CS 1675: Intro to Machine Learning Neural Networks

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Plan for this lecture

- Neural network basics
 - Definition and architecture
 - Biological inspiration
- Training
 - Loss functions
 - Backpropagation
 - Dealing with sparse data and overfitting
- Specialized variants (briefly)
 - Convolutional networks (CNNs) e.g. for images
 - Recurrent networks (RNNs) for sequences, language

Neural network definition

Figure 5.1 Network diagram for the twolayer neural network corresponding to (5.7). The input, hidden, and output variables are represented by nodes, and the weight parameters are represented by links between the nodes, in which the bias parameters are denoted by links coming from additional input and hidden variables x_0 and z_0 . Arrows denote the direction of information flow through the network during forward propagation.



 Nonlinear activation function h (e.g. sigmoid, tanh, RELU): $z_j = h(a_j)$ $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

i=1

Figure from Christopher Bishop



• Layer 3 (final)

 $a_k =$

Outputs

(binary)
$$y_k = \sigma(a_k) = \frac{1}{1 + \exp(-a_k)}$$
 (multiclass)
 $y_k = \frac{\exp(a_k)}{\sum_j \exp(a_j)}$
• Finally:

(binary)
$$y_k(\mathbf{x}, \mathbf{w}) = \sigma \left(\sum_{j=1}^M w_{kj}^{(2)} h\left(\sum_{i=1}^D w_{ji}^{(1)} x_i + w_{j0}^{(1)} \right) + w_{k0}^{(2)} \right)$$

Activation functions





Maxout $\max(w_1^T x + b_1, w_2^T x + b_2)$ **ELU** $f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha (\exp(x) - 1) & \text{if } x \le 0 \end{cases}$

A multi-layer neural network



- Nonlinear classifier
- Can approximate any continuous function to arbitrary accuracy given sufficiently many hidden units

Inspiration: Neuron cells

- Neurons
 - accept information from multiple inputs
 - transmit information to other neurons
- Multiply inputs by weights along edges
- Apply some function to the set of inputs at each node
- If output of function over threshold, neuron "fires"



Biological analog



Biological analog



Multilayer networks

- Cascade neurons together
- Output from one layer is the input to the next
- Each layer has its own sets of weights















Deep neural networks

- Lots of hidden layers
- Depth = power (usually)



How do we train them?

- No closed-form solution for the weights
- We will iteratively find such a set of weights that allow the outputs to match the desired outputs
- We want to minimize a loss function (a function of the weights in the network)
- For now let's simplify and assume there's a single layer of weights in the network

Classification goal

airplane	🛁 🔊 🐖 📈 🍬 – 🛃 🕅 🛶 💒
automobile	🔁 🖏 🚵 😂 🐝 😂 🦈 🐝
bird	in the second
cat	N N N N N N N N N N N N N N N N N N N
deer	
dog	1978 🔬 👟 🌦 🎊 🧑 📢 🔬 🎉
frog	NY NE CON SA
horse	
ship	🗃 🍻 🔤 🕍 🖕 🤌 📂 👛
truck	🚄 🍱 💒 🎆 🚝 🚞 🏹 🕋 🕌

Example dataset: CIFAR-10 10 labels 50,000 training images each image is 32x32x3 10,000 test images.

Classification scores

$$f(x,W) = Wx$$

 $f(x,W)$



10 numbers, indicating class scores

[32x32x3] array of numbers 0...1 (3072 numbers total)

Linear classifier



Example with an image with 4 pixels, and 3 classes (cat/dog/ship)



Andrej Karpathy

Linear classifier

Going forward: Loss function/Optimization



TODO:

- 1. Define a **loss function** that quantifies our unhappiness with the scores across the training data.
- 2. Come up with a way of efficiently finding the parameters that minimize the loss function.
 (optimization)

