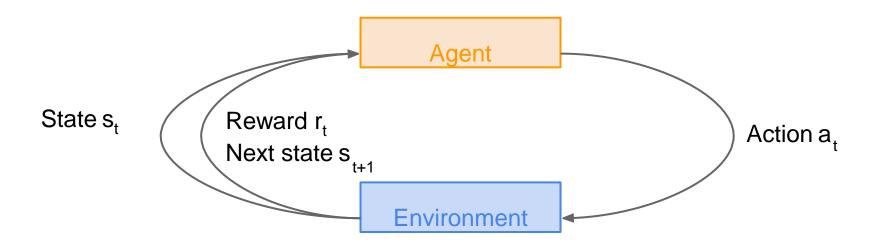
CS 1678/2078: Intro to Deep Learning Reinforcement Learning

Prof. Adriana Kovashka
University of Pittsburgh
April 8, 2024

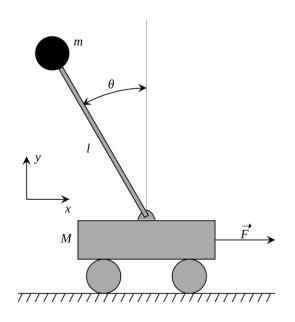
Plan for this lecture

- Basics: actions, states, rewards, MDP
- Different techniques (Q learning, policy gradients, actor-critic, etc.)
- Example applications

Reinforcement Learning



Cart-Pole Problem



Objective: Balance a pole on top of a movable cart

State: angle, angular speed, position, horizontal velocity

Action: horizontal force applied on the cart

Reward: 1 at each time step if the pole is upright

Atari Games



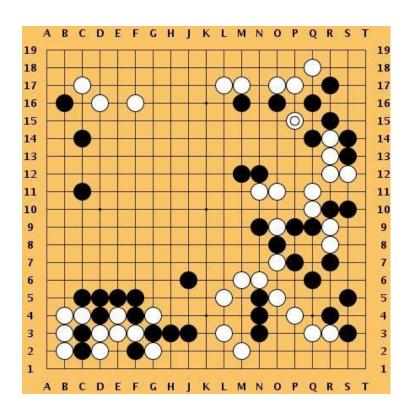
Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state

Action: Game controls e.g. Left, Right, Up, Down

Reward: Score increase/decrease at each time step

Go



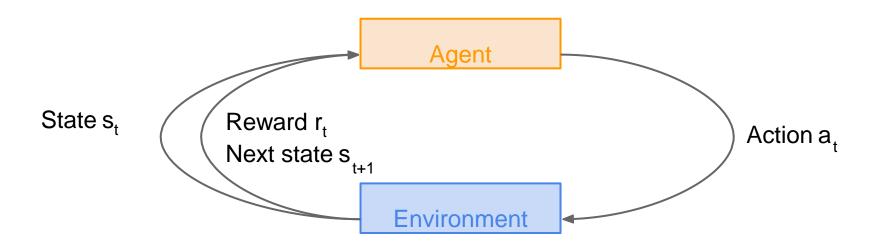
Objective: Win the game!

State: Position of all pieces

Action: Where to put the next piece down

Reward: 1 if win at the end of the game, 0 otherwise

How can we mathematically formalize the RL problem?



Markov Decision Process

- Mathematical formulation of the RL problem
- Markov property: Current state completely characterises the state of the world

Defined by: $(\mathcal{S},\mathcal{A},\mathcal{R},\mathbb{P},\gamma)$

 ${\cal S}\,$: set of possible states

 ${\cal A}\,$: set of possible actions

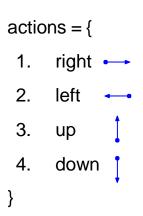
 \mathcal{R} : distribution of reward given (state, action) pair

