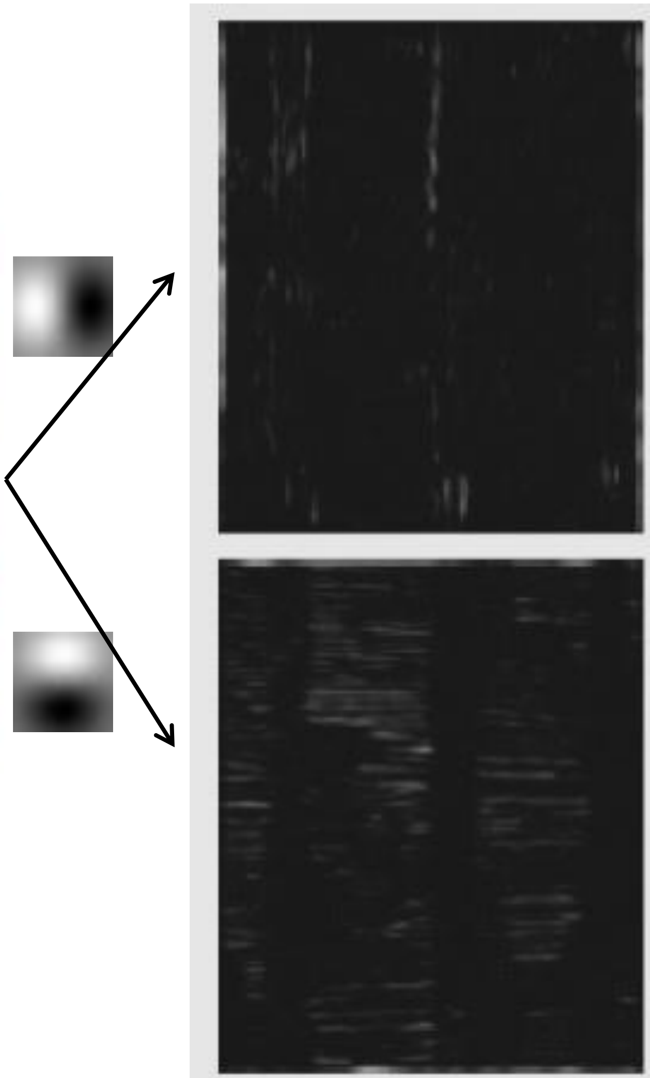
Texture representation: example

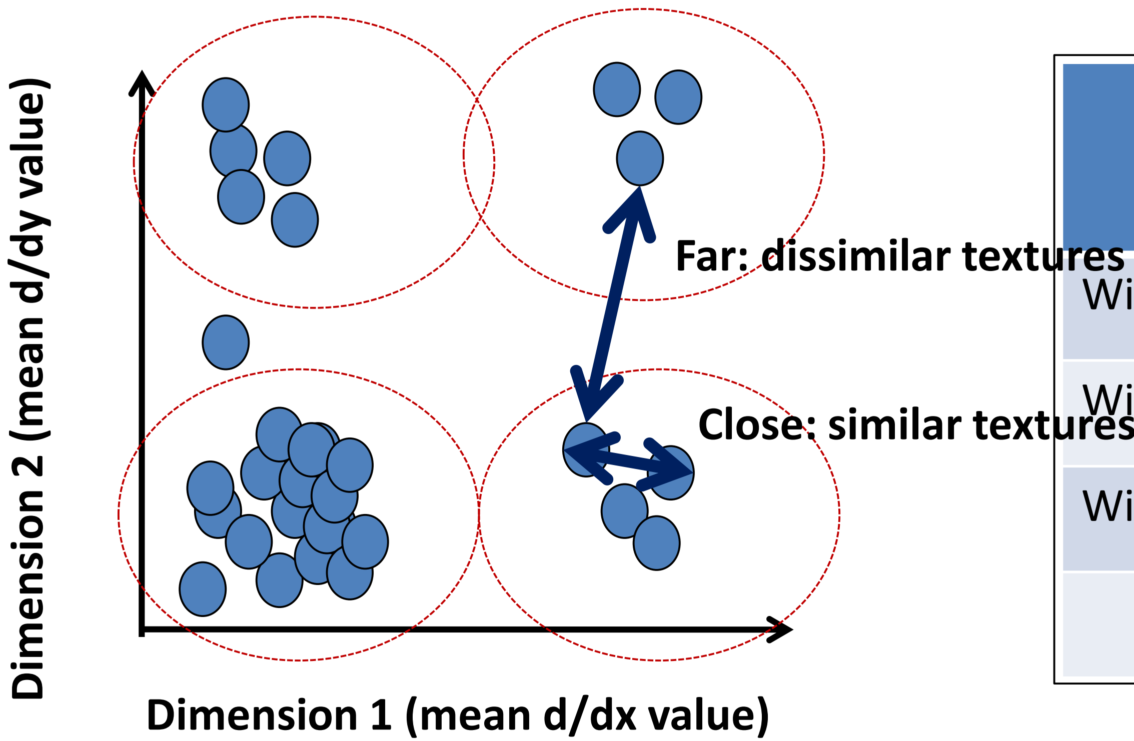
original image



derivative filter responses, squared

visualization of the assignment to texture "types"

Texture representation: example



	mean d/dx value	mean d/dy value
Win. #1	4	10
Win.#2	18	7
⋮		
Win.#9	20	20

⋮

statistics to summarize patterns in small windows

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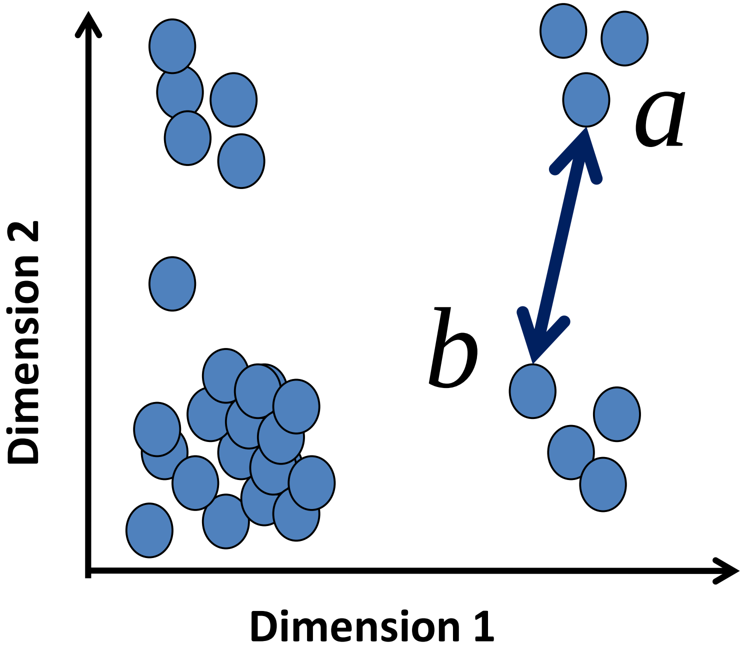
Computing distances using texture



$$D(a,b) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$$

$$= \sqrt{\sum_{i=1}^{d} (a_i - b_i)^2}$$

Euclidean distance (L_2)

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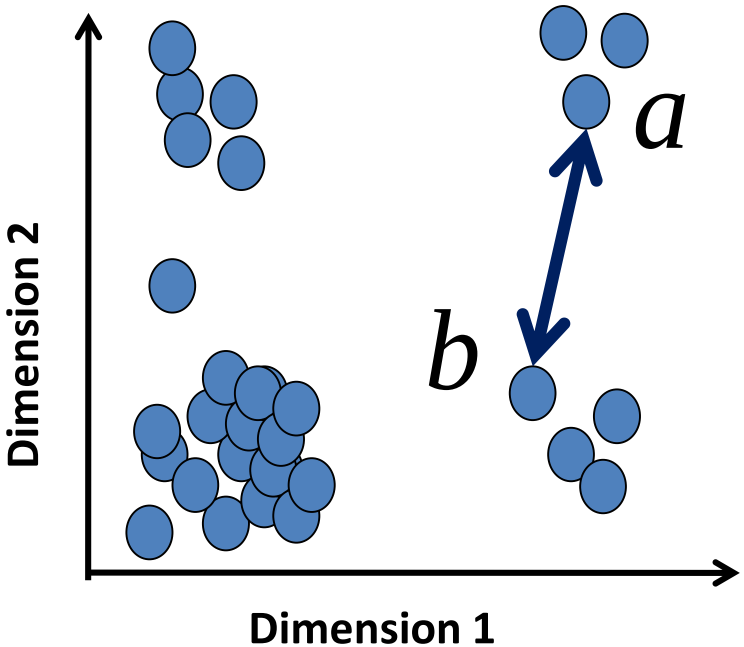
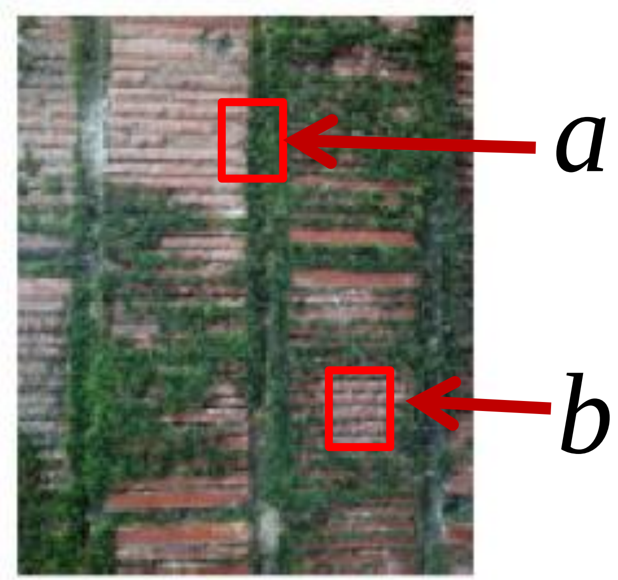
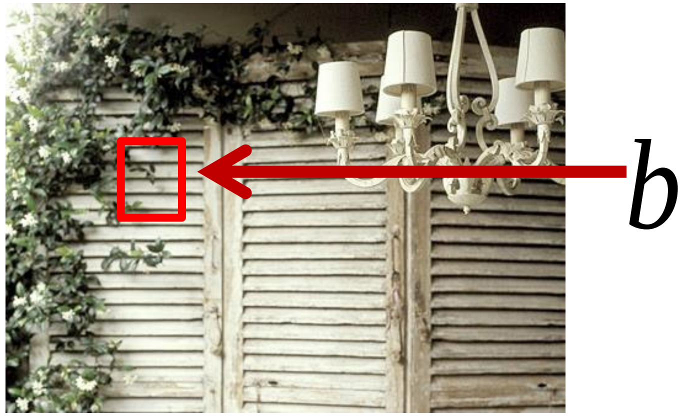
Texture representation: example







Distance reveals how dissimilar texture from window a is from texture in window b.

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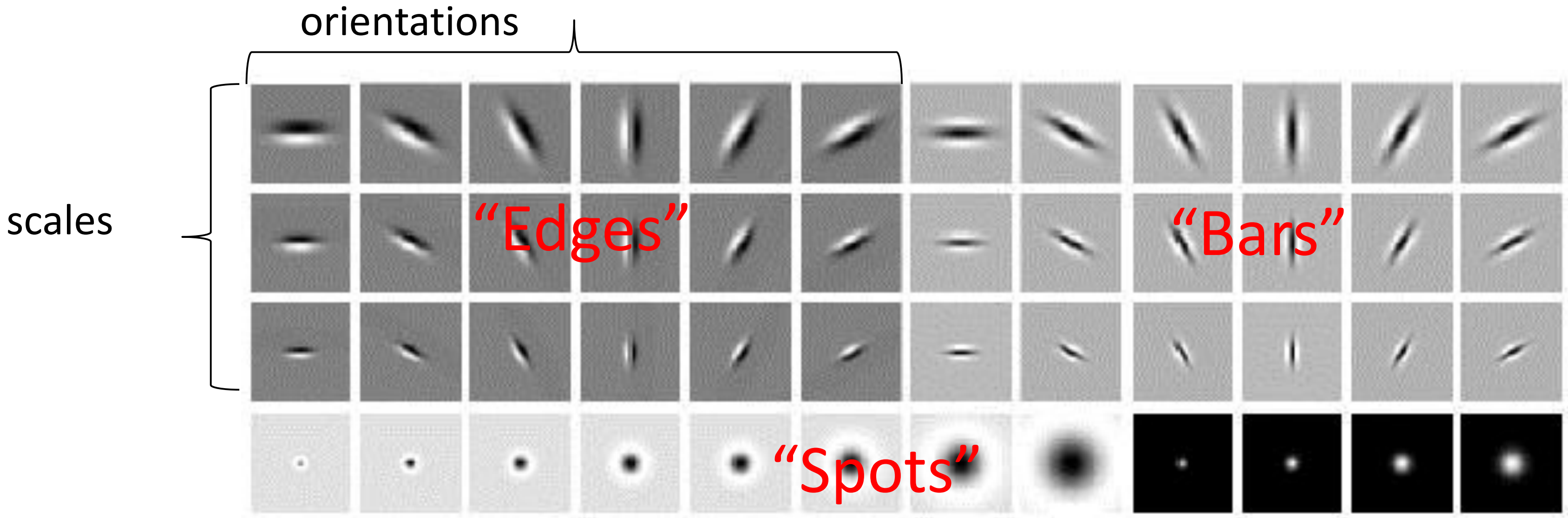
Filter banks

- Our previous example used two filters, and resulted in a 2-dimensional feature vector to describe texture in a window.
  - x and y derivatives revealed something about local structure.
- We can generalize to apply a collection of multiple (*d*) filters: a “filter bank”
- Then our feature vectors will be *d*-dimensional.

Adapted from Kristen Grauman

Filter banks

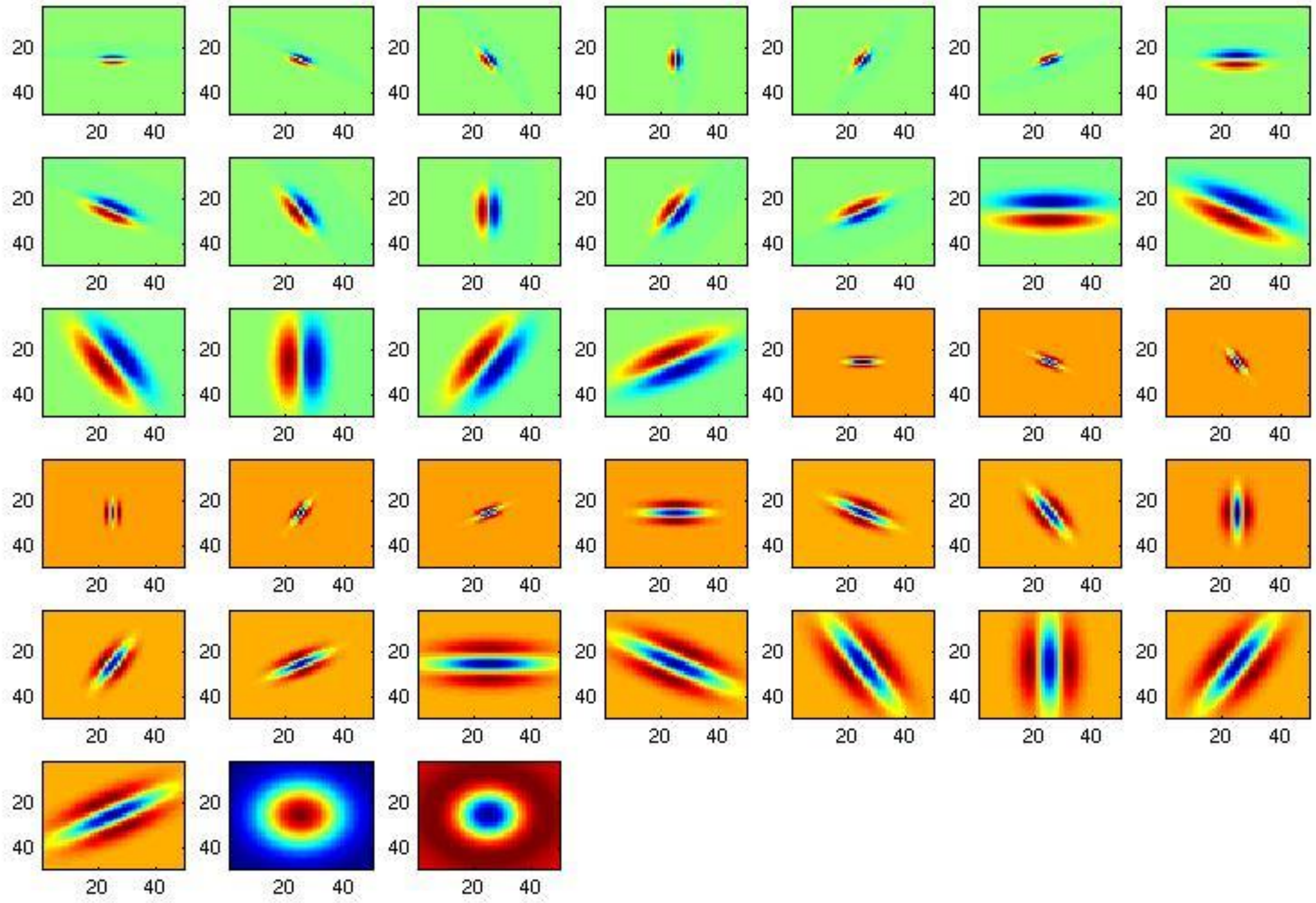


- What filters to put in the bank?
  - Typically we want a combination of scales and orientations, different types of patterns.

