Neural Net Examples

CS 1678 Intro to Deep Learning
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First architecture
Computing activations

• In all examples, \( x = [x_0 \ x_1 \ x_2] \), where \( x_0 = 1 \)
• Assume sigmoid activation function
• Initialize all weights to 0.1
• First example: \( x = [1 \ 1 \ 0] \)
• Second example: \( x = [1 \ 0 \ 1] \)
• Third example: \( x = [1 \ 1 \ 1] \)
Computing activations

• First example:
  • At hidden: $z_1 = ?$
  • At output: $y_1 = ? \ y_{\text{pred}} = ?$

• Second example:
  • At hidden: $z_1 = ?$
  • At output: $y_1 = ? \ y_{\text{pred}} = ?$

• Third example:
  • At hidden: $z_1 = ?$
  • At output: $y_1 = ? \ y_{\text{pred}} = ?$
Computing activations (answers)

• First example:
  • At hidden: \( z_1 = \frac{1}{1 + \exp(-(x_0 \times w^{(1)}_{10} + x_1 \times w^{(1)}_{11} + x_2 \times w^{(1)}_{12}))} \)
  • \( = \frac{1}{1 + \exp(-(1 \times 0.1 + 1 \times 0.1 + 0 \times 0.1))} = 0.5498 \)
  • At output: \( y_1 = \frac{1}{1 + \exp(-(z_0 \times w^{(2)}_{10} + z_1 \times w^{(2)}_{11}))} \)
  • \( = \frac{1}{1 + \exp(-(1 \times 0.1 + 0.5498 \times 0.1))} = 0.5387 \rightarrow y_{\text{pred}} = 1 \)

• Second example:
  • At hidden: \( z_1 = \frac{1}{1 + \exp(-(x_0 \times w^{(1)}_{10} + x_1 \times w^{(1)}_{11} + x_2 \times w^{(1)}_{12}))} \)
  • \( = \frac{1}{1 + \exp(-(1 \times 0.1 + 0 \times 0.1 + 1 \times 0.1))} = 0.5498 \)
  • At output: \( y_1 = \frac{1}{1 + \exp(-(z_0 \times w^{(2)}_{10} + z_1 \times w^{(2)}_{11}))} \)
  • \( = \frac{1}{1 + \exp(-(1 \times 0.1 + 0.5498 \times 0.1))} = 0.5387 \rightarrow y_{\text{pred}} = 1 \)
Computing activations (answers)

• Third example:
  • At hidden: \( z_1 = \frac{1}{1 + \exp(-(x_0 * w^{(1)}_{10} + x_1 * w^{(1)}_{11} + x_2 * w^{(1)}_{12}))} \)
  • \( = \frac{1}{1 + \exp(-(1*0.1+1*0.1+1*0.1))} = 0.5744 \)
  • At output: \( y_1 = \frac{1}{1 + \exp(-(z_0 * w^{(2)}_{10} + z_1 * w^{(2)}_{11}))} \)
  • \( = \frac{1}{1 + \exp(-(1*0.1+0.5744*0.1))} = 0.5393 \rightarrow y_{\text{pred}} = 1 \)
Second architecture
Computing activations

• In all examples, \( x = [x_0 \ x_1 \ x_2] \), where \( x_0 = 1 \)
• Assume sigmoid activation function
• Initialize all weights to 0.05
• First example: \( x = [1 \ 1 \ 0] \)
• Second example: \( x = [1 \ 0 \ 1] \)
• Third example: \( x = [1 \ 1 \ 1] \)
Computing activations

• First, second, third example:
  • At hidden:
    • $z_1 = ?$
    • $z_2 = ?$
  • At output:
    • $y_1 = ?$
    • $y_2 = ?$
    • $y_{\text{pred}} = [1 \ 1]$
Computing activations (answers)