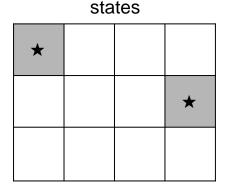
 γ : discount factor

Markov Decision Process

- At time step t=0, environment samples initial state $s_0 \sim p(s_0)$
- Then, for t=0 until done:
 - Agent selects action a,
 - Environment samples reward r, ~ R(. | s, a,)
 - Environment samples next state $s_{t+1} \sim P(. | s_t, a_t)$
 - Agent receives reward r, and next state s,+1
- A policy u is a function from S to A that specifies what action to take in each state
- **Objective**: find policy u* that maximizes cumulative discounted reward: $\sum \gamma^t r_t$

A simple MDP: Grid World

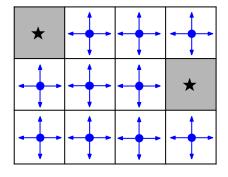




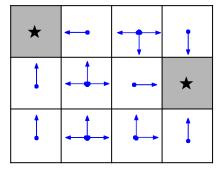
Set a negative "reward" for each transition (e.g. r = -1)

Objective: reach one of terminal states (greyed out) in least number of actions

A simple MDP: Grid World



Random Policy



Optimal Policy

The optimal policy u*

We want to find optimal policy u* that maximizes the sum of rewards.

How do we handle the randomness (initial state, transition probability...)?

The optimal policy u*

We want to find optimal policy u* that maximizes the sum of rewards.

How do we handle the randomness (initial state, transition probability...)? Maximize the **expected sum of rewards!**

Formally:
$$\pi^* = \arg\max_{\pi} \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | \pi\right]$$
 with $s_0 \sim p(s_0), a_t \sim \pi(\cdot|s_t), s_{t+1} \sim p(\cdot|s_t, a_t)$

Definitions: Value and Q-value functions

Following a policy produces sample trajectories (or paths) s₀, a₀, r₀, s₁, a₁, r₁, ...

How good is a state?

The **value function** at state s, is the expected cumulative reward from following the policy from state s:

$$V^{\pi}(s) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, \pi
ight]$$

How good is a state-action pair?

The **Q-value function** at state s and action a, is the expected cumulative reward from taking action a in state s and then following the policy:

$$Q^{\pi}(s,a) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi
ight]$$

Bellman equation

The optimal Q-value function Q* is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s,a) = \max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

Q* satisfies the following **Bellman equation**:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Intuition: if the optimal state-action values for the next time-step $Q^*(s',a')$ are known, then the optimal strategy is to take the action that maximizes the expected value of $r + \gamma Q^*(s',a')$

The optimal policy u* corresponds to taking the best action in any state as specified by Q*

Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s,a;\theta) \approx Q^*(s,a)$$
 function parameters (weights)

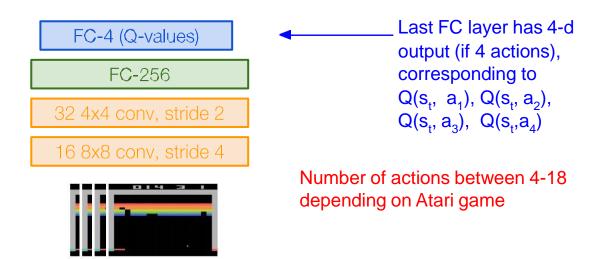
If the function approximator is a deep neural network => **deep q-learning**!

Q-network Architecture

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Q(s,a; heta) : neural network with weights heta

A single feedforward pass to compute Q-values for all actions from the current state => efficient!