Matlab code available for these examples:
http://www.robots.ox.ac.uk/~vgg/research/texclass/filters.html

Kristen Grauman

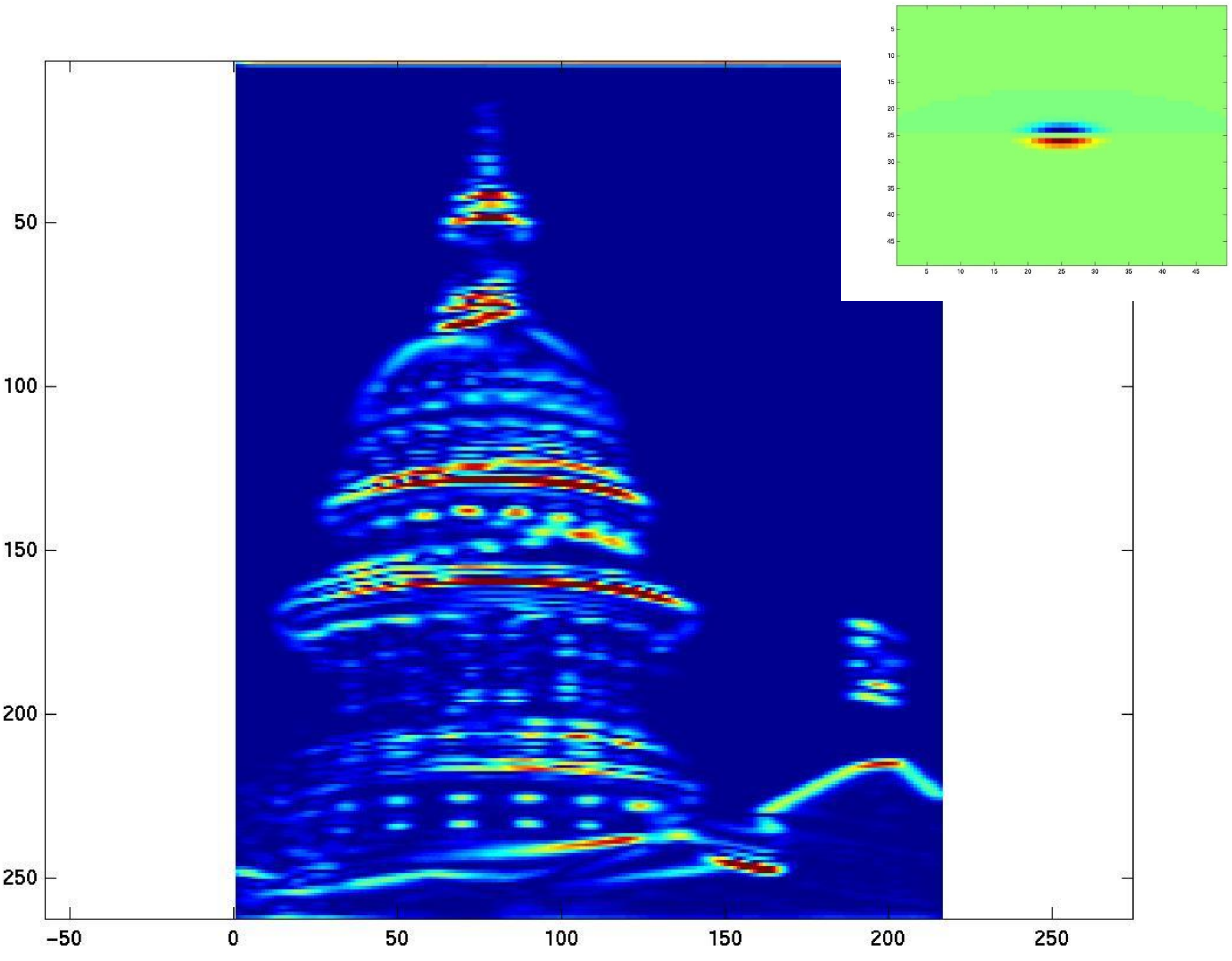
Filter bank



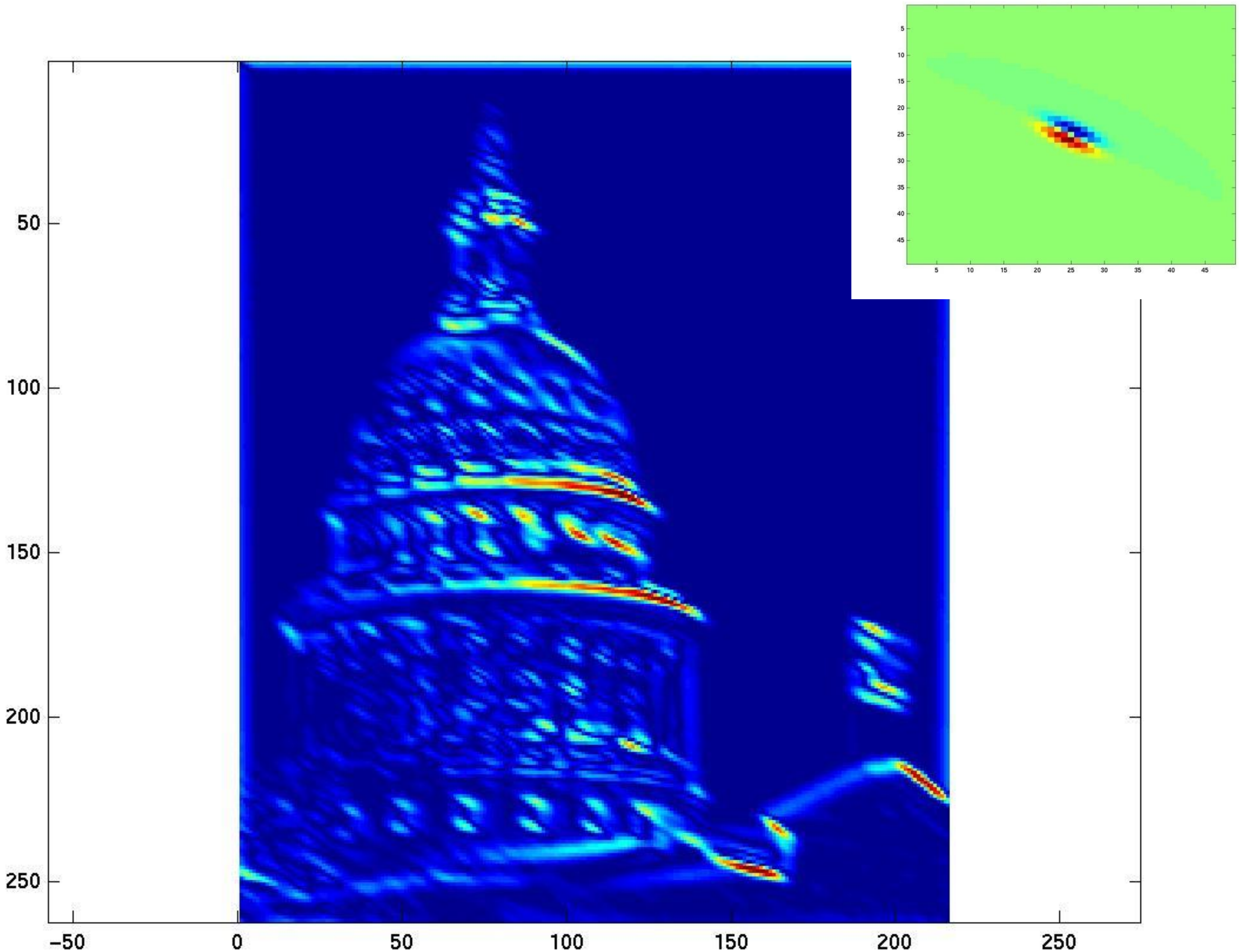
Kristen Grauman

Image from http://www.texasexplorer.com/austincap2.jpg

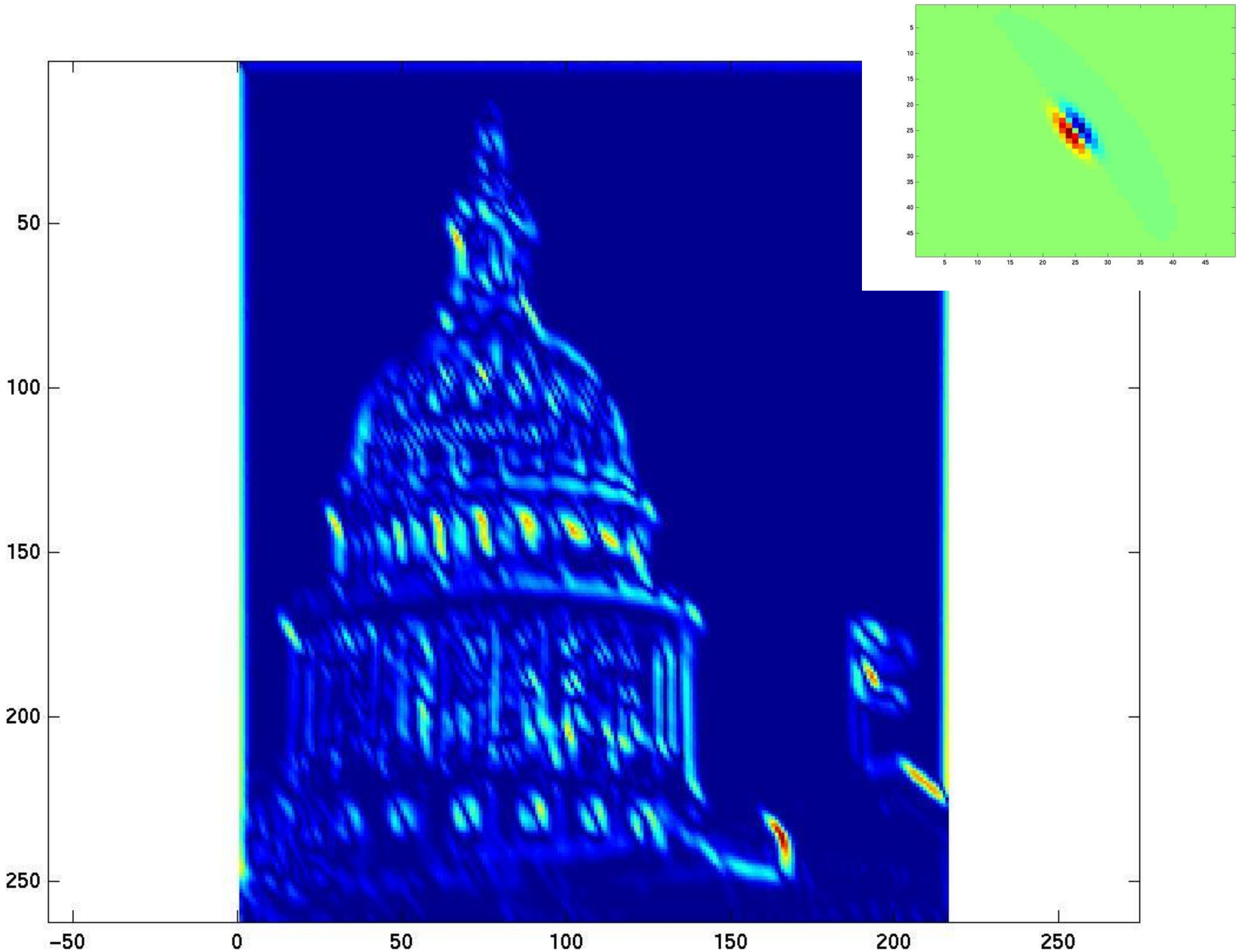
Kristen Grauman