- First example:
  - At hidden:
    - $z_1 = \frac{1}{1 + \exp(-x_0 w^{(1)}_{10} + x_1 w^{(1)}_{11} + x_2 w^{(1)}_{12})} = \frac{1}{1 + \exp(-(1*0.05+1*0.05+0*0.05))} = 0.5249$
    - $z_2 = \frac{1}{1 + \exp(-x_0 w^{(1)}_{20} + x_1 w^{(1)}_{21} + x_2 w^{(1)}_{22})} = \frac{1}{1 + \exp(-(1*0.05+0.5249*0.05+0.5249*0.05))} = 0.5249$
  - At output:
    - $y_1 = \frac{1}{1 + \exp(-(z_0 w^{(2)}_{10} + z_1 w^{(2)}_{11} + z_2 w^{(2)}_{12}))} = \frac{1}{1 + \exp(-(1*0.05+0.5249*0.05+0.5249*0.05))} = 0.5256$
    - $y_2 = \frac{1}{1 + \exp(-(z_0 w^{(2)}_{20} + z_1 w^{(2)}_{21} + z_2 w^{(2)}_{22}))} = \frac{1}{1 + \exp(-(1*0.05+0.5249*0.05+0.5249*0.05))} = 0.5256 \rightarrow y_{\text{pred}} = [1 \ 1]$
Computing activations (answers)

• Second example:
  • At hidden:
    • $z_1 = \frac{1}{1 + \exp(-(x_0 \times w^{(1)}_{10} + x_1 \times w^{(1)}_{11} + x_2 \times w^{(1)}_{12}))} = \frac{1}{1 + \exp(-(1 \times 0.05 + 0 \times 0.05 + 1 \times 0.05))} = 0.5249$
    • $z_2 = \frac{1}{1 + \exp(-(x_0 \times w^{(1)}_{20} + x_1 \times w^{(1)}_{21} + x_2 \times w^{(1)}_{22}))} = \frac{1}{1 + \exp(-(1 \times 0.05 + 0 \times 0.05 + 1 \times 0.05))} = 0.5249$
  • At output:
    • $y_1 = \frac{1}{1 + \exp(-(z_0 \times w^{(2)}_{10} + z_1 \times w^{(2)}_{11} + z_2 \times w^{(2)}_{12}))} = \frac{1}{1 + \exp(-(1 \times 0.05 + 0.5249 \times 0.05 + 0.5249 \times 0.05))} = 0.5256$
    • $y_2 = \frac{1}{1 + \exp(-(z_0 \times w^{(2)}_{20} + z_1 \times w^{(2)}_{21} + z_2 \times w^{(2)}_{22}))} = \frac{1}{1 + \exp(-(1 \times 0.05 + 0.5249 \times 0.05 + 0.5249 \times 0.05))} = 0.5256 \rightarrow y_{\text{pred}} = [1 1]$
Computing activations (answers)

• Third example:
  • At hidden:
    • $z_1 = 1 / [1 + \exp(-(x_0 * w^{(1)}_{10} + x_1 * w^{(1)}_{11} + x_2 * w^{(1)}_{12}))] = 1 / [1 + \exp(-(1*0.05+1*0.05+1*0.05))] = 0.5374$
    • $z_2 = 1 / [1 + \exp(-(x_0 * w^{(1)}_{20} + x_1 * w^{(1)}_{21} + x_2 * w^{(1)}_{22}))] = 1 / [1 + \exp(-(1*0.05+1*0.05+1*0.05))] = 0.5374$
  • At output:
    • $y_1 = 1 / [1 + \exp(-(z_0 * w^{(2)}_{10} + z_1 * w^{(2)}_{11} + z_2 * w^{(2)}_{12}))]$
      $= 1 / [1 + \exp(-(1*0.05+0.5374*0.05+0.5374*0.05))] = 0.5259$
    • $y_2 = 1 / [1 + \exp(-(z_0 * w^{(2)}_{20} + z_1 * w^{(2)}_{21} + z_2 * w^{(2)}_{22}))]$
      $= 1 / [1 + \exp(-(1*0.05+0.5374*0.05+0.5374*0.05))] = 0.5259 \rightarrow y_{pred} = [1 \ 1]
Training the first network

- Perform backpropagation using stochastic gradient descent (one sample at a time)
- Weights are initially all 0.1
- Learning rate is 0.3
- Sigmoid activation function at hidden and output
- \( d \frac{s(x)}{dx} = s(x) (1 - s(x)) \) 
- Samples have the following labels:
  - First example: \( x = [1\ 1\ 0], y = 1 \)
  - Second example: \( x = [1\ 0\ 1], y = 0 \)
  - Third example: \( x = [1\ 1\ 1], y = 1 \)
- Preview: What do you expect final weights to be?
Learning from first example