Current state s_t: 84x84x4 stack of last 4 frames (after RGB->grayscale conversion, downsampling, and cropping)

```
Algorithm 1 Deep Q-learning with Experience Replay
   Initialize replay memory \mathcal{D} to capacity N
   Initialize action-value function Q with random weights
   for episode = 1, M do
       Initialise sequence s_1 = \{x_1\} and preprocessed sequenced \phi_1 = \phi(s_1)
       for t = 1, T do
            With probability \epsilon select a random action a_t
            otherwise select a_t = \max_a Q^*(\phi(s_t), a; \theta)
            Execute action a_t in emulator and observe reward r_t and image x_{t+1}
             Set s_{t+1} = s_t, a_t, x_{t+1} and preprocess \phi_{t+1} = \phi(s_{t+1})
             Store transition (\phi_t, a_t, r_t, \phi_{t+1}) in \mathcal{D}
             Sample random minibatch of transitions (\phi_j, a_j, r_j, \phi_{j+1}) from \mathcal{D}
            \text{Set } y_j = \left\{ \begin{array}{ll} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{array} \right.
            Perform a gradient descent step on (y_j - Q(\phi_j, a_j; \theta))^2 according to equation 3
       end for
   end for
```

```
Algorithm 1 Deep Q-learning with Experience Replay
   Initialize replay memory \mathcal{D} to capacity N
                                                                                                      Initialize replay memory, Q-network
   Initialize action-value function Q with random weights
   for episode = 1, M do
       Initialise sequence s_1 = \{x_1\} and preprocessed sequenced \phi_1 = \phi(s_1)
       for t = 1, T do
            With probability \epsilon select a random action a_t
            otherwise select a_t = \max_a Q^*(\phi(s_t), a; \theta)
            Execute action a_t in emulator and observe reward r_t and image x_{t+1}
            Set s_{t+1} = s_t, a_t, x_{t+1} and preprocess \phi_{t+1} = \phi(s_{t+1})
            Store transition (\phi_t, a_t, r_t, \phi_{t+1}) in \mathcal{D}
            Sample random minibatch of transitions (\phi_j, a_j, r_j, \phi_{j+1}) from \mathcal{D}
            \text{Set } y_j = \left\{ \begin{array}{ll} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{array} \right.
            Perform a gradient descent step on (y_j - Q(\phi_j, a_j; \theta))^2 according to equation 3
       end for
   end for
```

```
Algorithm 1 Deep Q-learning with Experience Replay
   Initialize replay memory \mathcal{D} to capacity N
   Initialize action-value function Q with random weights
                                                                                       Play M episodes (full games)
  for episode = 1, M do
       Initialise sequence s_1 = \{x_1\} and preprocessed sequenced \phi_1 = \phi(s_1)
       for t = 1, T do
            With probability \epsilon select a random action a_t
            otherwise select a_t = \max_a Q^*(\phi(s_t), a; \theta)
            Execute action a_t in emulator and observe reward r_t and image x_{t+1}
            Set s_{t+1} = s_t, a_t, x_{t+1} and preprocess \phi_{t+1} = \phi(s_{t+1})
            Store transition (\phi_t, a_t, r_t, \phi_{t+1}) in \mathcal{D}
            Sample random minibatch of transitions (\phi_j, a_j, r_j, \phi_{j+1}) from \mathcal{D}
            \text{Set } y_j = \left\{ \begin{array}{ll} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{array} \right.
            Perform a gradient descent step on (y_j - Q(\phi_j, a_j; \theta))^2 according to equation 3
       end for
   end for
```

