Neural Net Examples

CS 1678/2078 Intro to Deep Learning

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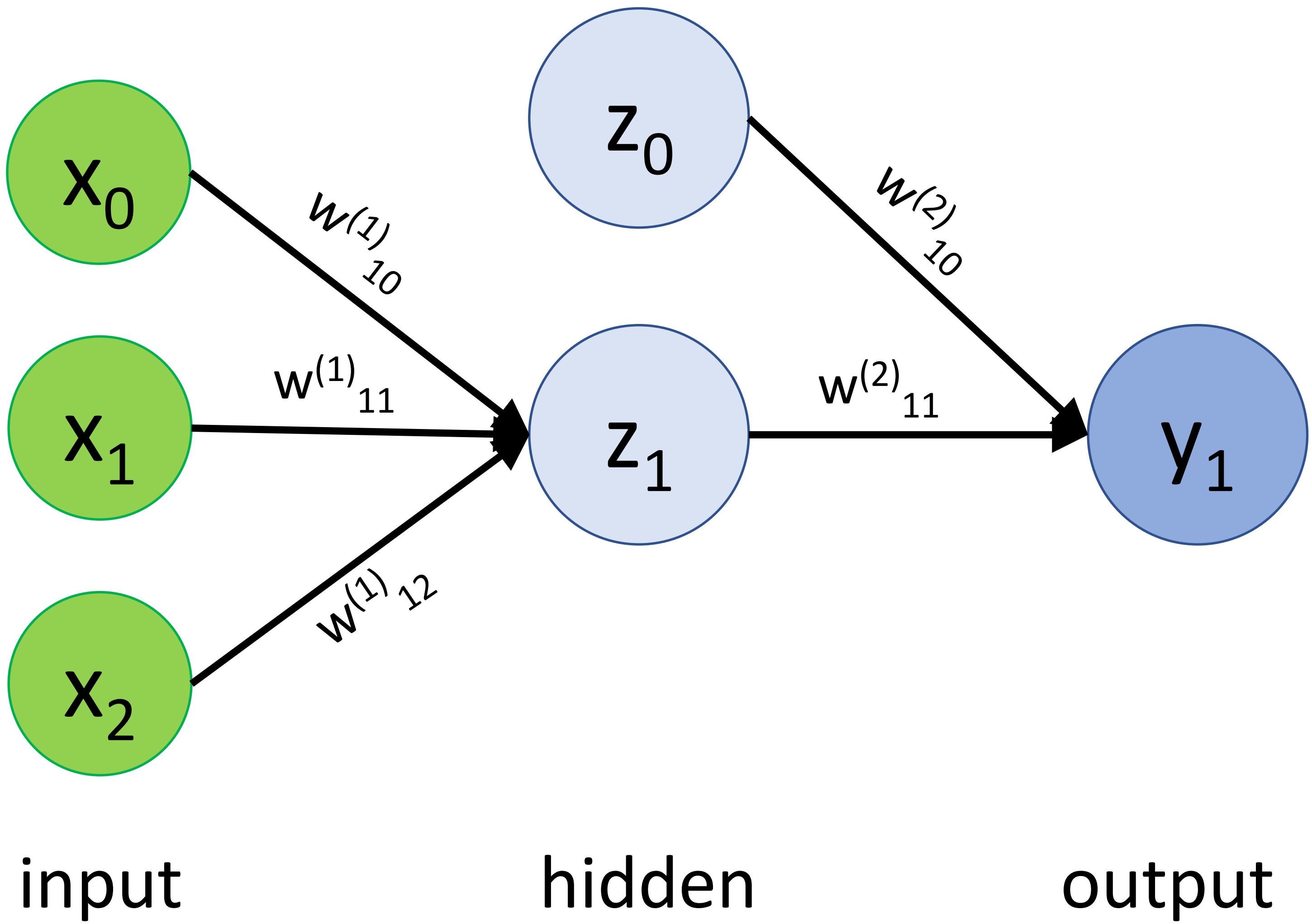
Slides by Adriana Kovashka, examples credit: Rebecca Hwa / Nick Dryslewski

First architecture



Computing activations

- In all examples, x = [x0 x1 x2], where x0 = 1
- Assume sigmoid activation function
- Initialize all weights to 0.1
- First example: x = [1 1 0]
- Second example: x = [1 0 1]
- Third example: x = [1 1 1]

Computing activations

- First example:
  - At hidden: z_1 = ?
  - At output: y_1 = ? y_{pred} = ?
- Second example:
  - At hidden: z_1 = ?
  - At output: y_1 = ? y_{pred} = ?
- Third example:
  - At hidden: z_1 = ?
  - At output: y_1 = ? y_{pred} = ?