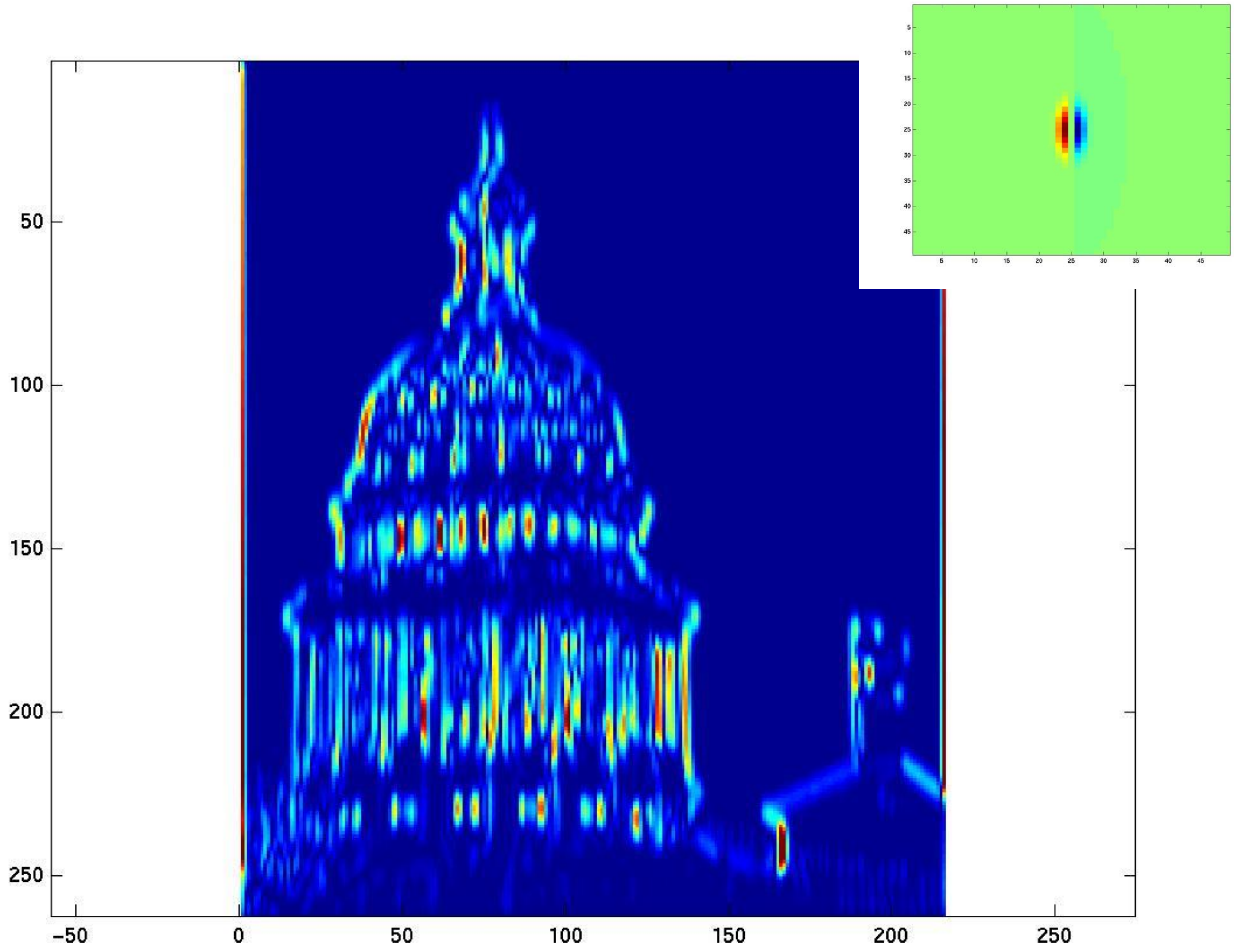
Kristen Grauman



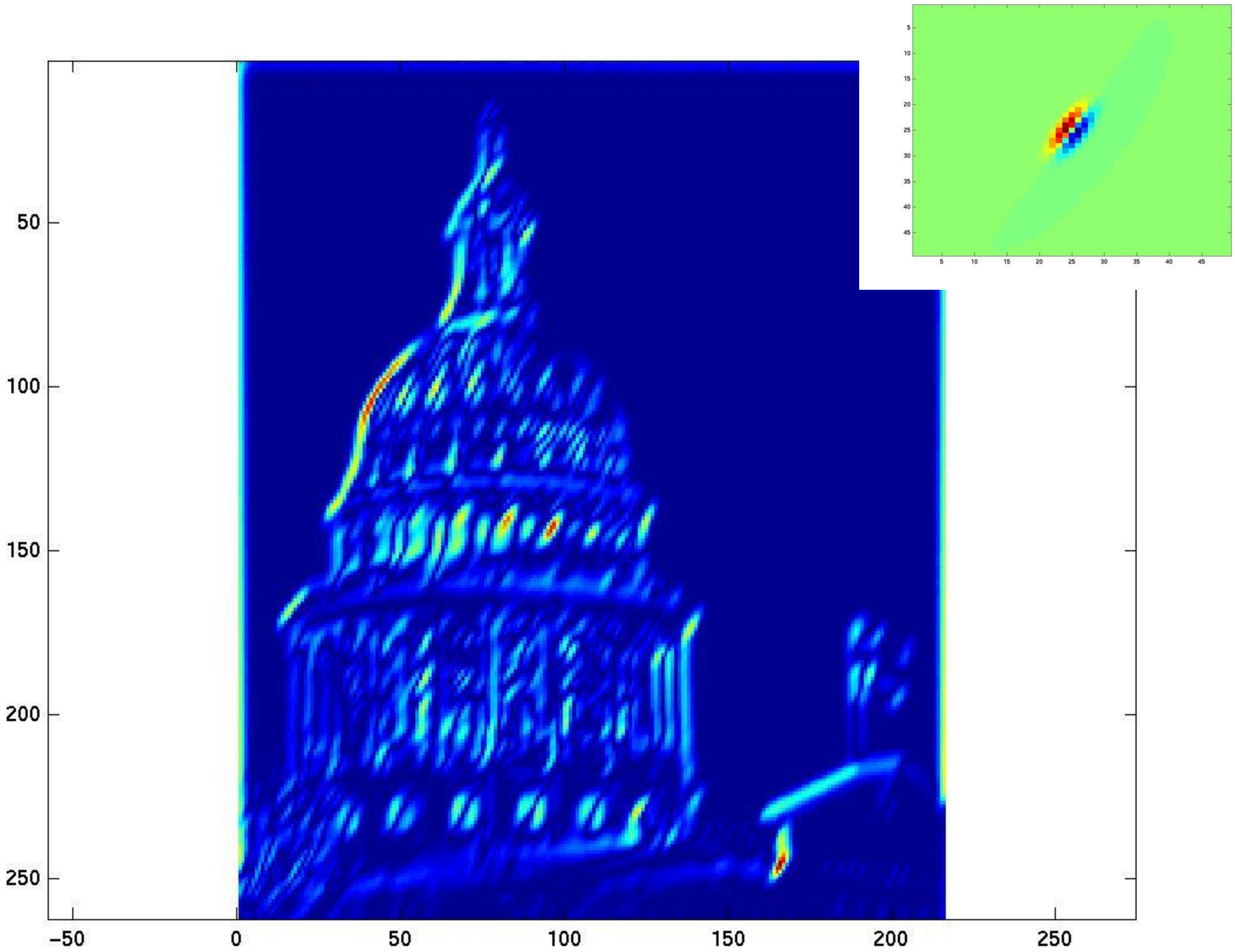
Kristen Grauman



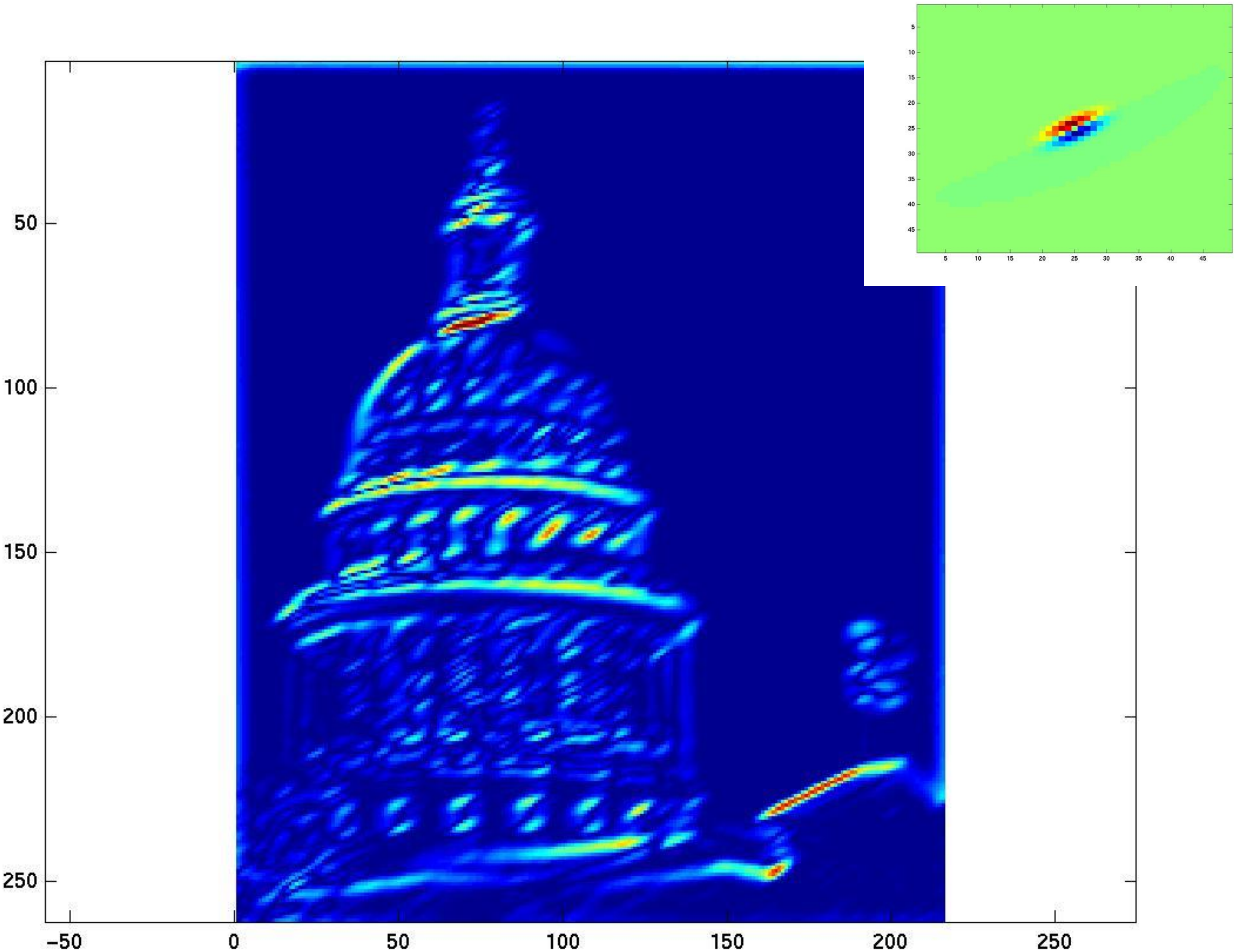
Kristen Grauman



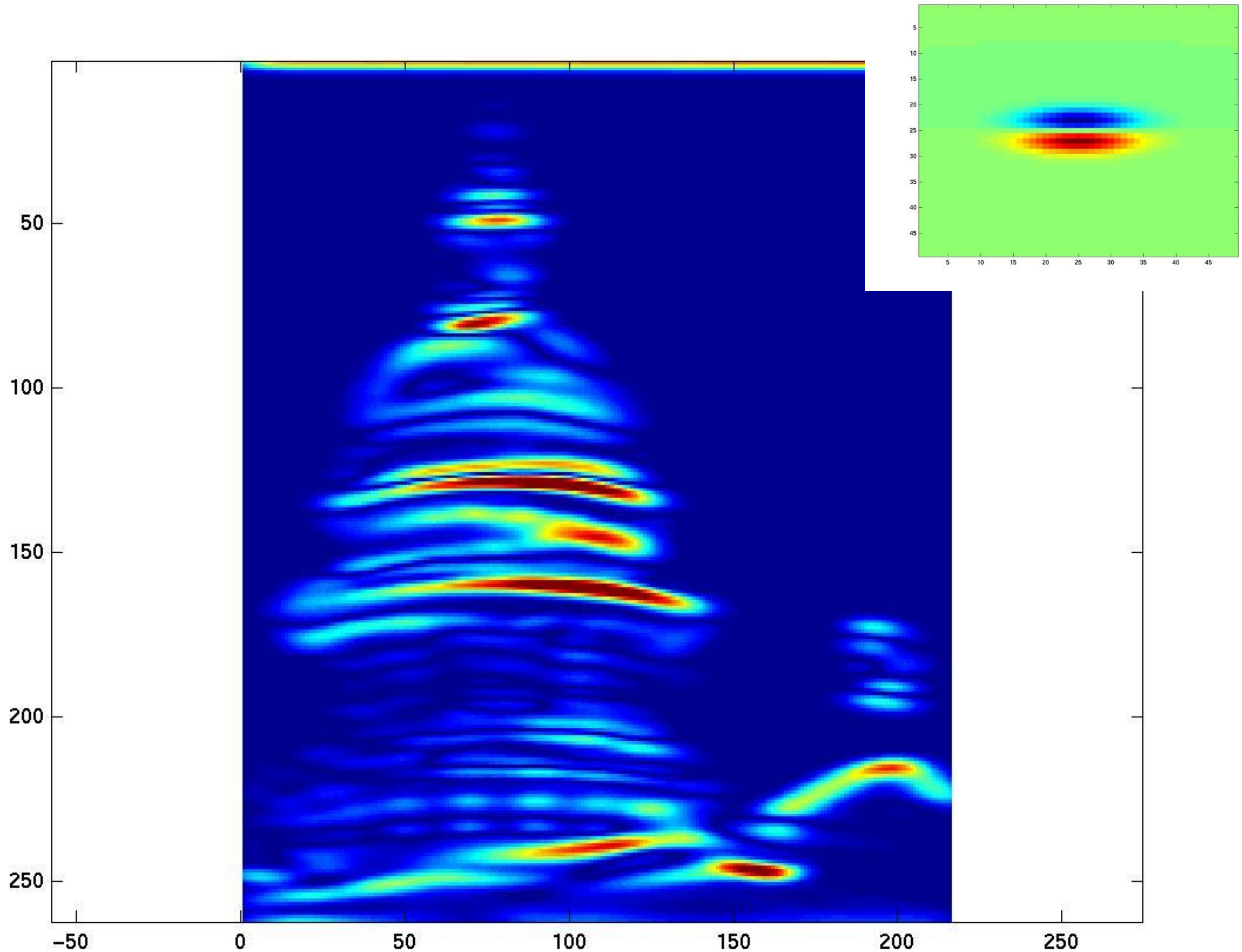
Kristen Grauman



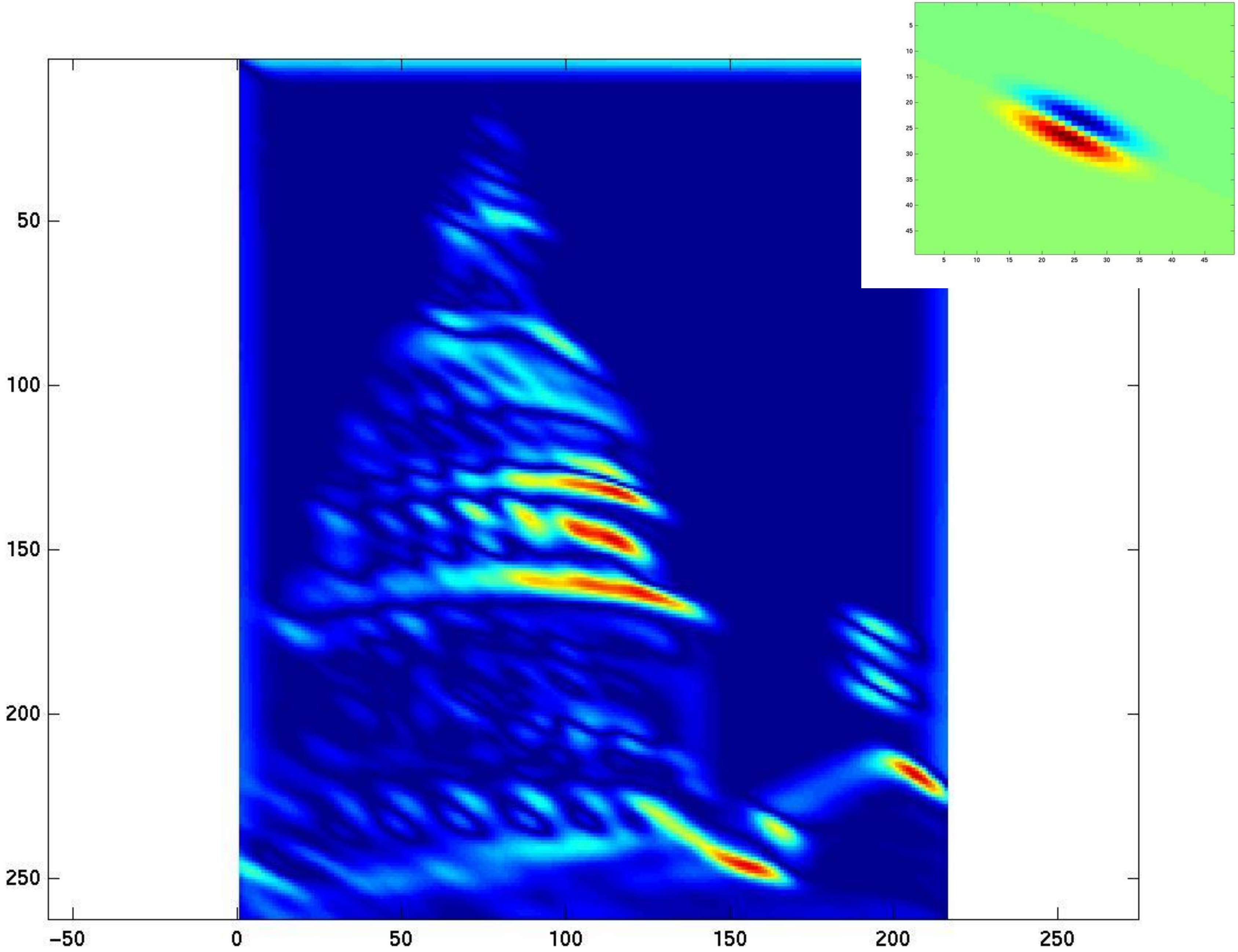
Kristen Grauman



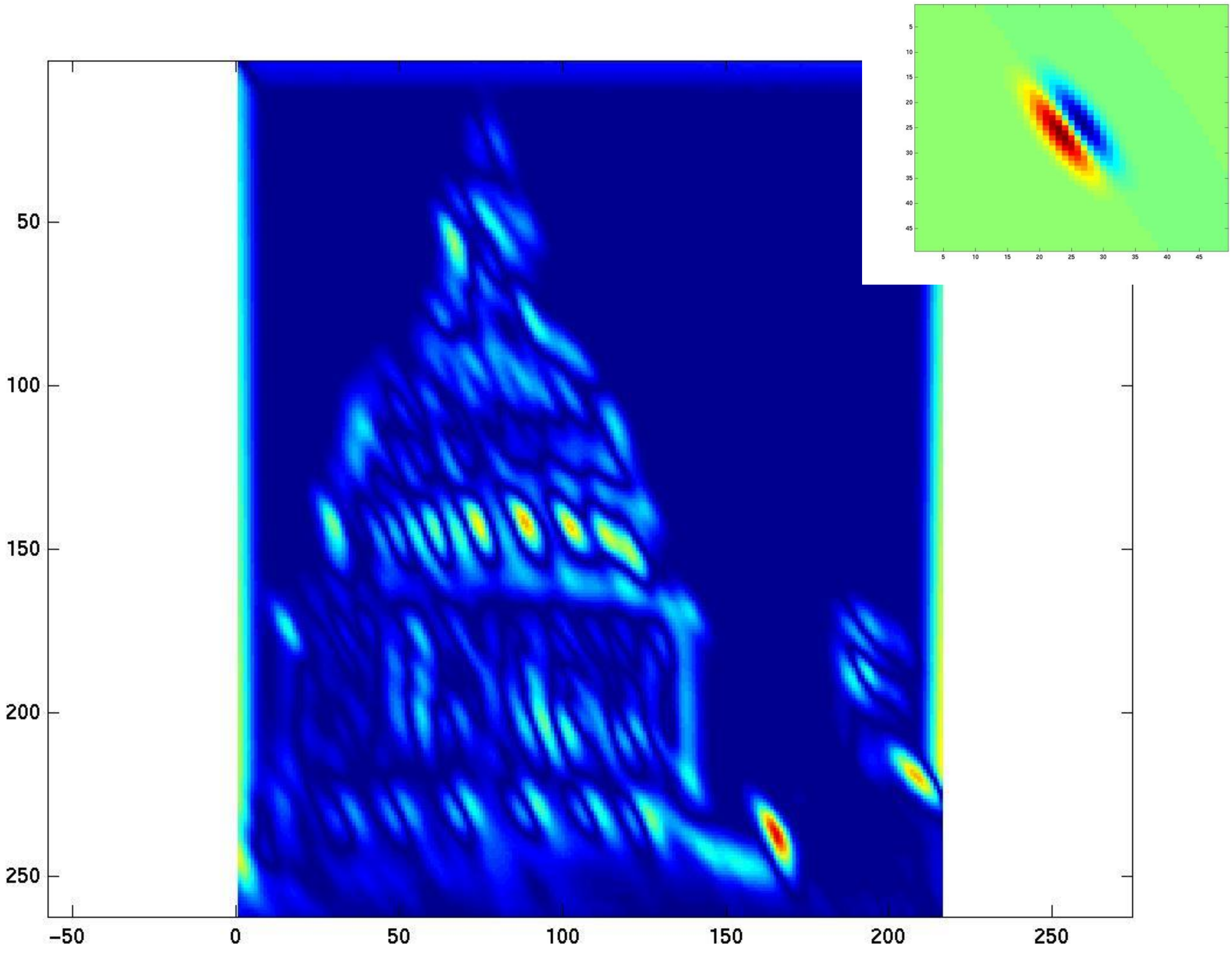
Kristen Grauman



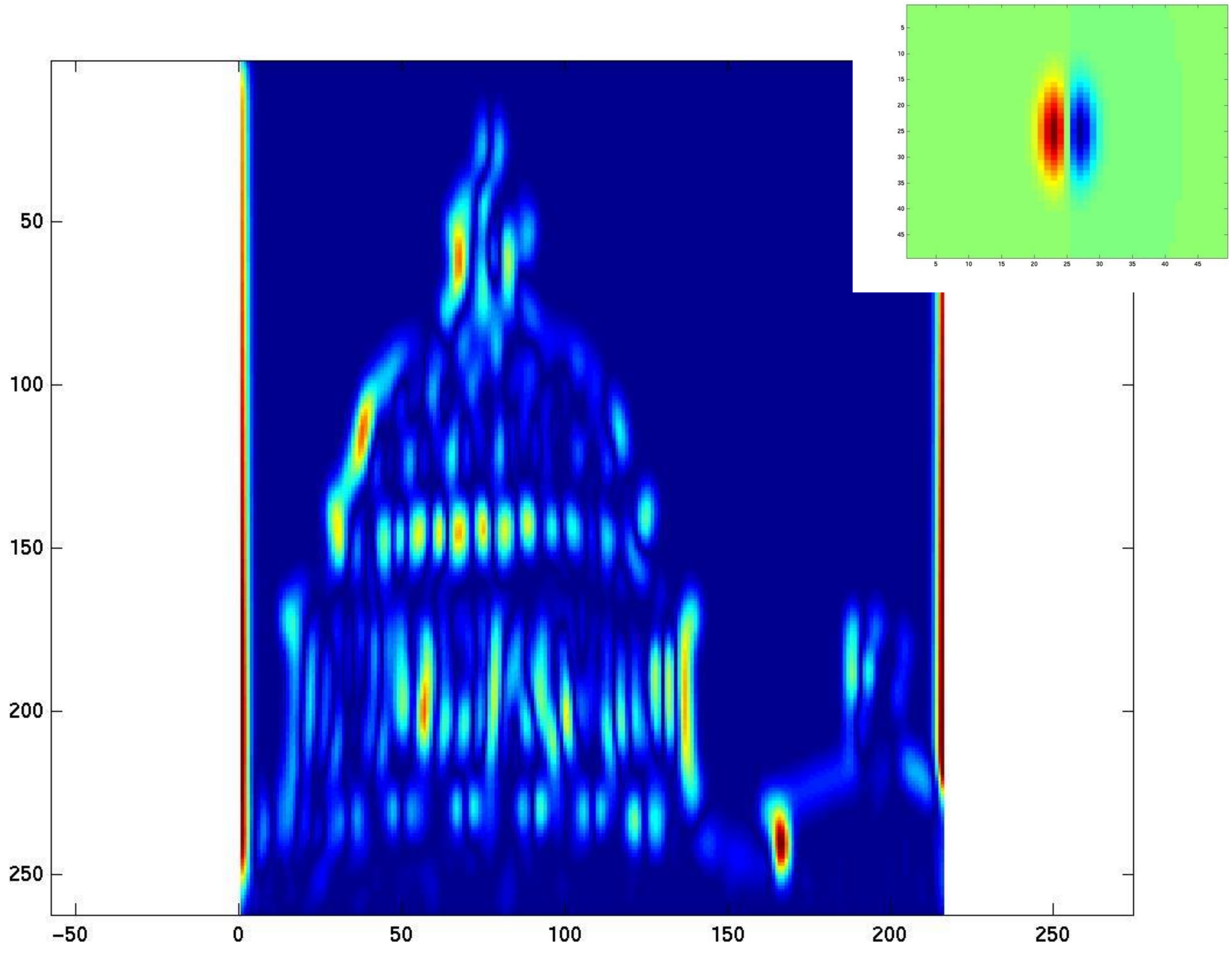