• First example: \( x = [1 \ 1 \ 0] \), \( y = 1 \)
• Weights are \( w^{(1)}_{10} = w^{(1)}_{11} = w^{(1)}_{12} = w^{(2)}_{10} = w^{(2)}_{11} = 0.1 \)
• Activations are \( z_1 = 0.5498 \), \( y_1 = 0.5387 \)
• Compute errors:
  • \( \delta_{y1} = ? \)
  • \( \delta_{z1} = ? \)
• Update weights:
  • \( w^{(2)}_{10} = w^{(2)}_{10} - ? \)
  • \( w^{(2)}_{11} = w^{(2)}_{11} - ? \)
  • \( w^{(1)}_{10} = w^{(1)}_{10} - ? \)
  • \( w^{(1)}_{11} = w^{(1)}_{11} - ? \)
  • \( w^{(1)}_{12} = w^{(1)}_{12} - ? \)
Learning from first example (answers)

• First example: $x = [1 \ 1 \ 0]$, $y = 1$
• Weights are $w^{(1)}_{10} = w^{(1)}_{11} = w^{(1)}_{12} = w^{(2)}_{10} = w^{(2)}_{11} = 0.1$
• Activations are $z_1 = 0.5498$, $y_1 = 0.5387$
• Compute errors:
  • $\delta_{y_1} = y_1 \cdot (1-y_1) \cdot (y_1-y_{\text{true}}) = 0.5387 \cdot (1-0.5387) \cdot (0.5387-1) = -0.1146$
  • $\delta_{z_1} = z_1 \cdot (1-z_1) \cdot (w^{(2)}_{11} \cdot \delta_{y_1}) = 0.5498 \cdot (1-0.5498) \cdot [0.1 \cdot -0.1146] = -0.0028$
• Update weights:
  • $w^{(2)}_{10} = w^{(2)}_{10} - 0.3 \cdot \delta_{y_1} \cdot x_0 = 0.1 + 0.3 \cdot 0.1146 \cdot 1 = 0.1343$
  • $w^{(2)}_{11} = w^{(2)}_{11} - 0.3 \cdot \delta_{y_1} \cdot z_1 = 0.1 + 0.3 \cdot 0.1146 \cdot 0.5498 = 0.1189$
  • $w^{(1)}_{10} = w^{(1)}_{10} - 0.3 \cdot \delta_{z_1} \cdot x_0 = 0.1 + 0.3 \cdot 0.0028 \cdot 1 = 0.1008$
  • $w^{(1)}_{11} = w^{(1)}_{11} - 0.3 \cdot \delta_{z_1} \cdot x_1 = 0.1 + 0.3 \cdot 0.0028 \cdot 1 = 0.1008$
  • $w^{(1)}_{12} = w^{(1)}_{12} - 0.3 \cdot \delta_{z_1} \cdot x_2 = 0.1 + 0.3 \cdot 0.0028 \cdot 0 = 0.1$
Learning from second example (answers)