Linear classifier

Suppose: 3 training examples, 3 classes. With some W the scores f(x, W) = Wx are:



cat	3.2	1.3	2.2
car	5.1	4.9	2.5
frog	-1.7	2.0	-3.1

Suppose: 3 training examples, 3 classes. With some W the scores f(x, W) = Wx are:



cat	3.2	1.3	2.2
car	5.1	4.9	2.5
frog	-1.7	2.0	-3.1

Hinge loss:

Given an example (x_i, y_i) where x_i is the image and where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the loss has the form:

$$L_i = \sum_{j
eq y_i} \max(0, s_j - s_{y_i} + 1)$$

Want:
$$S_{y_i} \ge S_j + 1$$

i.e. $S_j - S_{y_i} + 1 \le 0$

If true, loss is 0 If false, loss is magnitude of violation

Suppose: 3 training examples, 3 classes. With some W the scores f(x, W) = Wx are:



Hinge loss:

Given an example (x_i, y_i) where x_i is the image and where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the loss has the form:

 $\begin{aligned} L_i &= \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1) \\ &= \max(0, 5.1 - 3.2 + 1) \\ &+ \max(0, -1.7 - 3.2 + 1) \\ &= \max(0, 2.9) + \max(0, -3.9) \\ &= 2.9 + 0 \\ &= 2.9 \end{aligned}$

Suppose: 3 training examples, 3 classes. With some W the scores f(x, W) = Wx are:



Hinge loss:

Given an example (x_i, y_i) where x_i is the image and where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the loss has the form:

 $\begin{aligned} L_i &= \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1) \\ &= \max(0, 1.3 - 4.9 + 1) \\ &+ \max(0, 2.0 - 4.9 + 1) \\ &= \max(0, -2.6) + \max(0, -1.9) \\ &= 0 + 0 \\ &= 0 \end{aligned}$

Suppose: 3 training examples, 3 classes. With some W the scores f(x, W) = Wx are:



Hinge loss:

Given an example (x_i, y_i) where x_i is the image and where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the loss has the form:

$$\begin{split} L_i &= \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1) \\ &= \max(0, 2.2 - (-3.1) + 1) \\ &+ \max(0, 2.5 - (-3.1) + 1) \\ &= \max(0, 5.3 + 1) \\ &+ \max(0, 5.6 + 1) \\ &= 6.3 + 6.6 \\ &= 12.9 \end{split}$$

Suppose: 3 training examples, 3 classes. With some W the scores f(x, W) = Wx are:



cat	3.2	1.3	2.2	
car	5.1	4.9	2.5	
frog	-1.7	2.0	-3.1	
Losses:	2.9	0	12.9	-

Hinge loss:

Given an example (x_i, y_i) where x_i is the image and where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the loss has the form:

$$L_i = \sum_{j
eq y_i} \max(0, s_j - s_{y_i} + 1)$$

and the full training loss is the mean over all examples in the training data:

$$L = rac{1}{N} \sum_{i=1}^N L_i$$

L = (2.9 + 0 + 12.9)/3 = 15.8 / 3 = **5.3**

f(x,W) = Wx

 $L = rac{1}{N} \sum_{i=1}^{N} \sum_{j
eq y_i} \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1)$

Weight Regularization

 λ = regularization strength (hyperparameter)

$$L=rac{1}{N}\sum_{i=1}^N\sum_{j
eq y_i} \max(0,f(x_i;W)_j-f(x_i;W)_{y_i}+1)+\lambda R(W)$$

In common use: L2 regularization L1 regularization Dropout (will see later)

$$egin{aligned} R(W) &= \sum_k \sum_l W_{k,l}^2 \ R(W) &= \sum_k \sum_l |W_{k,l}| \end{aligned}$$

Another loss: Softmax (cross-entropy)



3.2

5.1

-1.7

scores = unnormalized log probabilities of the classes.

$$P(Y=k|X=x_i)=rac{e^{s_k}}{\sum_j e^{s_j}}$$
 where $oldsymbol{s}=f(x_i;W)$

Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

$$L_i = -\log P(Y=y_i|X=x_i)$$

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cat

car

frog

Another loss: Softmax (cross-entropy)