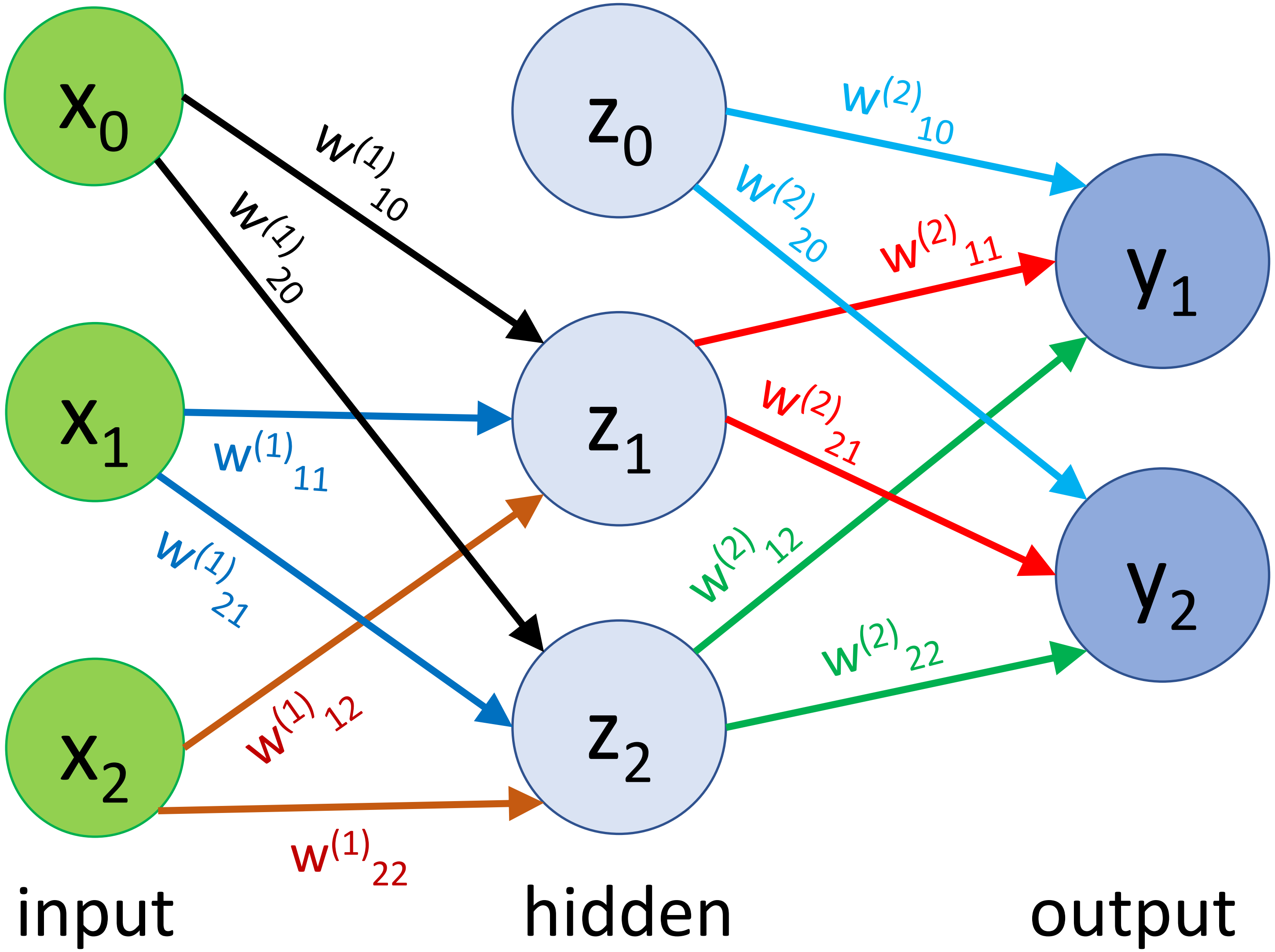
Computing activations (answers)