Kristen Grauman



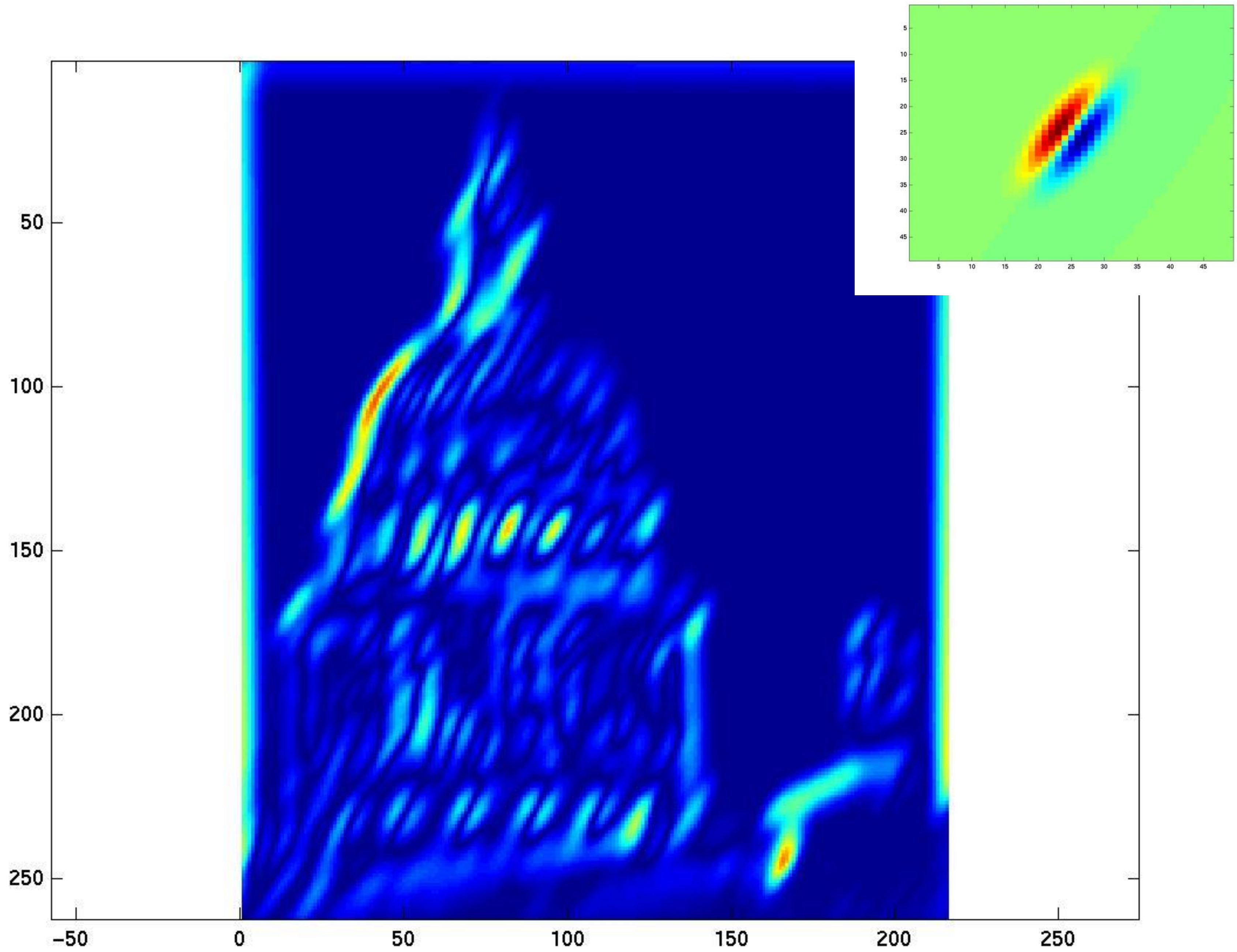
Kristen Grauman



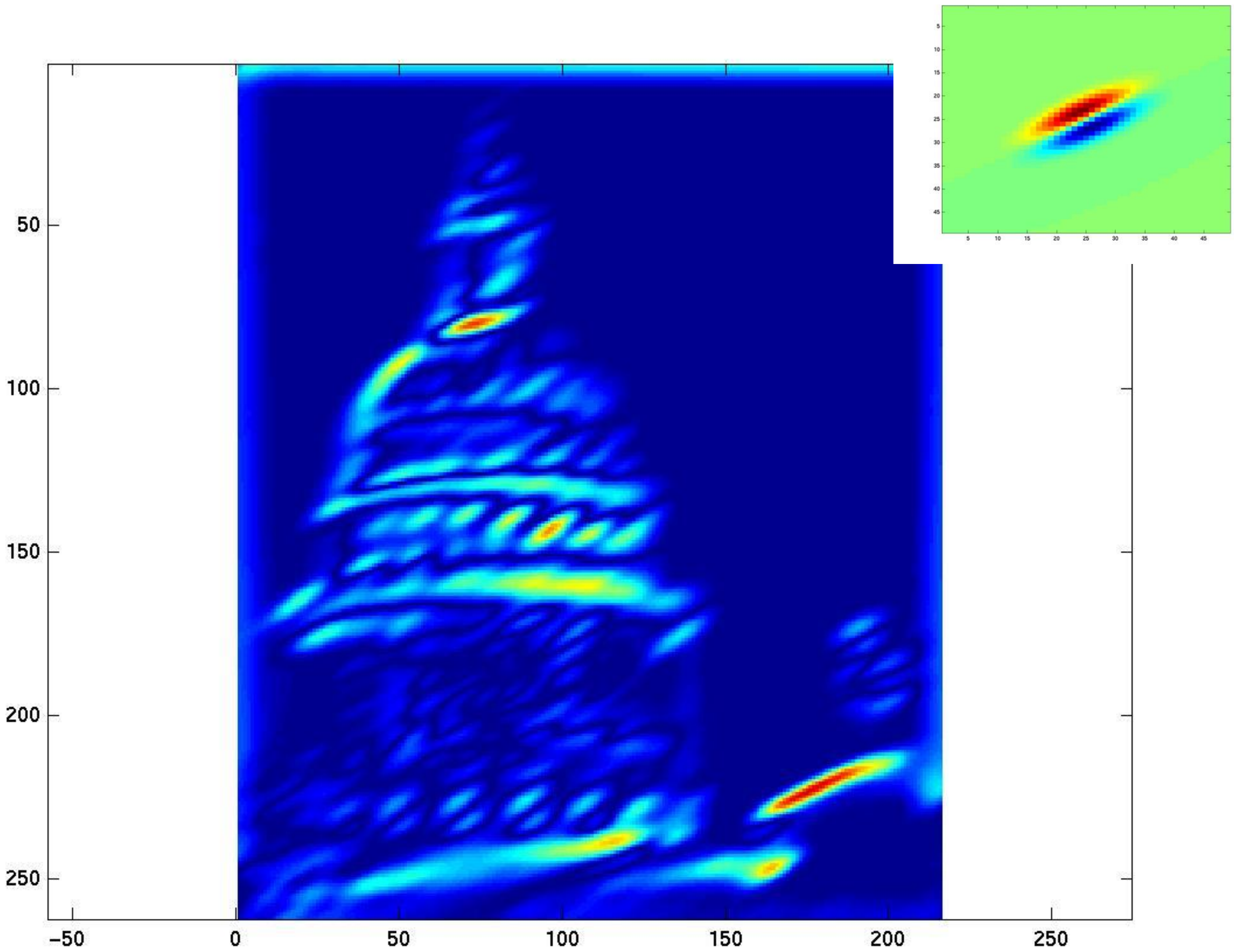
Kristen Grauman



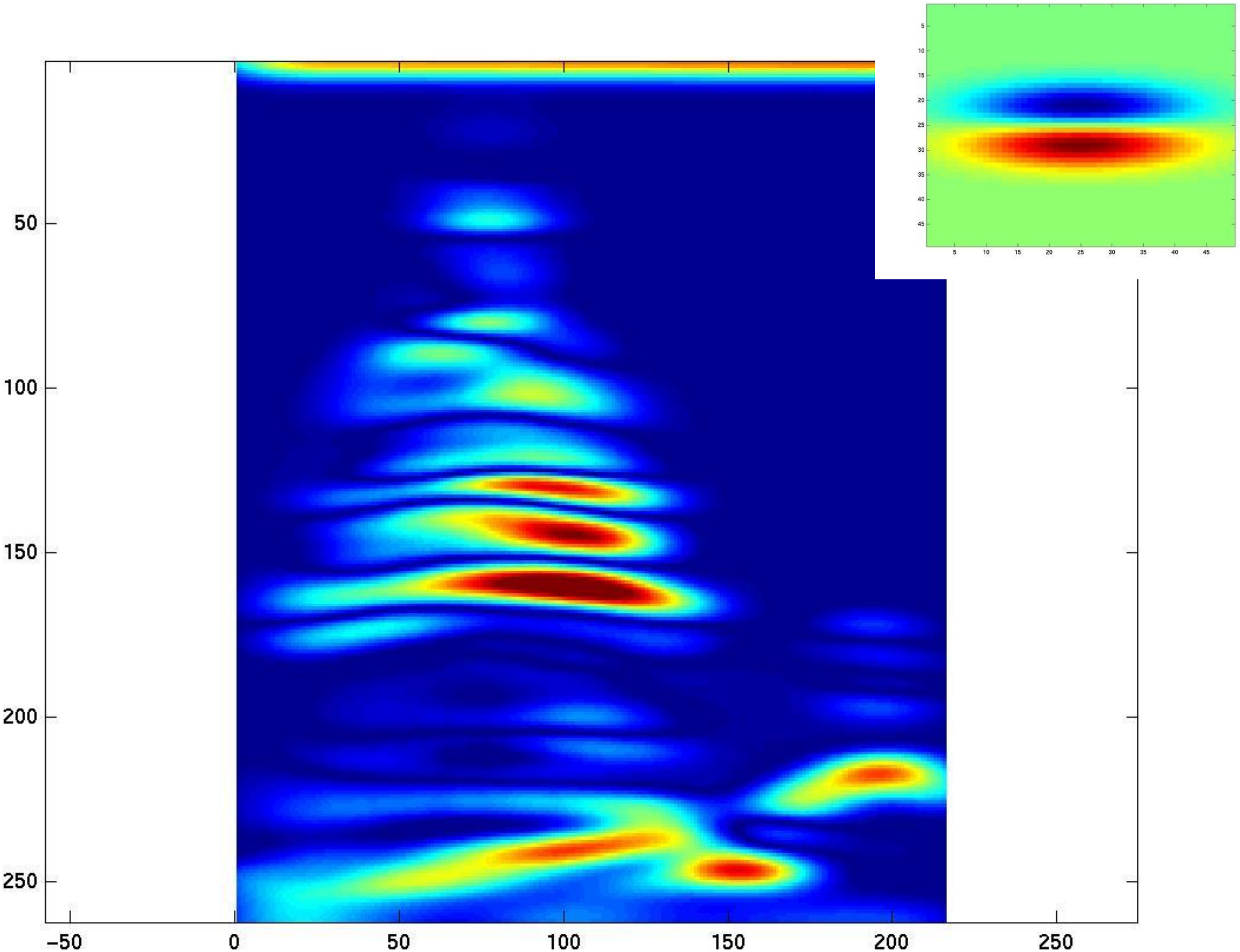
Kristen Grauman



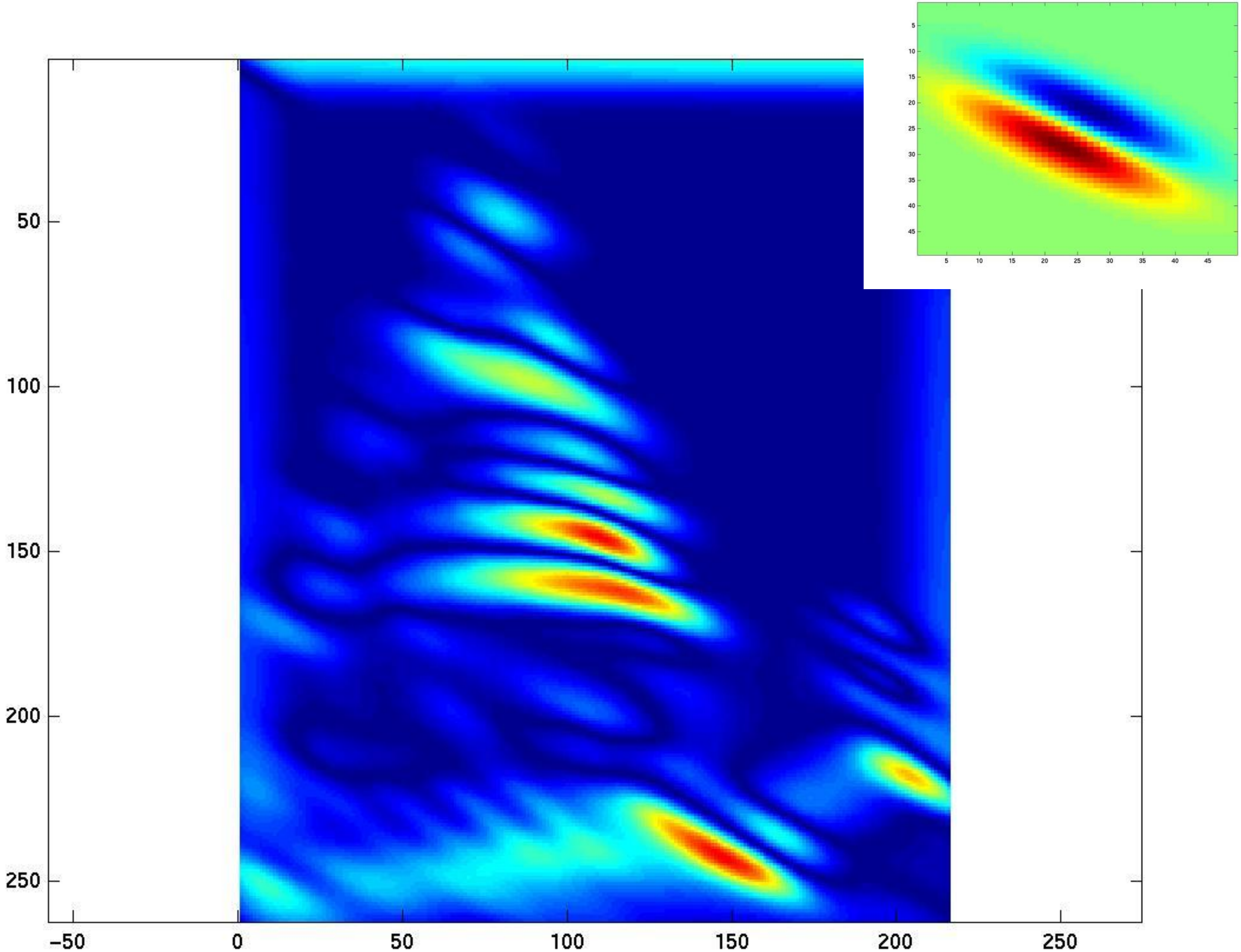
Kristen Grauman



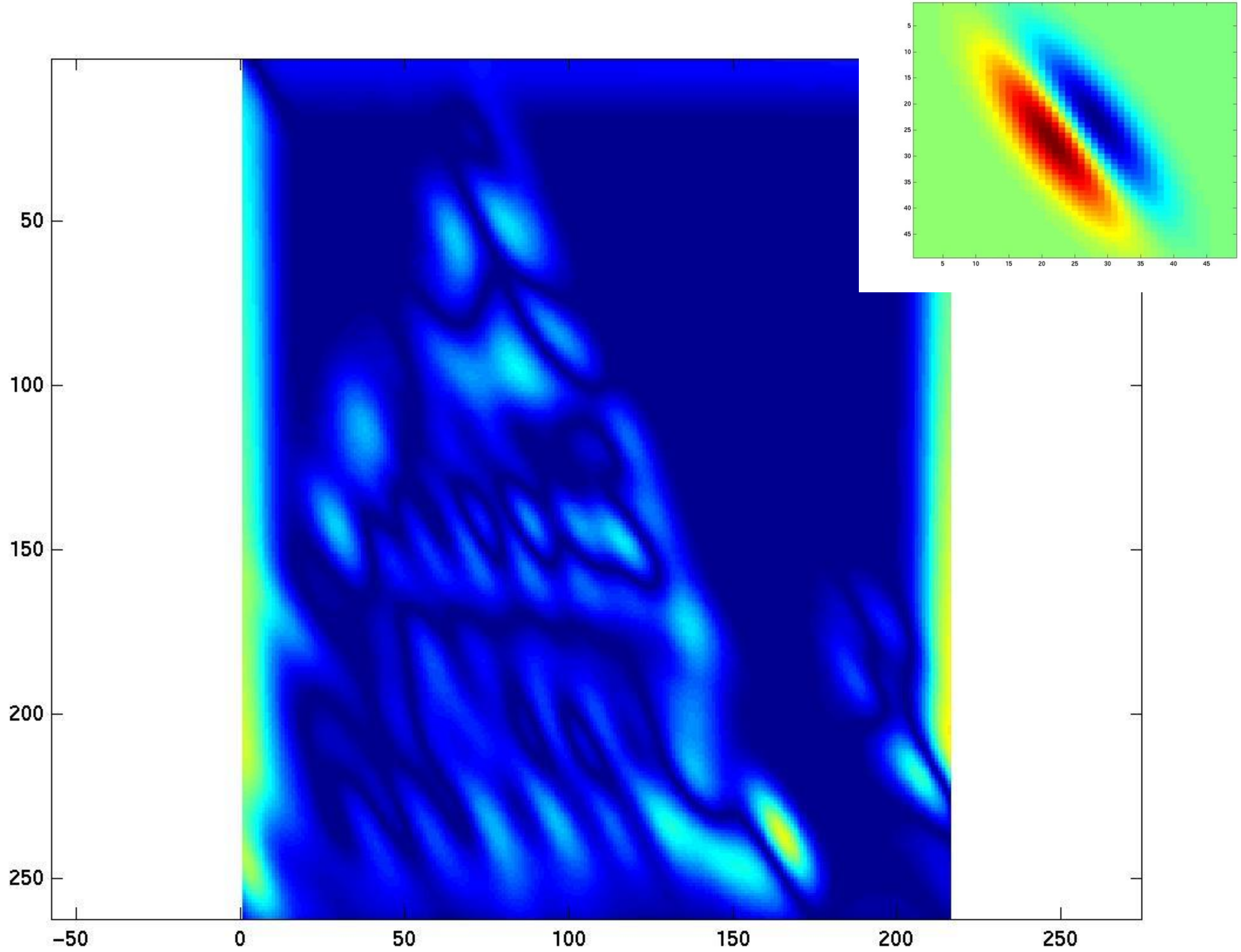
Kristen Grauman



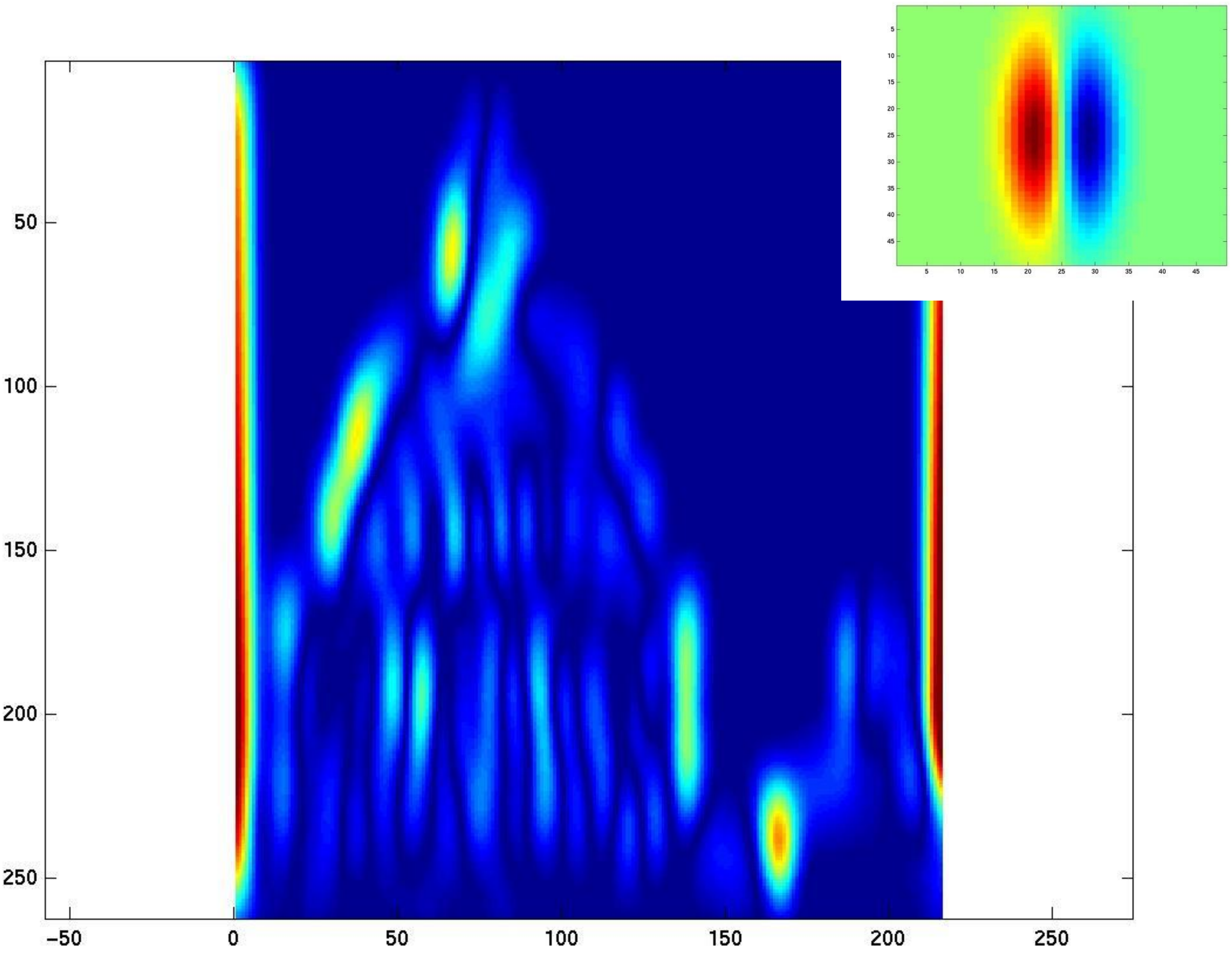