Other losses

• Triplet loss (Schroff, FaceNet, CVPR 2015)

$$\sum_{i=1}^{N} \left[\|f(x_{i}^{a}) - f(x_{i}^{p})\|_{2}^{2} - \|f(x_{i}^{a}) - f(x_{i}^{n})\|_{2}^{2} + \alpha \right]_{4}$$

a denotes anchor p denotes positive n denotes negative



Figure 3. The **Triplet Loss** minimizes the distance between an *an-chor* and a *positive*, both of which have the same identity, and maximizes the distance between the *anchor* and a *negative* of a different identity.

Anything you want! (almost)

To minimize loss, use gradient descent



Gradient descent in multi-layer nets

- How to update the weights at all layers?
- Answer: backpropagation of error from higher layers to lower layers


Computing gradient for each weight

• We need to move weights in direction opposite to gradient of loss wrt that weight:

 Loss depends on weights in an indirect way, so we'll use the chain rule and compute: dE/dw_{ii} = dE/dz_i dz_i/da_i da_i/dw_{ii}

(and similarly for dE/dw_{ki})

- The error (dE/dz_j) is hard to compute (indirect, need chain rule again)
- We'll simplify the computation by doing it step by step via *backpropagation* of error

Backpropagation: Graphic example

First calculate error of output units and use this to change the top layer of weights.



Backpropagation: Graphic example

Next calculate error for hidden units based on errors on the output units it feeds into.



Backpropagation: Graphic example

Finally update bottom layer of weights based on errors calculated for hidden units.









Generic example



Andrej Karpathy

Generic example



Andrej Karpathy



$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4



$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4
 $q = x + y$ $\frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$

$$f=qz \qquad \quad rac{\partial f}{\partial q}=z, rac{\partial f}{\partial z}=q$$

Want:
$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$$



$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4

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$$x = -2, y = 5, z = -4$$

$$y = 5, z = -4$$

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$$y = 5, z = -4$$

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$$y = 5, z = -4$$

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$$z = -4$$

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$$z = -4$$

Want:
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$$\frac{\partial f}{\partial z}$$

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Want:
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Want:
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$$f = qz \quad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

$$\overset{x -2}{y + \frac{q}{3}}$$

$$y = \frac{y}{4}$$

$$\frac{y - \frac{1}{4}}{3}$$

Chain rule: $\frac{\partial f}{\partial y}$

$$\frac{\partial f}{\partial y}$$

$$f(x, y, z) = (x + y)z$$

e.g. $x = -2, y = 5, z = -4$

$$q = x + y \qquad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \qquad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

$$\frac{\partial f}{\partial x}$$

Want:
$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$$

How to compute gradient in neural net?

• In a neural network:

$$a_j = \sum_i w_{ji} z_i \qquad z_j = h(a_j)$$

• Gradient is (using chain rule):



• Denote the "errors" as:

$$\delta_j \equiv \frac{\partial E_n}{\partial a_j}$$

• Also:

$$\frac{\partial a_j}{\partial w_{ji}} = z_i$$

How to compute gradient in neural net?

- For output units (identity output, squared error loss): $\delta_k = y_k t_k$
- For hidden units (using chain rule again):

$$\delta_j \equiv \frac{\partial E_n}{\partial a_j} = \sum_k \frac{\partial E_n}{\partial a_k} \frac{\partial a_k}{\partial a_j}$$

• Backprop formula:

Example

- Two layer network w/ tanh at hidden layer: $h(a) \equiv \tanh(a) = \frac{e^a - e^{-a}}{e^a + e^{-a}}$
- Derivative: $h'(a) = 1 h(a)^2$
- Minimize: $E_n = \frac{1}{2} \sum_{k=1}^{K} (y_k t_k)^2$
- Forward propagation:

$$a_{j} = \sum_{i=0}^{D} w_{ji}^{(1)} x_{i}$$
$$z_{j} = \tanh(a_{j})$$
$$y_{k} = \sum_{j=0}^{M} w_{kj}^{(2)} z_{j}$$