Algorithm 1 Deep Q-learning with Experience Replay Initialize replay memory \mathcal{D} to capacity NInitialize action-value function Q with random weights for episode = 1, M do Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$ Initialize state for t = 1, T do (starting game With probability ϵ select a random action a_t screen pixels) at the otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$ beginning of each Execute action a_t in emulator and observe reward r_t and image x_{t+1} episode Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$ Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D} Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D} Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$ Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3 end for end for

```
Algorithm 1 Deep Q-learning with Experience Replay
   Initialize replay memory \mathcal{D} to capacity N
   Initialize action-value function Q with random weights
  for episode = 1, M do
       Initialise sequence s_1 = \{x_1\} and preprocessed sequenced \phi_1 = \phi(s_1)
       for t = 1, T do
                                                                                                                           For each timestep t
            With probability \epsilon select a random action a_t
                                                                                                                           of the game
            otherwise select a_t = \max_a Q^*(\phi(s_t), a; \theta)
            Execute action a_t in emulator and observe reward r_t and image x_{t+1}
            Set s_{t+1} = s_t, a_t, x_{t+1} and preprocess \phi_{t+1} = \phi(s_{t+1})
            Store transition (\phi_t, a_t, r_t, \phi_{t+1}) in \mathcal{D}
            Sample random minibatch of transitions (\phi_j, a_j, r_j, \phi_{j+1}) from \mathcal{D}
           Set y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}
           Perform a gradient descent step on (y_j - Q(\phi_j, a_j; \theta))^2 according to equation 3
       end for
   end for
```

```
Algorithm 1 Deep Q-learning with Experience Replay
   Initialize replay memory \mathcal{D} to capacity N
   Initialize action-value function Q with random weights
  for episode = 1, M do
       Initialise sequence s_1 = \{x_1\} and preprocessed sequenced \phi_1 = \phi(s_1)
       for t = 1, T do
           With probability \epsilon select a random action a_t
                                                                                                                     With small probability,
           otherwise select a_t = \max_a Q^*(\phi(s_t), a; \theta)
                                                                                                                     select a random
           Execute action a_t in emulator and observe reward r_t and image x_{t+1}
                                                                                                                     action (explore),
            Set s_{t+1} = s_t, a_t, x_{t+1} and preprocess \phi_{t+1} = \phi(s_{t+1})
                                                                                                                     otherwise select
            Store transition (\phi_t, a_t, r_t, \phi_{t+1}) in \mathcal{D}
                                                                                                                     greedy action from
            Sample random minibatch of transitions (\phi_j, a_j, r_j, \phi_{j+1}) from \mathcal{D}
                                                                                                                     current policy
           Set y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}
           Perform a gradient descent step on (y_j - Q(\phi_j, a_j; \theta))^2 according to equation 3
       end for
   end for
```

```
Algorithm 1 Deep Q-learning with Experience Replay
   Initialize replay memory \mathcal{D} to capacity N
   Initialize action-value function Q with random weights
  for episode = 1, M do
       Initialise sequence s_1 = \{x_1\} and preprocessed sequenced \phi_1 = \phi(s_1)
       for t = 1, T do
            With probability \epsilon select a random action a_t
            otherwise select a_t = \max_a Q^*(\phi(s_t), a; \theta)
            Execute action a_t in emulator and observe reward r_t and image x_{t+1}
                                                                                                                           Take the action (a,),
            Set s_{t+1} = s_t, a_t, x_{t+1} and preprocess \phi_{t+1} = \phi(s_{t+1})
            Store transition (\phi_t, a_t, r_t, \phi_{t+1}) in \mathcal{D}
                                                                                                                           and observe the
            Sample random minibatch of transitions (\phi_j, a_j, r_j, \phi_{j+1}) from \mathcal{D}
                                                                                                                           reward r, and next
           Set y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}
                                                                                                                           state s<sub>t+1</sub>
           Perform a gradient descent step on (y_j - Q(\phi_j, a_j; \theta))^2 according to equation 3
       end for
   end for
```

```
Algorithm 1 Deep Q-learning with Experience Replay
   Initialize replay memory \mathcal{D} to capacity N
   Initialize action-value function Q with random weights
  for episode = 1, M do
       Initialise sequence s_1 = \{x_1\} and preprocessed sequenced \phi_1 = \phi(s_1)
       for t = 1, T do
            With probability \epsilon select a random action a_t
            otherwise select a_t = \max_a Q^*(\phi(s_t), a; \theta)
            Execute action a_t in emulator and observe reward r_t and image x_{t+1}
            Set s_{t+1} = s_t, a_t, x_{t+1} and preprocess \phi_{t+1} = \phi(s_{t+1})
                                                                                                                           Store transition in
            Store transition (\phi_t, a_t, r_t, \phi_{t+1}) in \mathcal{D}
                                                                                                                           replay memory
            Sample random minibatch of transitions (\phi_j, a_j, r_j, \phi_{j+1}) from \mathcal{D}
           Set y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}
           Perform a gradient descent step on (y_j - Q(\phi_j, a_j; \theta))^2 according to equation 3
       end for
   end for
```

```
Algorithm 1 Deep Q-learning with Experience Replay
   Initialize replay memory \mathcal{D} to capacity N
   Initialize action-value function Q with random weights
  for episode = 1, M do
       Initialise sequence s_1 = \{x_1\} and preprocessed sequenced \phi_1 = \phi(s_1)
       for t = 1, T do
           With probability \epsilon select a random action a_t
           otherwise select a_t = \max_a Q^*(\phi(s_t), a; \theta)
           Execute action a_t in emulator and observe reward r_t and image x_{t+1}
            Set s_{t+1} = s_t, a_t, x_{t+1} and preprocess \phi_{t+1} = \phi(s_{t+1})
            Store transition (\phi_t, a_t, r_t, \phi_{t+1}) in \mathcal{D}
            Sample random minibatch of transitions (\phi_j, a_j, r_j, \phi_{j+1}) from \mathcal D
                                                                                                                  Experience Replay:
           Set y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}
                                                                                                                  Sample a random
                                                                                                                  minibatch of transitions
           Perform a gradient descent step on (y_j - Q(\phi_j, a_j; \theta))^2 according to equation 3
                                                                                                                  from replay memory
       end for
                                                                                                                  and perform a gradient
   end for
                                                                                                                  descent step
```