Kristen Grauman



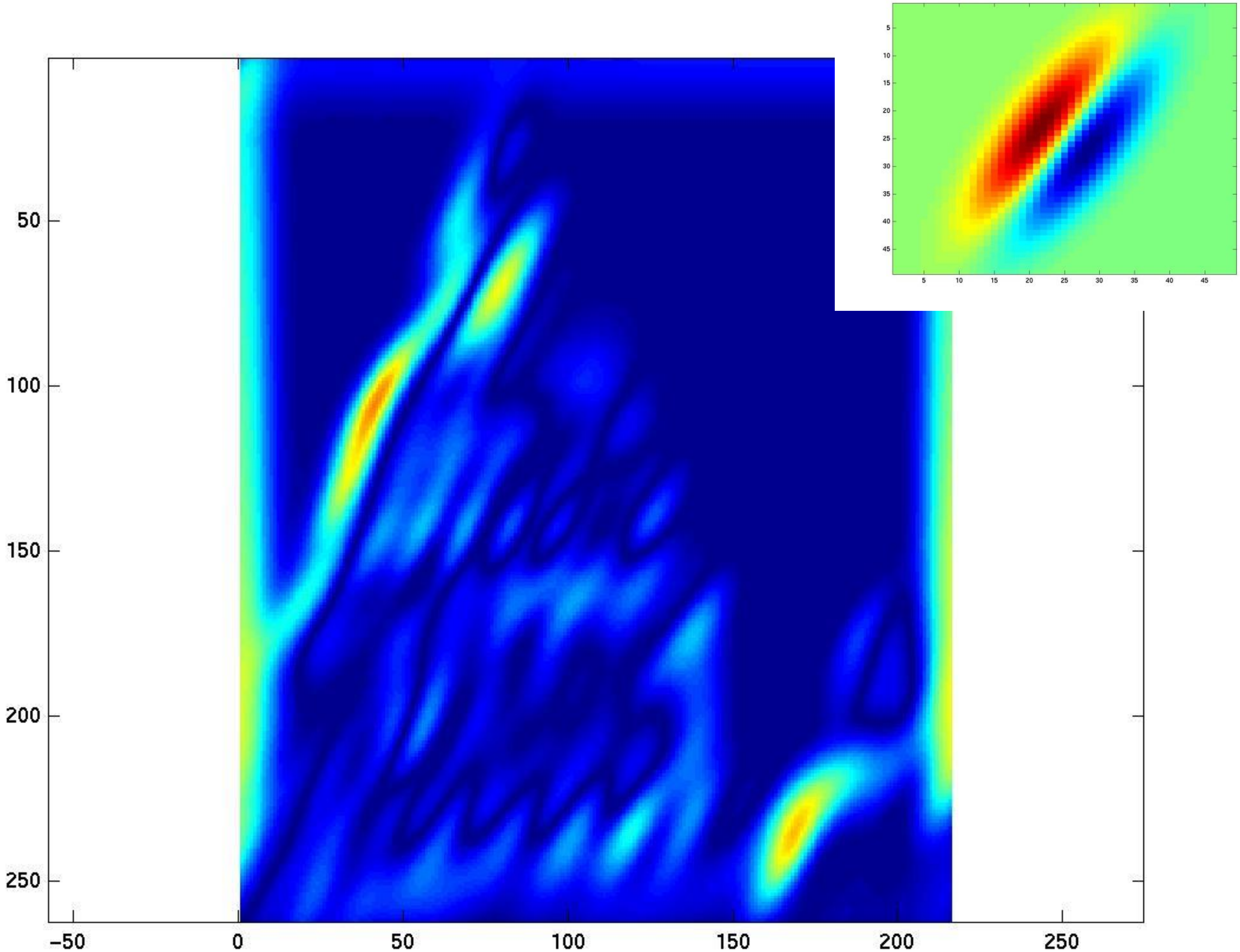
Kristen Grauman



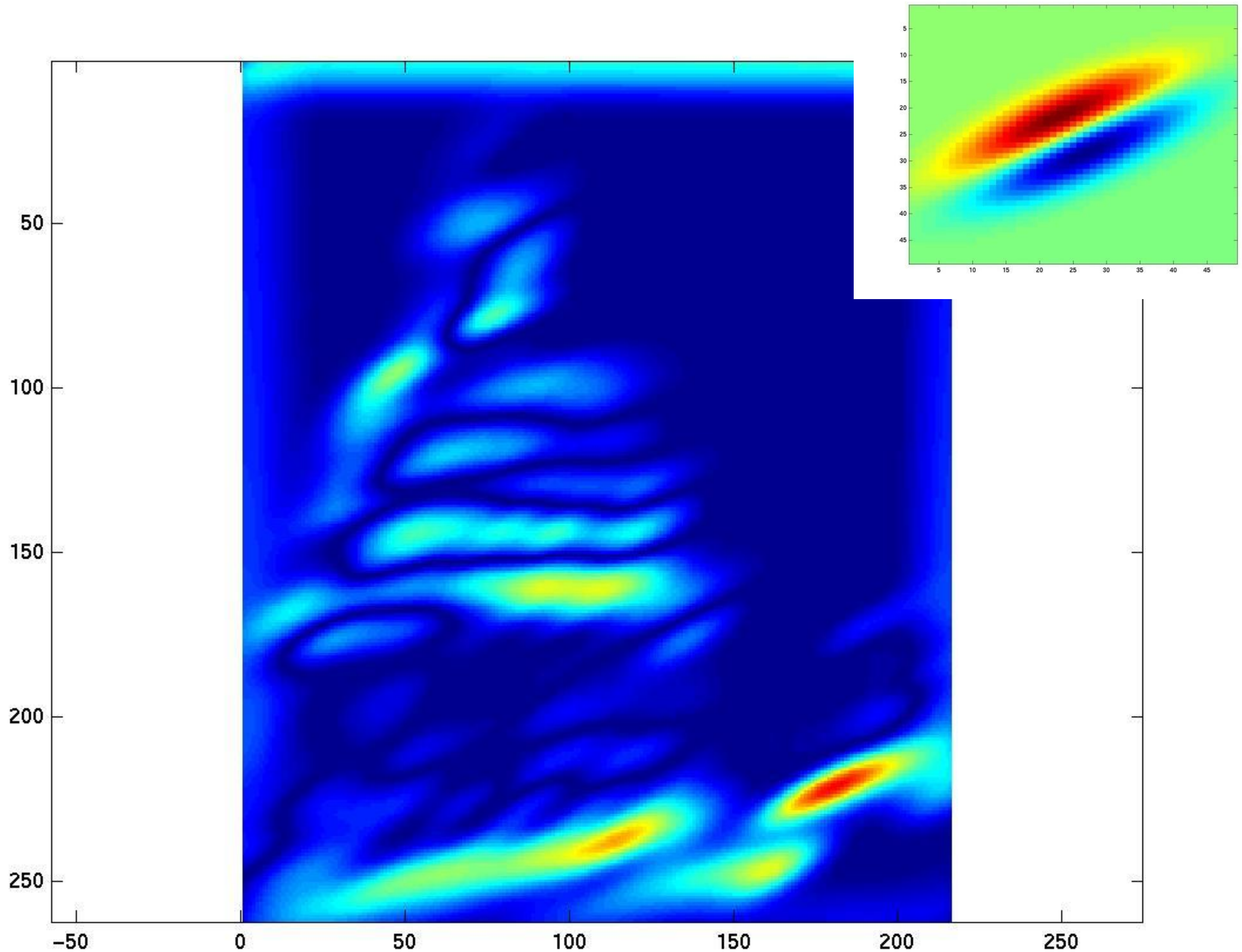
Kristen Grauman



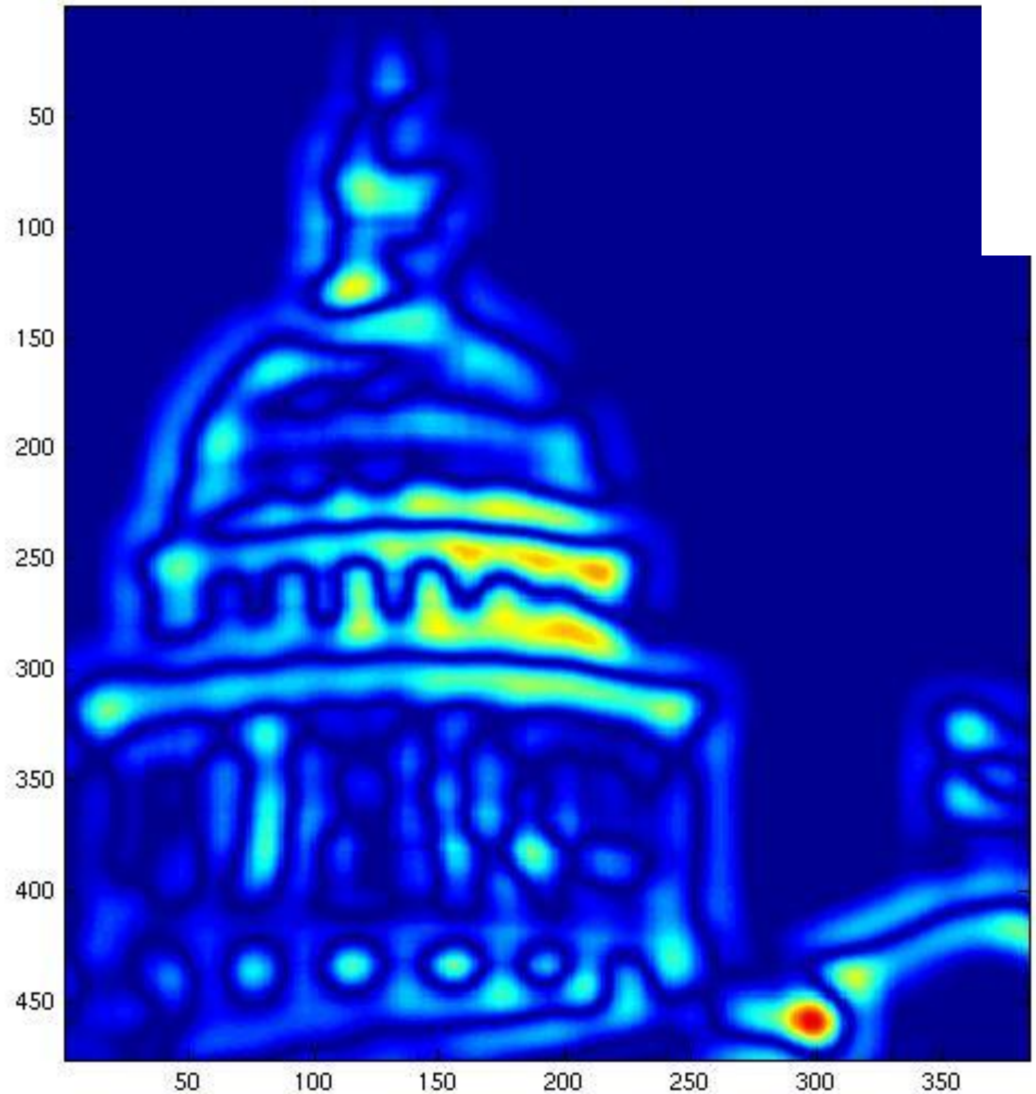
Kristen Grauman

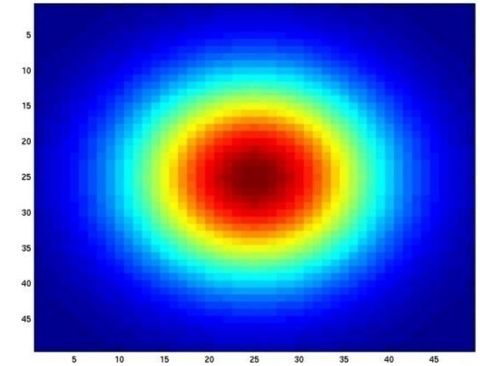


Kristen Grauman



Kristen Grauman

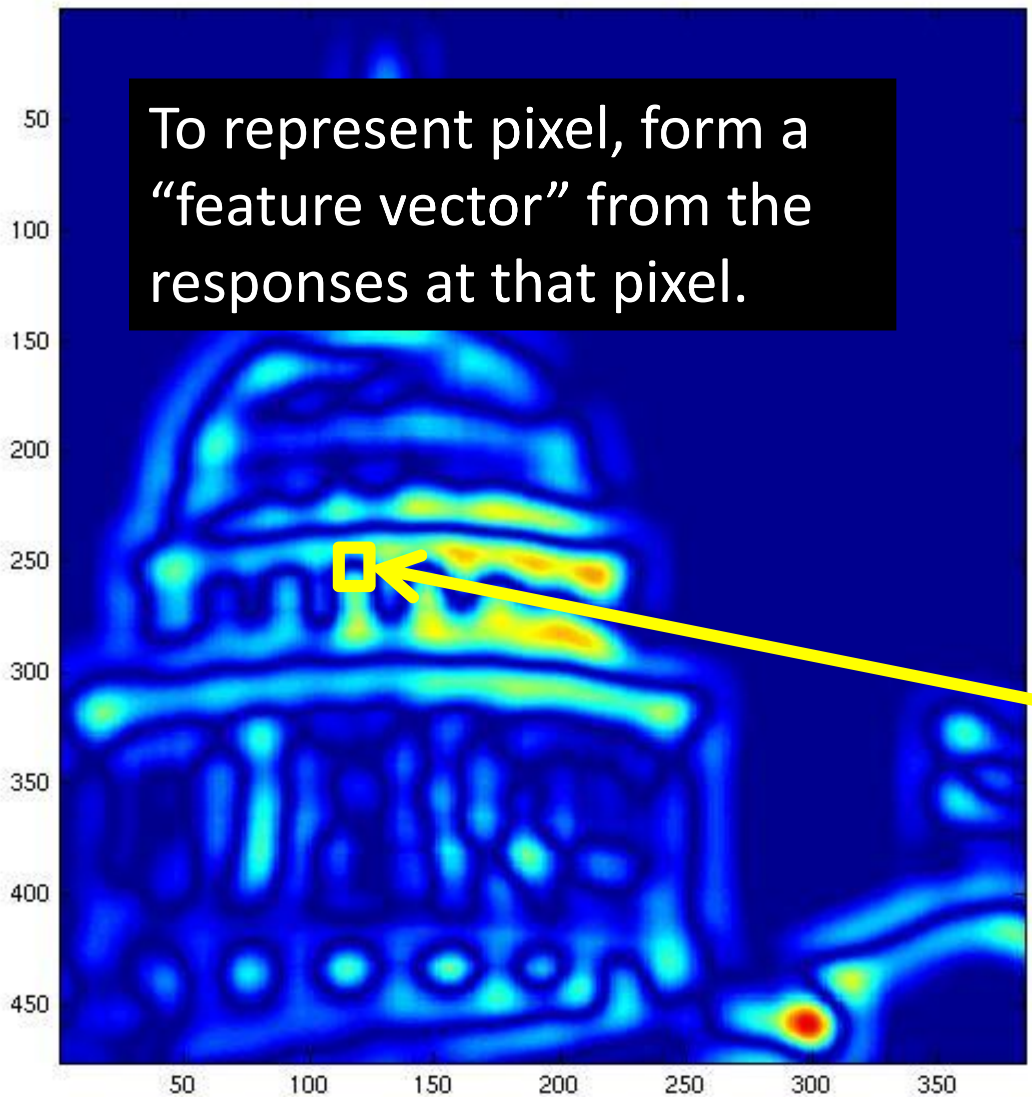




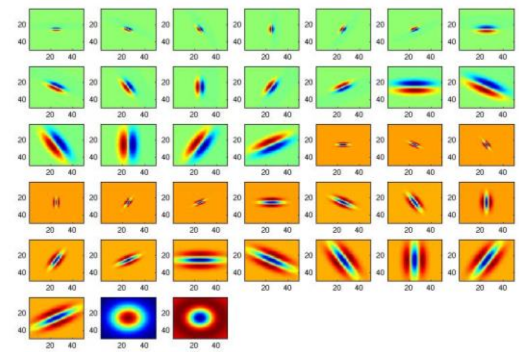
Kristen Grauman

Vectors of texture responses

To represent pixel, form a “feature vector” from the responses at that pixel.

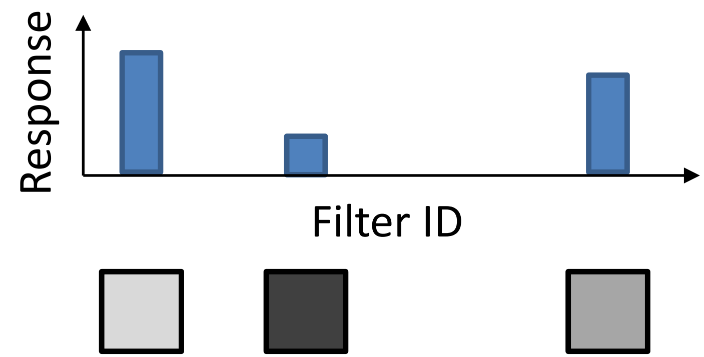


38 filters



1x38 feature vector representation

[r1, r2, …, r38]



Adapted from Kristen Grauman

Vectors of texture responses

To represent pixel, form a "feature vector" from the responses at that pixel.