Example

• Errors at output (identity function at output):

$$\delta_k = y_k - t_k$$

• Errors at hidden units:

(

$$\delta_j = h'(a_j) \sum_k w_{kj} \delta_k$$

$$\delta_j = (1 - z_j^2) \sum_{k=1}^{\infty} w_{kj} \delta_k$$

K

• Derivatives wrt weights:

$$\frac{\partial E_n}{\partial w_{ji}^{(1)}} = \delta_j x_i, \qquad \frac{\partial E_n}{\partial w_{kj}^{(2)}} = \delta_k z_j$$

Same example with graphic and math

First calculate error of output units and use this to change the top layer of weights.



Same example with graphic and math

Next calculate error for hidden units based on errors on the output units it feeds into.



Same example with graphic and math

Finally update bottom layer of weights based on errors calculated for hidden units.



Example: algorithm for sigmoid, sqerror

- Initialize all weights to small random values
- Until convergence (e.g. all training examples' error small, or error stops decreasing) repeat:
 - For each (x, t=class(x)) in training set:
 - Calculate network outputs: y_k
 - Compute errors (gradients wrt activations) for each unit:

$$\begin{split} & \gg \delta_k &= y_k \ (1-y_k) \ (y_k - t_k) & \text{for output units} \\ & \gg \delta_j &= z_j \ (1-z_j) \ \sum_k w_{kj} \ \delta_k & \text{for hidden units} \\ & - \text{Update weights:} \end{split}$$

 $w_{kj} = w_{kj} - \eta \ \delta_k \ z_j$ for output units $w_{ji} = w_{ji} - \eta \ \delta_j \ x_i$ for hidden units

Recall:
$$\mathbf{w}_{ji} = \mathbf{w}_{ji} - \eta \, \mathbf{dE}/\mathbf{dz}_j \, \mathbf{dz}_j/\mathbf{da}_j \, \mathbf{da}_j/\mathbf{dw}_{ji}$$

$$\delta_j = h'(a_j) \sum_k w_{kj} \delta_k$$

Adapted from R. Hwa, R. Mooney

Comments on training algorithm

- Not guaranteed to converge to zero training error, may converge to local optima or oscillate indefinitely.
- However, in practice, does converge to low error for many large networks on real data.
- Thousands of epochs (epoch = network sees all training data once) may be required, hours or days to train.
- To avoid local-minima problems, run several trials starting with different random weights (*random restarts*), and take results of trial with lowest training set error.
- May be hard to set learning rate and to select number of hidden units and layers.
- Neural networks had fallen out of fashion in 90s, early 2000s; back with a new name and significantly improved performance (deep networks trained with dropout and lots of data).

Dealing with sparse data

- Deep neural networks require lots of data, and can overfit easily
- The more weights you need to learn, the more data you need
- That's why with a deeper network, you need more data for training than for a shallower network
- Ways to prevent overfitting include:
 - Using a validation set to stop training or pick parameters
 - Regularization
 - Transfer learning
 - Data augmentation

Over-training prevention

• Running too many epochs can result in over-fitting.



 Keep a hold-out validation set and test accuracy on it after every epoch. Stop training when additional epochs actually increase validation error.

Determining best number of hidden units

- Too few hidden units prevents the network from adequately fitting the data.
- Too many hidden units can result in over-fitting.



• Use internal cross-validation to empirically determine an optimal number of hidden units.