https://arxiv.org/pdf/1312.5602.pdf

Policy Gradients

What is a problem with Q-learning?
The Q-function can be very complicated!

Example: a robot grasping an object has a very high-dimensional state => hard to learn exact value of every (state, action) pair

But the policy can be much simpler: just close your hand Can we learn a policy directly, e.g. finding the best policy from a collection of policies?

Policy Gradients

Formally, let's define a class of parameterized policies: $\Pi = \{\pi_{\theta}, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

$$J(heta) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | \pi_ heta
ight]$$

We want to find the optimal policy $\ heta^* = rg \max_{ heta} J(heta)$

How can we do this?

Gradient ascent on policy parameters!

REINFORCE Algorithm (Williams 1992)

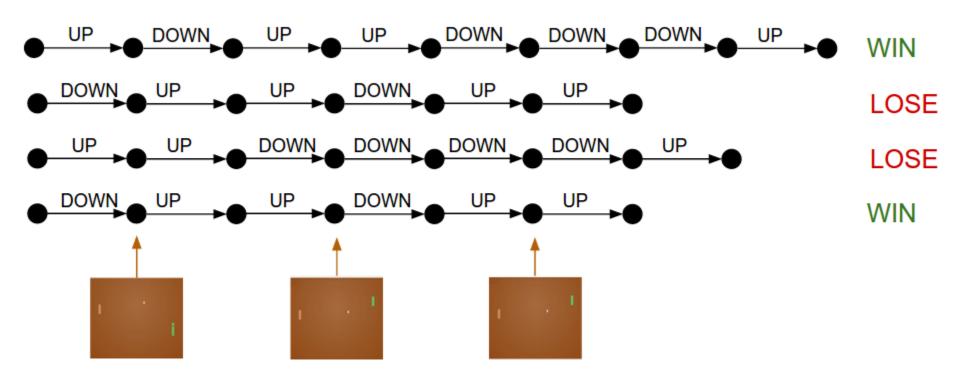
Gradient estimator:
$$\nabla_{\theta} J(\theta) pprox \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

Interpretation:

- If $r(\tau)$ is high, push up the probabilities of the actions seen
- If $r(\tau)$ is low, push down the probabilities of the actions seen

Might seem simplistic to say that if a trajectory is good then all its actions were good. But in expectation, it averages out!

Policy Gradients



REINFORCE Algorithm (Williams 1992)

Gradient estimator:
$$\nabla_{\theta}J(\theta) pprox \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t|s_t)$$

Interpretation:

- If $r(\tau)$ is high, push up the probabilities of the actions seen
- If $r(\tau)$ is low, push down the probabilities of the actions seen

Might seem simplistic to say that if a trajectory is good then all its actions were good. But in expectation, it averages out!

However, this also suffers from high variance because credit assignment is really hard. Can we help the estimator?