To represent *image*, compute statistics over all pixel feature vectors, e.g. their mean.

$$[r_{(1,1)}^{1},\ r_{(1,1)}^{2},\ \ldots,\ r_{(1,1)}^{38}]$$

$$[r_{(1,2)}^{1},\ r_{(1,2)}^{2},\ \ldots,\ r_{(1,2)}^{38}]$$

Pixel location (row, column)

Filter ID

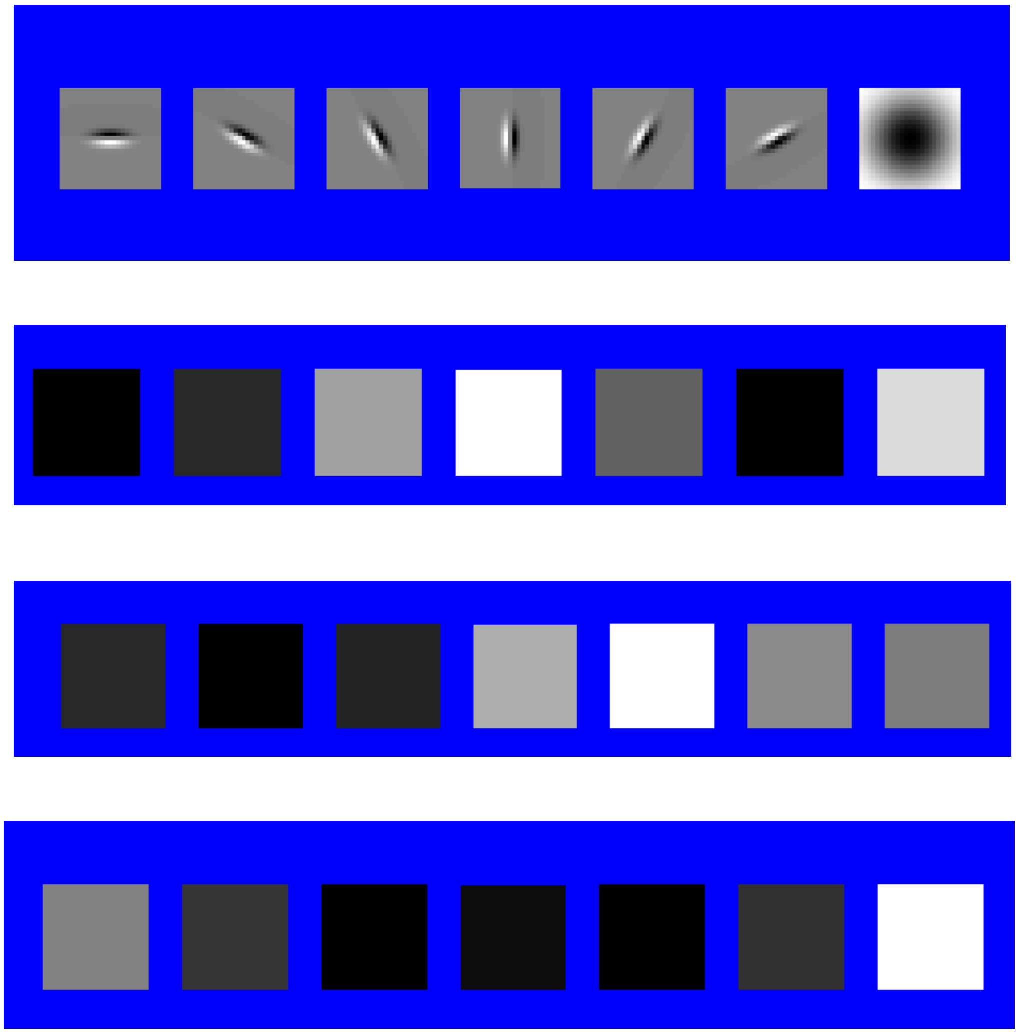
...

$$[r_{(W,H)}^{1},\ r_{(W,H)}^{2},\ \ldots,\ r_{(W,H)}^{38}]$$

$$[\mathrm{mean}(r_{(:)}^{1}),\ \mathrm{mean}(r_{(:)}^{2}),\ \ldots,\ \mathrm{mean}(r_{(:)}^{38})]$$

You try: Can you match the texture to the response?

Filters



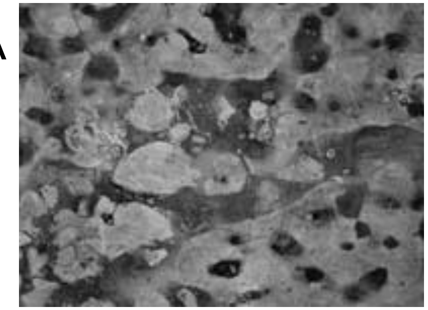
1

2

3

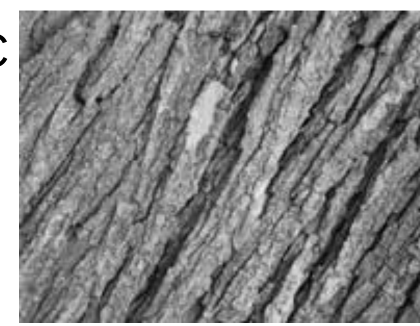
Mean abs responses

A



B

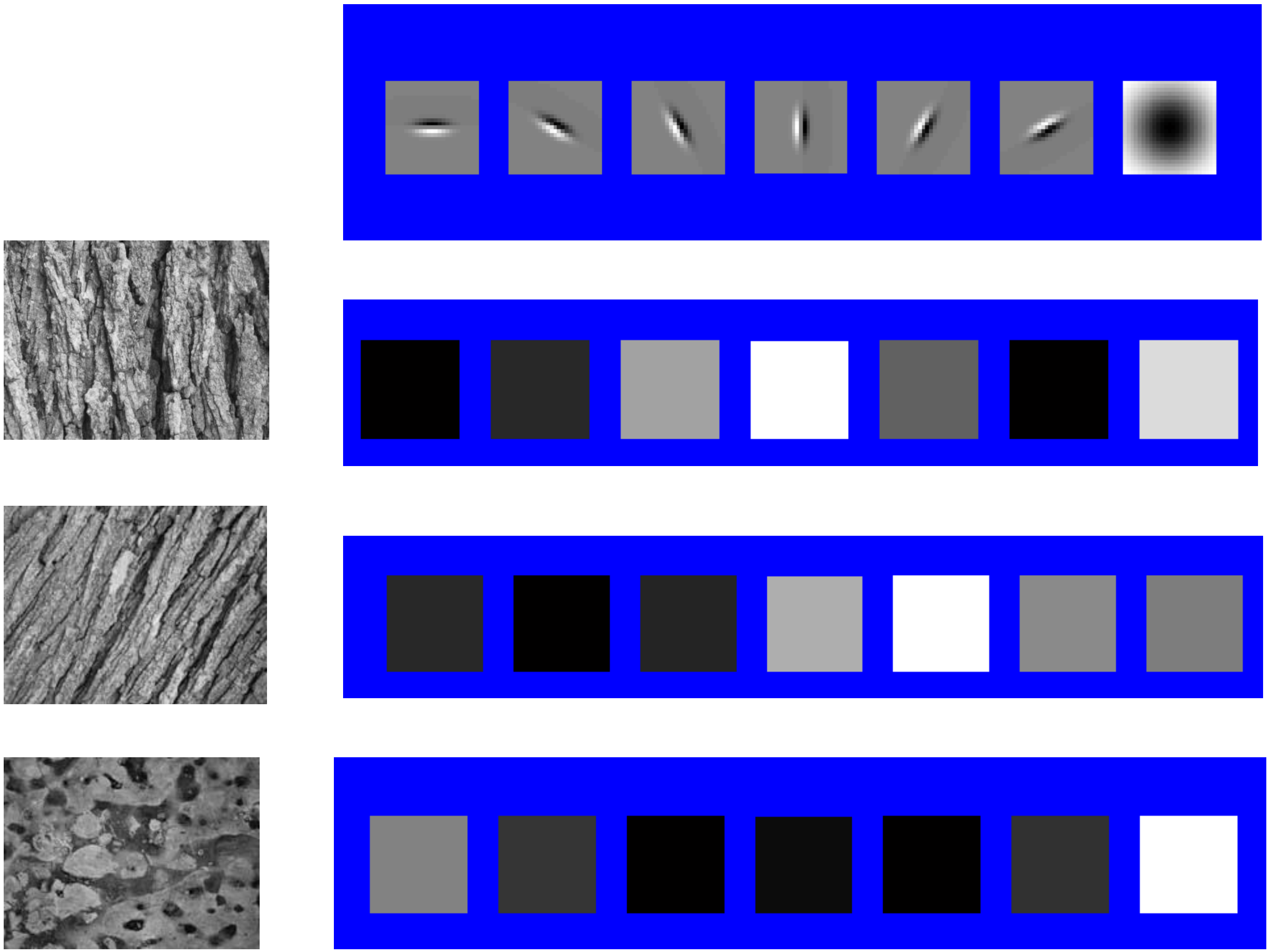
C



Derek Hoiem

Representing texture by mean abs response

Filters



Mean abs responses

Derek Hoiem

Classifying materials, “stuff”

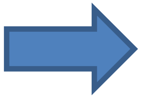
Figure by Varma & Zisserman

Plan for next two lectures

- Filters: math and properties
- Types of filters
  - Linear
    - Smoothing
    - Other
  - Non-linear
    - Median
- Texture representation with filters
- Anti-aliasing for image subsampling

Sampling

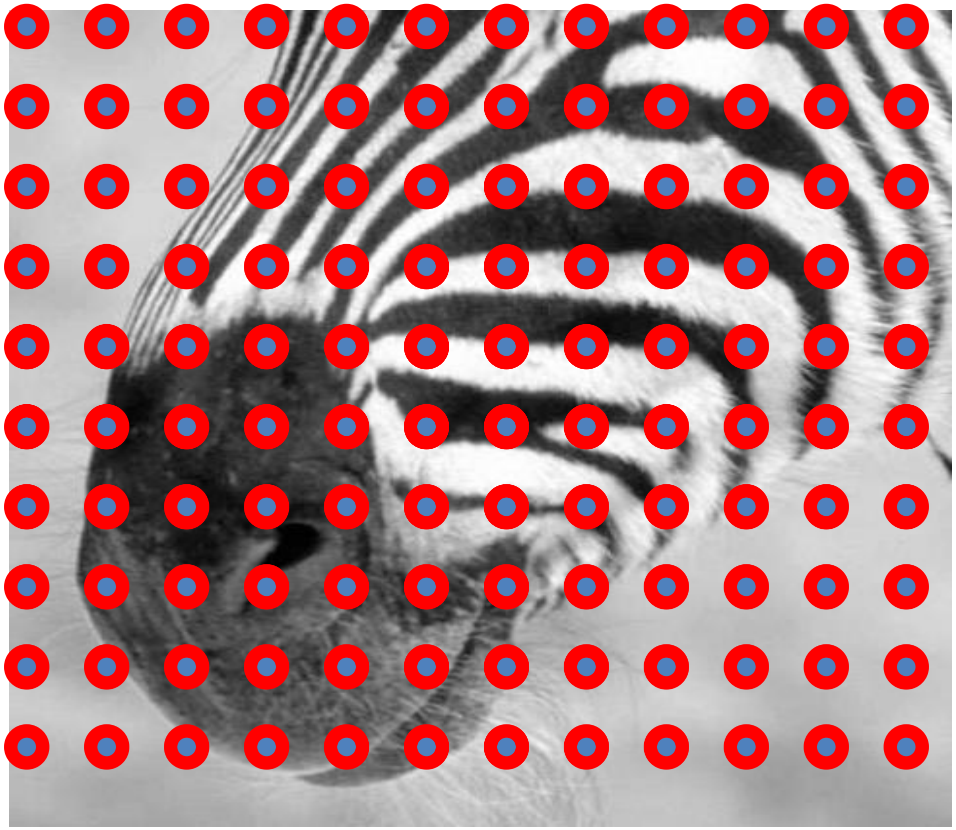
Why does a lower resolution image still make sense to us? What do we lose?



Derek Hoiem

Image: http://www.flickr.com/photos/igorms/136916757/

Subsampling by a factor of 2

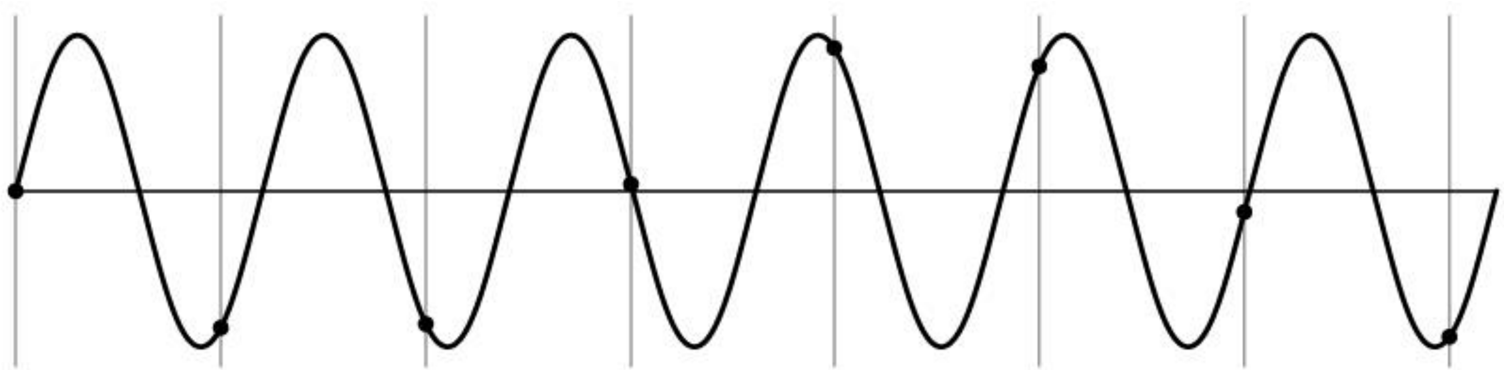


Throw away every other row and column to create a 1/2 size image

Derek Hoiem

Aliasing problem

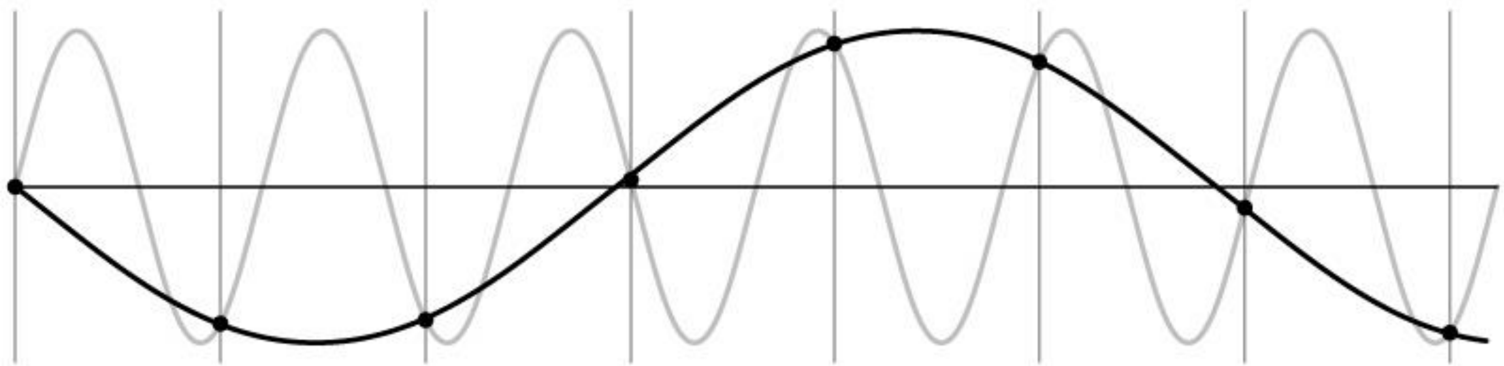
- 1D example (sinewave):