- First example:
  - At hidden: $z_1 = 1 / [1 + \exp(-(x_0*w^{(1)}_{10}+x_1*w^{(1)}_{11} +x_2*w^{(1)}_{12}))]$
  - $= 1 / [1 + \exp(-(1*0.1+1*0.1+0*0.1))] = 0.5498$
  - At output: $y_1 = 1 / [1 + \exp(-(z_0*w^{(2)}_{10} +z_1*w^{(2)}_{11}))]$
  - $= 1 / [1 + \exp(-(1*0.1+0.5498*0.1))] = 0.5387 \rightarrow y_{pred} = 1$
- Second example:
  - At hidden: $z_1 = 1 / [1 + \exp(-(x_0*w^{(1)}_{10}+x_1*w^{(1)}_{11} +x_2*w^{(1)}_{12}))]$
  - $= 1 / [1 + \exp(-(1*0.1+0*0.1+1*0.1))] = 0.5498$
  - At output: $y_1 = 1 / [1 + \exp(-(z_0*w^{(2)}_{10} +z_1*w^{(2)}_{11}))]$
  - $= 1 / [1 + \exp(-(1*0.1+0.5498*0.1))] = 0.5387 \rightarrow y_{pred} = 1$

Computing activations (answers)

- Third example:
  - At hidden: z_1 = 1 / [1 + exp(-(x_0*$w^{(1)}_{10}$+x_1*$w^{(1)}_{11}$ +x_2*$w^{(1)}_{12}$))]
  - = 1 / [1 + exp(-(1*0.1+1*0.1+1*0.1))] = 0.5744
  - At output: y_1 = 1 / [1 + exp(-(z_0*$w^{(2)}_{10}$ +z_1*$w^{(2)}_{11}$))]
  - = 1 / [1 + exp(-(1*0.1+0.5744*0.1))] = 0.5393 → y_{pred} = 1

Second architecture



Computing activations

- In all examples, x = [x0 x1 x2], where x0 = 1
- Assume sigmoid activation function
- Initialize all weights to 0.05
- First example: x = [1 1 0]
- Second example: x = [1 0 1]
- Third example: x = [1 1 1]

Computing activations

- First, second, third example:
  - At hidden:
    - z_1 = ?
    - z_2 = ?
  - At output:
    - y_1 = ?
    - y_2 = ?
    - y_{pred} = [1 1]

Computing activations (answers)

- First example:
  - At hidden:
    - $z_1 = 1 / [1 + \exp(-(x_0 * w^{(1)}_{10} + x_1 * w^{(1)}_{11} + x_2 * w^{(1)}_{12}))] = 1 / [1 + \exp(-(1*0.05+1*0.05+0*0.05))] = 0.5249$
    - $z_2 = 1 / [1 + \exp(-(x_0 * w^{(1)}_{20} + x_1 * w^{(1)}_{21} + x_2 * w^{(1)}_{22}))] = 1 / [1 + \exp(-(1*0.05+1*0.05+0*0.05))] = 0.5249$
  - At output:
    - $y_1 = 1 / [1 + \exp(-(z_0 * w^{(2)}_{10} + z_1 * w^{(2)}_{11} + z_2 * w^{(2)}_{12}))] = 1 / [1 + \exp(-(1*0.05+0.5249*0.05+0.5249*0.05))] = 0.5256$
    - $y_2 = 1 / [1 + \exp(-(z_0 * w^{(2)}_{20} + z_1 * w^{(2)}_{21} + z_2 * w^{(2)}_{22}))] = 1 / [1 + \exp(-(1*0.05+0.5249*0.05+0.5249*0.05))] = 0.5256 \rightarrow y_{pred} = [1\ 1]$

Computing activations (answers)

- Second example:
  - At hidden:
    - $z_1 = 1 / [1 + \exp(-(x_0 * w^{(1)}_{10} + x_1 * w^{(1)}_{11} + x_2 * w^{(1)}_{12}))] =$
      $1 / [1 + \exp(-(1*0.05+0*0.05+1*0.05))] = 0.5249$
    - $z_2 = 1 / [1 + \exp(-(x_0 * w^{(1)}_{20} + x_1 * w^{(1)}_{21} + x_2 * w^{(1)}_{22}))] =$
      $1 / [1 + \exp(-(1*0.05+0*0.05+1*0.05))] = 0.5249$
  - At output:
    - $y_1 = 1 / [1 + \exp(-(z_0 * w^{(2)}_{10} + z_1 * w^{(2)}_{11} + z_2 * w^{(2)}_{12}))]$
      $= 1 / [1 + \exp(-(1*0.05+0.5249*0.05+0.5249*0.05))] =$
      0.5256
    - $y_2 = 1 / [1 + \exp(-(z_0 * w^{(2)}_{20} + z_1 * w^{(2)}_{21} + z_2 * w^{(2)}_{22}))]$
      $= 1 / [1 + \exp(-(1*0.05+0.5249*0.05+0.5249*0.05))] =$
      $0.5256 \rightarrow y_{pred} = [1\ 1]$