- Second example: \( x = [1 \ 0 \ 1] \), \( y = 0 \)
- Weights are \( w^{(1)}_{10} = w^{(1)}_{11} = 0.1008 \), \( w^{(1)}_{12} = 0.1 \), \( w^{(2)}_{10} = 0.1343 \), \( w^{(2)}_{11} = 0.1189 \)
- Activations are (recompute with new weights):
  - \( z_1 = \frac{1}{1 + \exp(-(x_0 \cdot w^{(1)}_{10} + x_1 \cdot w^{(1)}_{11} + x_2 \cdot w^{(1)}_{12}))} = \frac{1}{1 + \exp(-1*0.1008+0*0.1008+1*0.1))} = 0.55 \)
  - \( y_1 = \frac{1}{1 + \exp(-z_0 \cdot w^{(2)}_{10} + z_1 \cdot w^{(2)}_{11}))} = \frac{1}{1 + \exp(-1*0.1343+0.55*0.1189))} = 0.5498 \)
- Compute errors:
  - \( \delta_{y_1} = y_1 \cdot (1-y_1) \cdot (y_1-y_{true}) = 0.5498 \cdot (1-0.5498) \cdot (0.5498-0) = 0.1361 \)
  - \( \delta_{z_1} = z_1 \cdot (1-z_1) \cdot (w^{(2)}_{11} \cdot \delta_{y_1}) = 0.55 \cdot (1-0.55) \cdot [0.1189 \cdot 0.1361] = 0.004 \)
- Update weights:
  - \( w^{(2)}_{10} = w^{(2)}_{10} - 0.3 \cdot \delta_{y_1} \cdot z_0 = 0.1343 - 0.3 \cdot 0.1361 \cdot 1 = 0.0935 \)
  - \( w^{(2)}_{11} = w^{(2)}_{11} - 0.3 \cdot \delta_{y_1} \cdot z_1 = 0.1189 - 0.3 \cdot 0.1361 \cdot 0.55 = 0.0964 \)
  - \( w^{(1)}_{10} = w^{(1)}_{10} - 0.3 \cdot \delta_{z_1} \cdot x_0 = 0.1008 - 0.3 \cdot 0.004 \cdot 1 = 0.0996 \)
  - \( w^{(1)}_{11} = w^{(1)}_{11} - 0.3 \cdot \delta_{z_1} \cdot x_1 = 0.1008 - 0.3 \cdot 0.004 \cdot 0 = 0.1008 \)
  - \( w^{(1)}_{12} = w^{(1)}_{12} - 0.3 \cdot \delta_{z_1} \cdot x_2 = 0.1 - 0.3 \cdot 0.004 \cdot 1 = 0.0988 \)
Learning from third example (answers)

• Third example: \( x = [1 \ 1 \ 1], \ y = 1 \)

  • Weights are \( w^{(1)}_{10} = 0.0996, \ w^{(1)}_{11} = 0.1008, \ w^{(1)}_{12} = 0.0988, \ w^{(2)}_{10} = 0.0935, \ w^{(2)}_{11} = 0.0964 \)

• Activations are (recompute with new weights):
  - \( z_1 = 1 / [1 + \exp(-(x_0 \cdot w^{(1)}_{10} + x_1 \cdot w^{(1)}_{11} + x_2 \cdot w^{(1)}_{12}))] = 1 / [1 + \exp(- (1 \cdot 0.0996 + 1 \cdot 0.1008 + 1 \cdot 0.0988))] = 0.5742 \)
  - \( y_1 = 1 / [1 + \exp(-(z_0 \cdot w^{(2)}_{10} + z_1 \cdot w^{(2)}_{11}))] = 1 / [1 + \exp(- (1 \cdot 0.0935 + 0.5735 \cdot 0.0964))] = 0.5371 \)

• Compute errors:
  - \( \delta_y = y_1 \cdot (1-y_1) \cdot (y_1-y_{true}) = 0.5371 \cdot (1-0.5371) \cdot (0.5371-1) = -0.1151 \)
  - \( \delta_z = z_1 \cdot (1-z_1) \cdot (w^{(2)}_{11} \cdot \delta_y) = 0.5742 \cdot (1-0.5742) \cdot [0.0964 \cdot -0.1151] = -0.0027 \)

• Update weights:
  - \( w^{(2)}_{10} = w^{(2)}_{10} - 0.3 \cdot \delta_y \cdot z_0 = 0.0935 + 0.3 \cdot 0.1151 \cdot 1 = 0.1280 \)
  - \( w^{(2)}_{11} = w^{(2)}_{11} - 0.3 \cdot \delta_y \cdot z_1 = 0.0964 + 0.3 \cdot 0.1151 \cdot 0.5735 = 0.1162 \)
  - \( w^{(1)}_{10} = w^{(1)}_{10} - 0.3 \cdot \delta_z \cdot x_0 = 0.0996 + 0.3 \cdot 0.0027 \cdot 1 = 0.1004 \)
  - \( w^{(1)}_{11} = w^{(1)}_{11} - 0.3 \cdot \delta_z \cdot x_1 = 0.1008 + 0.3 \cdot 0.0027 \cdot 1 = 0.1016 \)
  - \( w^{(1)}_{12} = w^{(1)}_{12} - 0.3 \cdot \delta_z \cdot x_2 = 0.0988 + 0.3 \cdot 0.0027 \cdot 1 = 0.0996 \)
Recap

- Do the $w^{(1)}$ weights we obtained make sense?