Effect of number of neurons



more neurons = more capacity

Regularization

- L1, L2 regularization (weight decay)
- Dropout
 - Randomly turn off some neurons
 - Allows individual neurons to independently be responsible for performance



Dropout: A simple way to prevent neural networks from overfitting [Srivastava JMLR 2014]

Adapted from Jia-bin Huang
Effect of regularization

Do not use size of neural network as a regularizer. Use stronger regularization instead:



(you can play with this demo over at ConvNetJS: <u>http://cs.stanford.</u> edu/people/karpathy/convnetjs/demo/classify2d.html)

Transfer learning

- If you have sparse data in your domain of interest (target), but have rich data in a disjoint yet related domain (source),
- You can train the early layers on the source domain, and only the last few layers on the *target domain:*



Set these to the already learned Learn these on your own task weights from another network

Transfer learning



Another option: use network as feature extractor, train SVM/LR on extracted features for target task

Transfer learning



Another solution: Data augmentation

Create *virtual* training samples; if images:

- Horizontal flip
- Random crop
- Color casting
- Geometric distortion



Packages

TensorFlow

Torch / PyTorch

Keras

Caffe and Caffe Model Zoo

Learning Resources

http://deeplearning.net/

http://cs231n.stanford.edu (CNNs, vision)

http://cs224d.stanford.edu/ (RNNs, language)

Summary

- Feed-forward network architecture
- Training deep neural nets
 - We need an objective function that measures and guides us towards good performance
 - We need a way to minimize the loss function: (stochastic, mini-batch) gradient descent
 - We need backpropagation to propagate error towards all layers and change weights at those layers
- Practices for preventing overfitting, training with little data

Convolutional Neural Networks

"Shallow" vs. "deep" vision architectures





Deep learning: "Deep" architecture Image/ Video Pixels
Layer 1 ... Layer N Simple classifier Class

Lana Lazebnik

Example: CNN features for detection

R-CNN: Regions with CNN features



Object detection system overview. Our system (1) takes an input image, (2) extracts around 2000 bottom-up region proposals, (3) computes features for each proposal using a large convolutional neural network (CNN), and then (4) classifies each region using class-specific linear SVMs. R-CNN achieves a mean average precision (mAP) of 53.7% on PASCAL VOC 2010. For comparison, Uijlings et al. (2013) report 35.1% mAP using the same region proposals, but with a spatial pyramid and bag-of-visual-words approach. The popular deformable part models perform at 33.4%.

R. Girshick, J. Donahue, T. Darrell, and J. Malik, <u>Rich Feature Hierarchies for Accurate</u> <u>Object Detection and Semantic Segmentation</u>, CVPR 2014.

Lana Lazebnik

Convolutional Neural Networks (CNN)

- Neural network with specialized connectivity structure
- Stack multiple stages of feature extractors
- Higher stages compute more global, more invariant, *more abstract* features
- Classification layer at the end





Y. LeCun, L. Bottou, Y. Bengio, and P. Haffner, <u>Gradient-based learning applied to document</u> recognition, Proceedings of the IEEE 86(11): 2278–2324, 1998.

Adapted from Lana Lazebnik

Convolutional Neural Networks (CNN)

- Feed-forward feature extraction:
 - 1. Convolve input with learned filters
 - 2. Apply non-linearity
 - 3. Spatial pooling (downsample)
- Supervised training of convolutional filters by back-propagating classification error



1. Convolution

- Apply learned filter weights
- One feature map per filter
- Stride can be greater than
 1 (faster, less memory)





Feature Map

Adapted from Rob Fergus

2. Non-Linearity

- Per-element (independent)
- Options:
 - Tanh
 - Sigmoid
 - Rectified linear unit (ReLU)
 - Avoids saturation issues



3. Spatial Pooling

- Sum or max over non-overlapping / overlapping regions
- Role of pooling:
 - Invariance to small transformations
 - Larger receptive fields (neurons see more of input)







Sum



3. Spatial Pooling

- Sum or max over non-overlapping / overlapping regions
- Role of pooling:
 - Invariance to small transformations
 - Larger receptive fields (neurons see more of input)





32x32x3 image



5x5x3 filter

Convolve the filter with the image i.e. "slide over the image spatially, computing dot products"

Convolution Layer



Convolution Layer



activation map



Convolution Layer

consider a second, green filter

32x32x3 image 5x5x3 filter convolve (slide) over all spatial locations

28

activation maps

For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:



We stack these up to get a "new image" of size 28x28x6!

Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



Preview: ConvNet is a sequence of Convolutional Layers, interspersed with activation functions



Preview

[From recent Yann LeCun slides]



Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]



The First Popular Architecture: AlexNet



Recurrent Neural Networks



vanilla neural networks

Andrej Karpathy



e.g. image captioning image -> sequence of words



e.g. **sentiment classification** sequence of words -> sentiment







many to many



e.g. **machine translation** seq of words -> seq of words



e.g. video classification on frame level

Recurrent Neural Network



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Recurrent Neural Network


Recurrent Neural Network



Recurrent Neural Network

We can process a sequence of vectors \mathbf{x} by applying a recurrence formula at every time step:

$$h_t = f_W(h_{t-1}, x_t)$$

Notice: the same function and the same set of parameters are used at every time step.



(Vanilla) Recurrent Neural Network

The state consists of a single "hidden" vector h:



Character-level language model example

Vocabulary: [h,e,l,o]



Character-level language model example

Vocabulary: [h,e,l,o]



Character-level language model example

$$h_t = anh(W_{hh}h_{t-1} + W_{xh}x_t)$$



Character-level language model example

Vocabulary: [h,e,l,o]



Extensions

- Vanishing gradient problem makes it hard to model long sequences
 - Multiplying together many values between 0 and 1 (range of gradient of sigmoid, tanh)
- One solution: Use RELU
- Another solution: Use RNNs with gates
 - Adaptively decide how much of memory to keep
 - Gated Recurrent Units (GRUs), Long Short Term
 Memories (LSTMs)

Generating poetry with RNNs



Generating poetry with RNNs

tyntd-iafhatawiaoihrdemot lytdws e ,tfti, astai f ogoh eoase rrranbyne 'nhthnee e plia tklrgd t o idoe ns,smtt h ne etie h,hregtrs nigtike,aoaenns lng				
train more				
"Tmont thithey" fomesscerliund Keushey. Thom here sheulke, anmerenith ol sivh I lalterthend Bleipile shuwy fil on aseterlome coaniogennc Phe lism thond hon at. MeiDimorotion in ther thize."				
train more				
Aftair fall unsuch that the hall for Prince Velzonski's that me of her hearly, and behs to so arwage fiving were to it beloge, pavu say falling misfort how, and Gogition is so overelical and ofter.				
train more				
"Why do what that day," replied Natasha, and wishing to himself the fact the princess, Princess Mary was easier, fed in had oftened him. Pierre aking his soul came to the packs and drove up his father-in-law women.				

More info: <u>http://karpathy.github.io/2015/05/21/rnn-effectiveness/</u>

Generating poetry with RNNs

PANDARUS:

Alas, I think he shall be come approached and the day When little srain would be attain'd into being never fed, And who is but a chain and subjects of his death, I should not sleep.

Second Senator:

They are away this miseries, produced upon my soul, Breaking and strongly should be buried, when I perish The earth and thoughts of many states.

DUKE VINCENTIO:

Well, your wit is in the care of side and that.

Second Lord:

They would be ruled after this chamber, and my fair nues begun out of the fact, to be conveyed, Whose noble souls I'll have the heart of the wars.

Clown:

Come, sir, I will make did behold your worship.

VIOLA:

I'll drink it.

VIOLA:

Why, Salisbury must find his flesh and thought That which I am not aps, not a man and in fire, To show the reining of the raven and the wars To grace my hand reproach within, and not a fair are hand, That Caesar and my goodly father's world; When I was heaven of presence and our fleets, We spare with hours, but cut thy council I am great, Murdered and by thy master's ready there My power to give thee but so much as hell: Some service in the noble bondman here, Would show him to her wine.

KING LEAR:

O, if you were a feeble sight, the courtesy of your law, Your sight and several breath, will wear the gods With his heads, and my hands are wonder'd at the deeds, So drop upon your lordship's head, and your opinion Shall be against your honour.