Computing activations (answers)

- Third example:
  - At hidden:
    - $z_1 = 1 / [1 + \exp(-(x_0*w^{(1)}_{10}+x_1*w^{(1)}_{11}+x_2*w^{(1)}_{12}))] = 1 / [1 + \exp(-(1*0.05+1*0.05+1*0.05))] = 0.5374$
    - $z_2 = 1 / [1 + \exp(-(x_0*w^{(1)}_{20}+x_1*w^{(1)}_{21}+x_2*w^{(1)}_{22}))] = 1 / [1 + \exp(-(1*0.05+1*0.05+1*0.05))] = 0.5374$
  - At output:
    - $y_1 = 1 / [1 + \exp(-(z_0*w^{(2)}_{10}+z_1*w^{(2)}_{11}+z_2*w^{(2)}_{12}))] = 1 / [1 + \exp(-(1*0.05+0.5374*0.05+0.5374*0.05))] = 0.5259$
    - $y_2 = 1 / [1 + \exp(-(z_0*w^{(2)}_{20}+z_1*w^{(2)}_{21}+z_2*w^{(2)}_{22}))] = 1 / [1 + \exp(-(1*0.05+0.5374*0.05+0.5374*0.05))] = 0.5259 \rightarrow y_{pred} = [1\ 1]$

Training the first network

- Perform backpropagation using stochastic gradient descent (one sample at a time)
- Weights are initially all 0.1
- Learning rate is 0.3
- Sigmoid activation function at hidden and output
- d s(x) / dx = s(x) (1 – s(x)) dx
- Samples have the following labels:
  - First example: x = [1 1 0], y = 1
  - Second example: x = [1 0 1], y = 0
  - Third example: x = [1 1 1], y = 1
- Preview: What do you expect final weights to be?

Learning from first example

- First example: $x = [1\ 1\ 0]$, $y = 1$
- Weights are $w^{(1)}{}_{10} = w^{(1)}{}_{11} = w^{(1)}{}_{12} = w^{(2)}{}_{10} = w^{(2)}{}_{11} = 0.1$
- Activations are $z_1 = 0.5498$, $y_1 = 0.5387$
- Compute errors:
  - $\delta_{y1} = ?$
  - $\delta_{z1} = ?$
- Update weights:
  - $w^{(2)}{}_{10} = w^{(2)}{}_{10} - ?$
  - $w^{(2)}{}_{11} = w^{(2)}{}_{11} - ?$
  - $w^{(1)}{}_{10} = w^{(1)}{}_{10} - ?$
  - $w^{(1)}{}_{11} = w^{(1)}{}_{11} - ?$
  - $w^{(1)}{}_{12} = w^{(1)}{}_{12} - ?$

Learning from first example (answers)

- First example: $x = [1\ 1\ 0]$, $y = 1$
- Weights are $w^{(1)}_{10} = w^{(1)}_{11} = w^{(1)}_{12} = w^{(2)}_{10} = w^{(2)}_{11} = 0.1$
- Activations are $z_1 = 0.5498$, $y_1 = 0.5387$
- Compute errors:
  - $\delta_{y1} = y_1*(1-y_1)*(y_1-y_{true}) = 0.5387*(1-0.5387)*(0.5387-1) = -0.1146$
  - $\delta_{z1} = z_1*(1-z_1)*(w^{(2)}_{11}*\delta_{y1}) = 0.5498*(1-0.5498)*[0.1*-0.1146] = -0.0028$
- Update weights:
  - $w^{(2)}_{10} = w^{(2)}_{10} - 0.3*\delta_{y1}*z_0 = 0.1 + 0.3*0.1146*1 = 0.1343$
  - $w^{(2)}_{11} = w^{(2)}_{11} - 0.3*\delta_{y1}*z_1 = 0.1 + 0.3*0.1146*0.5498 = 0.1189$
  - $w^{(1)}_{10} = w^{(1)}_{10} - 0.3*\delta_{z1}*x_0 = 0.1 + 0.3*0.0028*1 = 0.1008$
  - $w^{(1)}_{11} = w^{(1)}_{11} - 0.3*\delta_{z1}*x_1 = 0.1 + 0.3*0.0028*1 = 0.1008$
  - $w^{(1)}_{12} = w^{(1)}_{12} - 0.3*\delta_{z1}*x_2 = 0.1 + 0.3*0.0028*0 = 0.1$

Learning from second example (answers)