Variance Reduction

Gradient estimator: $\nabla_{\theta} J(\theta) pprox \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$

First idea: Push up probabilities of an action seen, only by the cumulative future reward from that state

$$abla_{ heta}J(heta)pprox \sum_{t\geq 0}\left(\sum_{t'\geq t}r_{t'}
ight)
abla_{ heta}\log\pi_{ heta}(a_t|s_t)$$

Second idea: Use discount factor γ to ignore delayed effects

$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

Variance Reduction: Baseline

Problem: The raw value of a trajectory isn't necessarily meaningful. For example, if rewards are all positive, you keep pushing up probabilities of actions.

What is important then? Whether a reward is better or worse than what you expect to get

Idea: Introduce a baseline function dependent on the state. Concretely, estimator is now:

$$abla_{ heta} J(heta) pprox \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} - b(s_t) \right)
abla_{ heta} \log \pi_{ heta}(a_t|s_t)$$

How to choose the baseline?

Want to push up the probability of an action from a state, if this action was better than the **expected value of what we should get from that state**.

Intuitively, we are happy with an action a_t in a state s_t if $Q^{\pi}(s_t, a_t) - V^{\pi}(s_t)$ is large. On the contrary, we are unhappy with an action if it's small.

Using this, we get the estimator:

$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} (Q^{\pi_{\theta}}(s_t, a_t) - V^{\pi_{\theta}}(s_t)) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

Actor-Critic Algorithm

We can combine Policy Gradients and Q-learning by training both an **actor** (the policy) and a **critic** (the Q-function).

- The actor decides which action to take, and the critic tells the actor how good its action was and how it should adjust
- Also alleviates the task of the critic as it only has to learn the values of (state, action) pairs generated by the policy
- Can also incorporate Q-learning tricks e.g. experience replay
- Define by the advantage function how much an action was better than expected $A^{\pi}(s,a) = Q^{\pi}(s,a) V^{\pi}(s)$

Policy Gradients tl;dr

- Objective: $\sum i Ai \log p(yi|xi)$
- x_i = state
- y_i = sampled action
- A_i = "advantage" e.g. +1/-1 for win/lose in simplest version, or discounted, or improvement over "baseline"

Policy Gradients vs Q-Learning

- Policy gradients suffers from high variance and instability; might want to make gradients smaller (e.g. relative to a baseline)
- Policy gradients can handle continuous action spaces (Gaussian policy)
- Estimating exact value of state-action pairs vs choosing what actions to take (value not important)
- Step-by-step (did I correctly estimate the reward at this time) vs delayed feedback (run policy and wait until game terminates)

RL for navigation

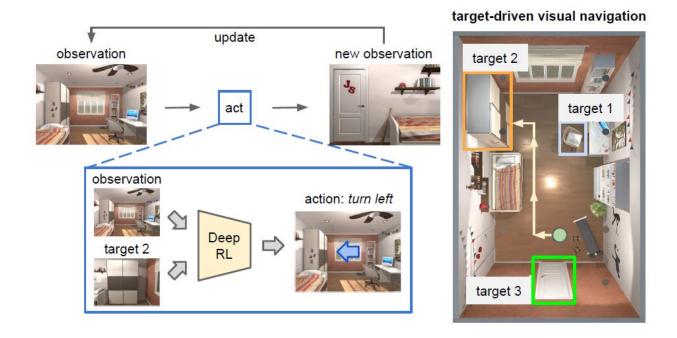
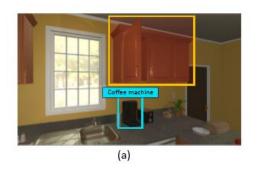


Fig. 1. The goal of our deep reinforcement learning model is to navigate towards a visual target with a minimum number of steps. Our model takes the current observation and the image of the target as input and generates an action in the 3D environment as the output. Our model learns to navigate to different targets in a scene without re-training.