Source: S. Marschner

Aliasing problem

- 1D example (sinewave):



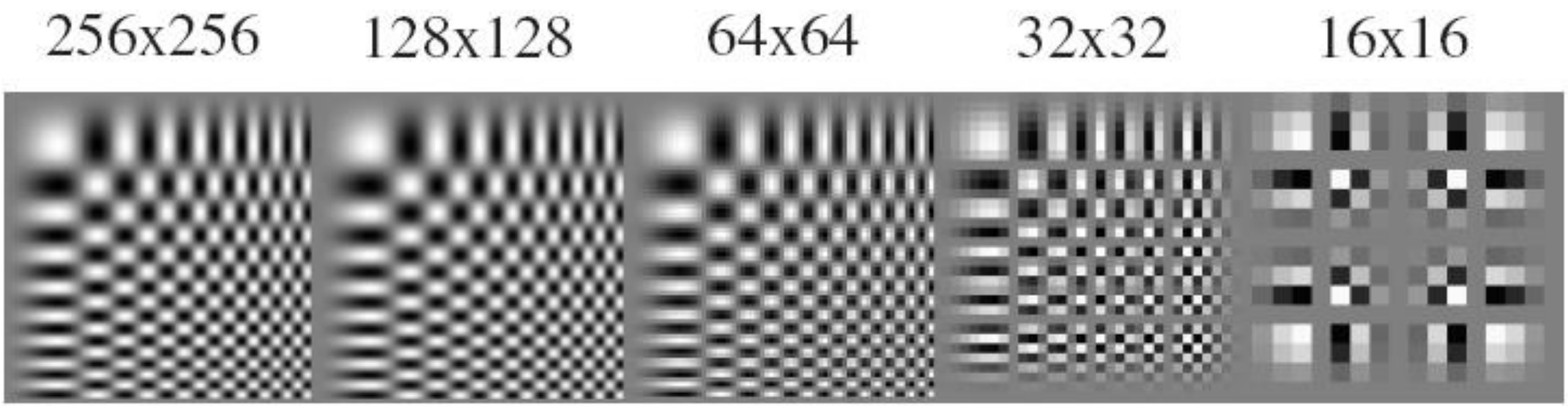
Source: S. Marschner

Aliasing problem

- Sub-sampling may be dangerous….
- Characteristic errors may appear:
  - "Wagon wheels rolling the wrong way in movies"
  - "Striped shirts look funny on color television"

Source: D. Forsyth

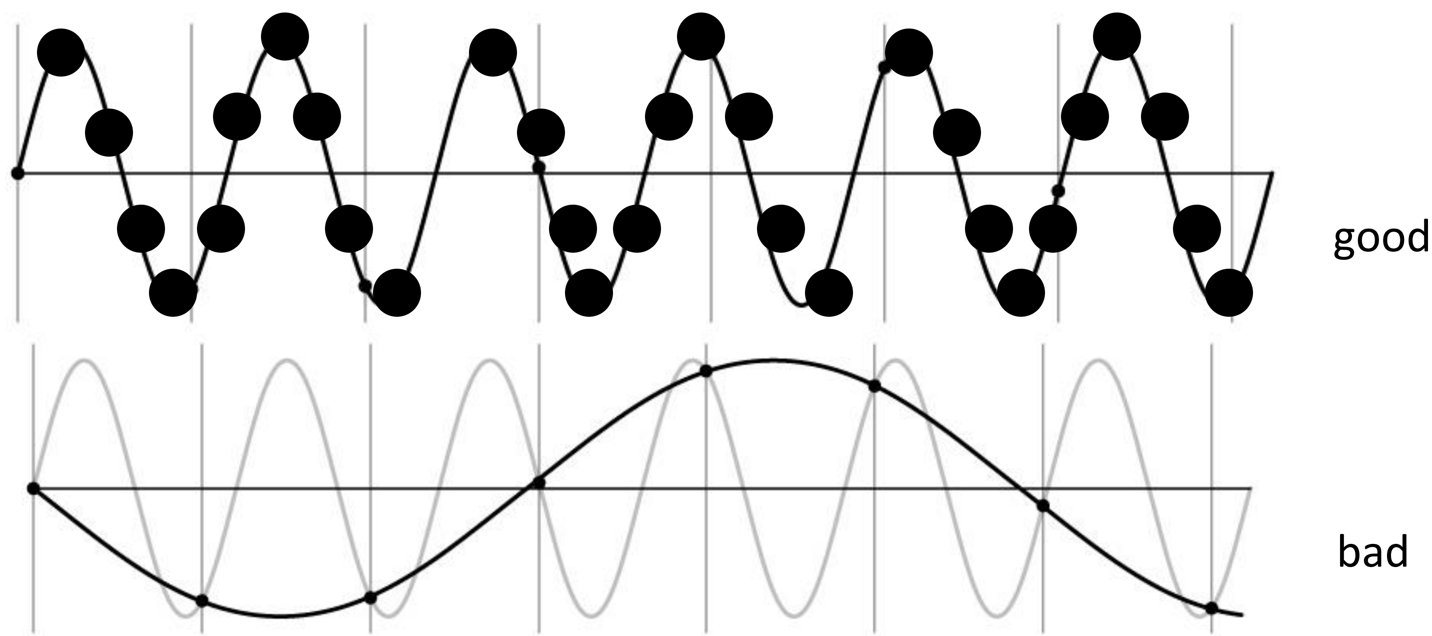
Sampling and aliasing



Derek Hoiem

Nyquist-Shannon Sampling Theorem

- When sampling a signal at discrete intervals, the sampling frequency must be $\geq 2 \times f_{max}$
- f_{max} = max frequency of the input signal
- This will allows to reconstruct the original perfectly from the sampled version



Derek Hoiem

Anti-aliasing

Solutions:

- Sample more often

- Get rid of high frequencies
  - What are these in the case of images?
  - Will lose information, but it’s better than aliasing
  - Apply a smoothing filter

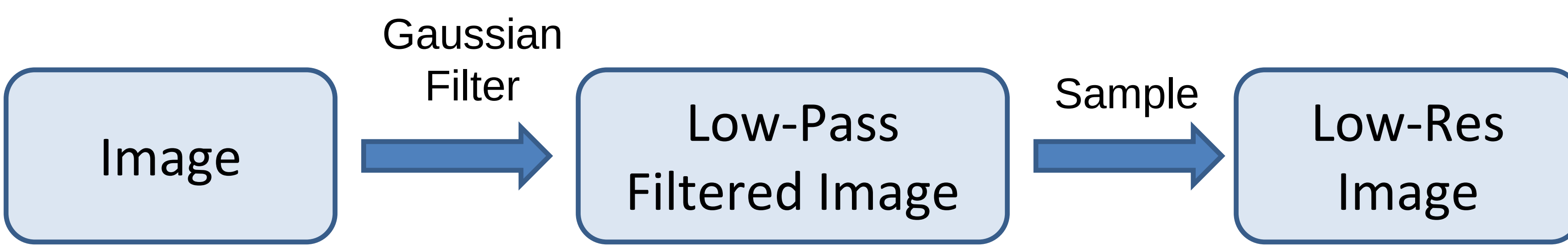
Derek Hoiem

Algorithm for downsampling by factor of 2

1. Start with image(h, w)
2. Apply low-pass filter

   im_blur = imfilter(image, fspecial('gaussian', 7, 1))
3. Sample every other pixel

   im_small = im_blur(1:2:end, 1:2:end);



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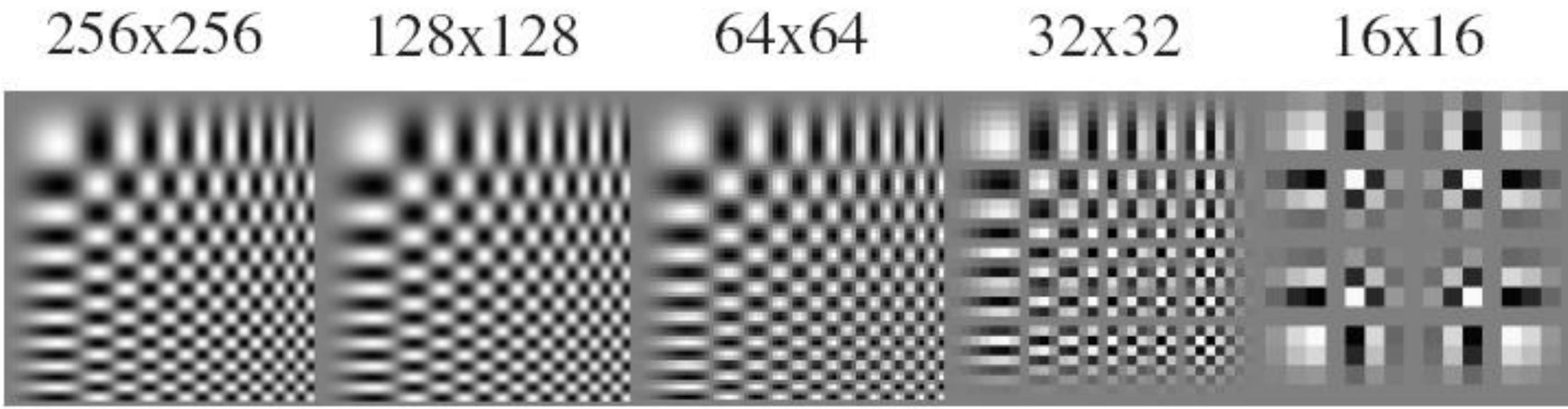
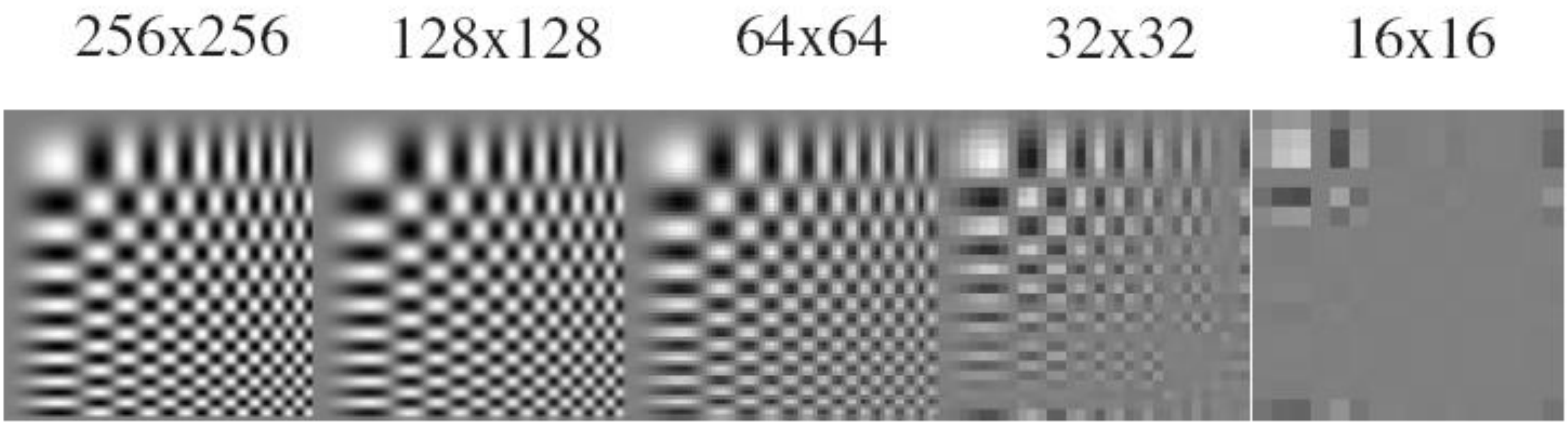
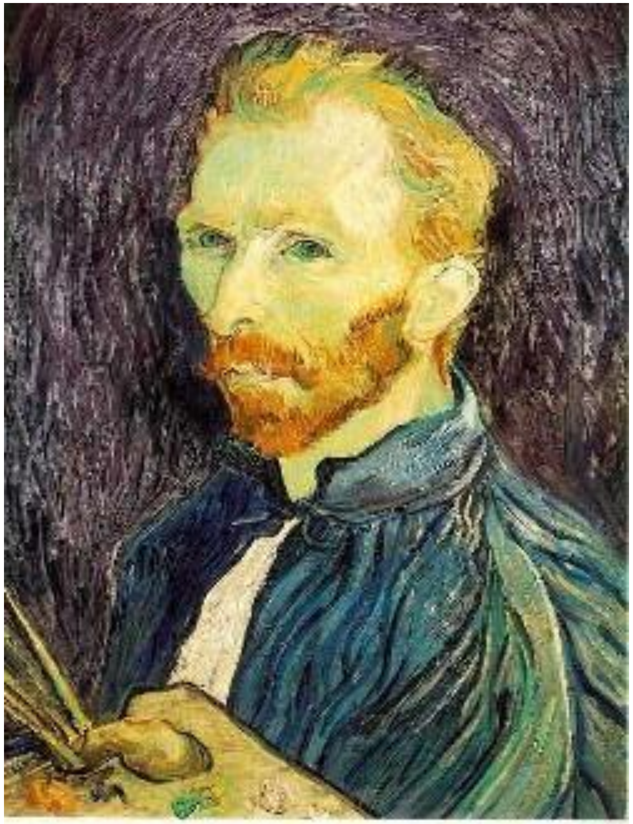
Anti-aliasing





Forsyth and Ponce 2002

Subsampling without pre-filtering

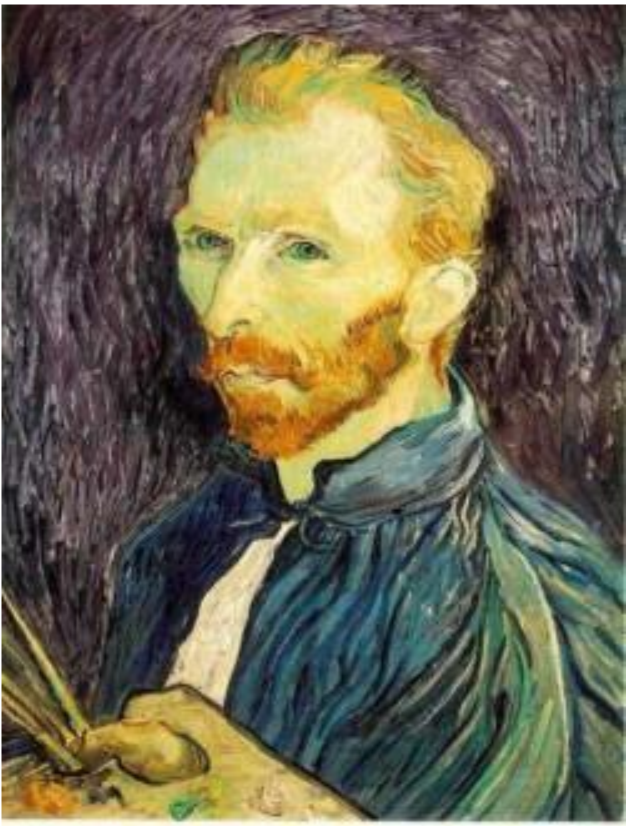

1/2


1/4 (2x zoom)


1/8 (4x zoom)

Slide by Steve Seitz

Subsampling with Gaussian pre-filtering



Gaussian 1/2

G 1/4

G 1/8

Slide by Steve Seitz

Summary

- Filters useful for
  - Enhancing images (smoothing, removing noise), e.g.
    - Box filter (linear)
    - Gaussian filter (linear)
    - Median filter
  - Detecting patterns (e.g. gradients)
- Texture is a useful property that is often indicative of materials, appearance cues
  - Texture representations summarize repeating patterns of local structure
- Can use filtering to reduce the effects of subsampling