- Second example: x = [1 0 1], y = 0
- Weights are $w^{(1)}_{10} = w^{(1)}_{11} = 0.1008$, $w^{(1)}_{12} = 0.1$, $w^{(2)}_{10} = 0.1343$, $w^{(2)}_{11} = 0.1189$
- Activations are (recompute with new weights):
  - $z_1 = 1 / [1 + \exp(-(x_0 * w^{(1)}_{10} + x_1 * w^{(1)}_{11} + x_2 * w^{(1)}_{12}))] = 1 / [1 + \exp(-(1*0.1008+0*0.1008+1*0.1))] = 0.55$
  - $y_1 = 1 / [1 + \exp(-(z_0 * w^{(2)}_{10} + z_1 * w^{(2)}_{11}))] = 1 / [1 + \exp(-(1*0.1343+0.55*0.1189))] = 0.5498$
- Compute errors:
  - $\delta_{y1} = y_1 * (1 - y_1) * (y_1 - y_{true}) = 0.5498*(1-0.5498)*(0.5498-0) = 0.1361$
  - $\delta_{z1} = z_1 * (1 - z_1) * (w^{(2)}_{11} * \delta_{y1}) = 0.55*(1-0.55)*[0.1189*0.1361] = 0.004$
- Update weights:
  - $w^{(2)}_{10} = w^{(2)}_{10} - 0.3 * \delta_{y1} * z_0 = 0.1343 - 0.3*0.1361*1 = 0.0935$
  - $w^{(2)}_{11} = w^{(2)}_{11} - 0.3 * \delta_{y1} * z_1 = 0.1189 - 0.3*0.1361*0.55 = 0.0964$
  - $w^{(1)}_{10} = w^{(1)}_{10} - 0.3 * \delta_{z1} * x_0 = 0.1008 - 0.3*0.004*1 = 0.0996$
  - $w^{(1)}_{11} = w^{(1)}_{11} - 0.3 * \delta_{z1} * x_1 = 0.1008 - 0.3*0.004*0 = 0.1008$
  - $w^{(1)}_{12} = w^{(1)}_{12} - 0.3 * \delta_{z1} * x_2 = 0.1 - 0.3*0.004*1 = 0.0988$

Learning from third example (answers)

- Third example: x = [1 1 1], y = 1
- Weights are $w^{(1)}_{10}$ = 0.0996, $w^{(1)}_{11}$ = 0.1008, $w^{(1)}_{12}$ = 0.0988, $w^{(2)}_{10}$ = 0.0935, $w^{(2)}_{11}$ = 0.0964
- Activations are (recompute with new weights):
  - $z_1 = 1 / [1 + \exp(-(x_0*w^{(1)}_{10}+x_1*w^{(1)}_{11}+x_2*w^{(1)}_{12}))] = 1 / [1 + \exp(-(1*0.0996+1*0.1008+1*0.0988))] = 0.5742$
  - $y_1 = 1 / [1 + \exp(-(z_0*w^{(2)}_{10}+z_1*w^{(2)}_{11}))] = 1 / [1 + \exp(-(1*0.0935+0.5735*0.0964))] = 0.5371$
- Compute errors:
  - $\delta_{y1} = y_1*(1-y_1)*(y_1-y_{true}) = 0.5371*(1-0.5371)*(0.5371-1) = -0.1151$
  - $\delta_{z1} = z_1*(1-z_1)*(w^{(2)}_{11}*\delta_{y1}) = 0.5742*(1-0.5742)*[0.0964*-0.1151] = -0.0027$
- Update weights:
  - $w^{(2)}_{10} = w^{(2)}_{10} - 0.3*\delta_{y1}*z_0 = 0.0935 + 0.3*0.1151*1 = 0.1280$
  - $w^{(2)}_{11} = w^{(2)}_{11} - 0.3*\delta_{y1}*z_1 = 0.0964 + 0.3*0.1151*0.5735 = 0.1162$
  - $w^{(1)}_{10} = w^{(1)}_{10} - 0.3*\delta_{z1}*x_0 = 0.0996 + 0.3*0.0027*1 = 0.1004$
  - $w^{(1)}_{11} = w^{(1)}_{11} - 0.3*\delta_{z1}*x_1 = 0.1008 + 0.3*0.0027*1 = 0.1016$
  - $w^{(1)}_{12} = w^{(1)}_{12} - 0.3*\delta_{z1}*x_2 = 0.0988 + 0.3*0.0027*1 = 0.0996$

Recap

- Do the $w^{(1)}$ weights we obtained make sense?