RL for navigation



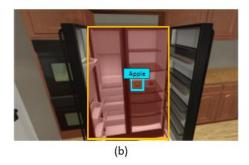


Figure 1: Our goal is to use scene priors to improve navigation in unseen scenes and towards novel objects. (a) There is no mug in the field of view of the agent, but the likely location for finding a mug is the cabinet near the coffee machine. (b) The agent has not seen a mango before, but it infers that the most likely location for finding a mango is the fridge since similar objects such as apple appear there as well. The most likely locations are shown with the orange box.

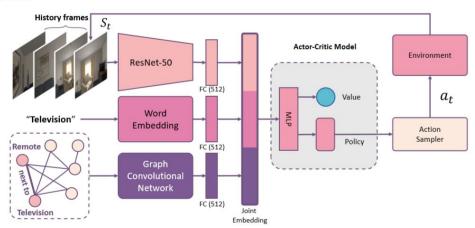


Figure 2: **Overview of the architecture.** Our model to incorporate semantic knowledge into semantic navigation. Specifically, we learn a policy network that decides an action based on the visual features of the current state, the semantic target category feature and the features extracted from the knowledge graph. We extract features from the parts of the knowledge graph that are activated.

RL for question-answering

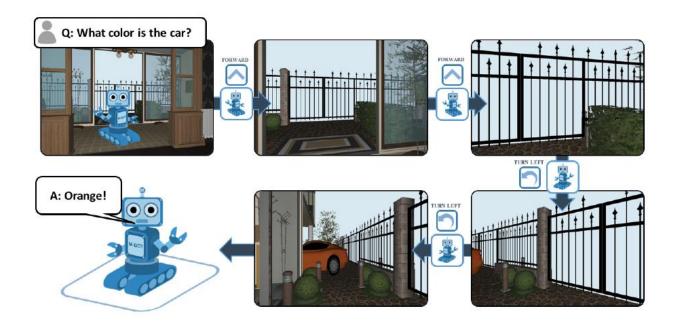


Figure 1: Embodied Question Answering – EmbodiedQA– tasks agents with navigating rich 3D environments in order to answer questions. These agents must jointly learn language understanding, visual reasoning, and goal-driven navigation to succeed.

RL for question-answering

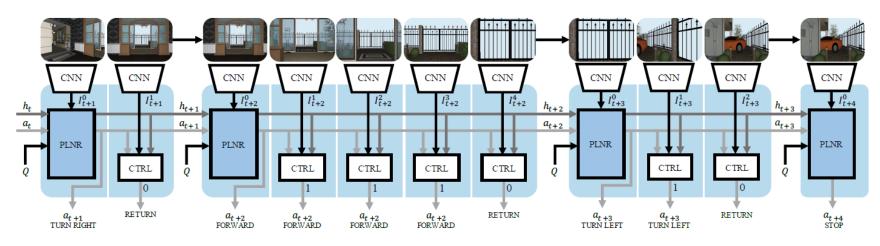


Figure 4: Our PACMAN navigator decomposes navigation into a planner and a controller. The planner selects actions and the controller executes these actions a variable number of times. This enables the planner to operate on shorter timescales, strengthening gradient flows.

RL for object detection

Sequence of attended regions to localize the object

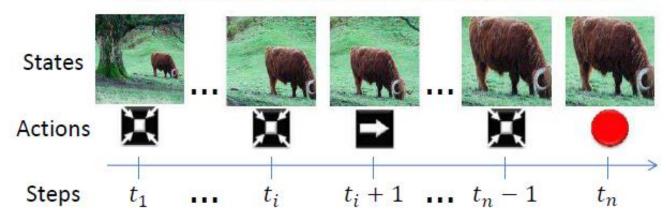


Figure 1. A sequence of actions taken by the proposed algorithm to localize a cow. The algorithm attends regions and decides how to transform the bounding box to progressively localize the object.

RL for object detection



Figure 2. Illustration of the actions in the proposed MDP, giving 4 degrees of freedom to the agent for transforming boxes.

$$R_a(s,s') = sign\left(IoU(b',g) - IoU(b,g)\right)$$
 $R_{\omega}(s,s') = \begin{cases} +\eta & \text{if } IoU(b,g) \ge \tau \\ -\eta & \text{otherwise} \end{cases}$