Generating textbooks with RNNs

open source textbook on algebraic geometry

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Latex source

Generating textbooks with RNNs

For $\bigoplus_{n=1,...,m}$ where $\mathcal{L}_{m_{\bullet}} = 0$, hence we can find a closed subset \mathcal{H} in \mathcal{H} and any sets \mathcal{F} on X, U is a closed immersion of S, then $U \to T$ is a separated algebraic space.

Proof. Proof of (1). It also start we get

 $S = \operatorname{Spec}(R) = U \times_X U \times_X U$

and the comparicoly in the fibre product covering we have to prove the lemma generated by $\coprod Z \times_U U \to V$. Consider the maps M along the set of points Sch_{fppf} and $U \to U$ is the fibre category of S in U in Section, ?? and the fact that any U affine, see Morphisms, Lemma ??. Hence we obtain a scheme S and any open subset $W \subset U$ in Sh(G) such that $Spec(R') \to S$ is smooth or an

$$U = \bigcup U_i \times_{S_i} U_i$$

which has a nonzero morphism we may assume that f_i is of finite presentation over S. We claim that $\mathcal{O}_{X,x}$ is a scheme where $x, x', s'' \in S'$ such that $\mathcal{O}_{X,x'} \to \mathcal{O}'_{X',x'}$ is separated. By Algebra, Lemma ?? we can define a map of complexes $\operatorname{GL}_{S'}(x'/S'')$ and we win.

To prove study we see that $\mathcal{F}|_U$ is a covering of \mathcal{X}' , and \mathcal{T}_i is an object of $\mathcal{F}_{X/S}$ for i > 0 and \mathcal{F}_p exists and let \mathcal{F}_i be a presheaf of \mathcal{O}_X -modules on \mathcal{C} as a \mathcal{F} -module. In particular $\mathcal{F} = U/\mathcal{F}$ we have to show that

$$\widetilde{M}^{\bullet} = \mathcal{I}^{\bullet} \otimes_{\operatorname{Spec}(k)} \mathcal{O}_{S,s} - i_X^{-1} \mathcal{F})$$

is a unique morphism of algebraic stacks. Note that

Arrows = $(Sch/S)_{fppf}^{opp}, (Sch/S)_{fppf}$

and

 $V = \Gamma(S, \mathcal{O}) \longmapsto (U, \operatorname{Spec}(A))$

is an open subset of X. Thus U is affine. This is a continuous map of X is the inverse, the groupoid scheme S.

Proof. See discussion of sheaves of sets.

The result for prove any open covering follows from the less of Example ??. It may replace S by $X_{spaces, \acute{e}tale}$ which gives an open subspace of X and T equal to S_{Zar} , see Descent, Lemma ??. Namely, by Lemma ?? we see that R is geometrically regular over S.

Lemma 0.1. Assume (3) and (3) by the construction in the description.

Suppose $X = \lim |X|$ (by the formal open covering X and a single map $\underline{\operatorname{Proj}}_X(\mathcal{A}) = \operatorname{Spec}(B)$ over U compatible with the complex

 $Set(\mathcal{A}) = \Gamma(X, \mathcal{O}_{X, \mathcal{O}_X}).$

When in this case of to show that $\mathcal{Q} \to C_{Z/X}$ is stable under the following result in the second conditions of (1), and (3). This finishes the proof. By Definition ?? (without element is when the closed subschemes are catenary. If T is surjective we may assume that T is connected with residue fields of S. Moreover there exists a closed subspace $Z \subset X$ of X where U in X' is proper (some defining as a closed subset of the uniqueness it suffices to check the fact that the following theorem

(1) f is locally of finite type. Since S = Spec(R) and Y = Spec(R).

Proof. This is form all sheaves of sheaves on X. But given a scheme U and a surjective étale morphism $U \to X$. Let $U \cap U = \coprod_{i=1,...,n} U_i$ be the scheme X over S at the schemes $X_i \to X$ and $U = \lim_i X_i$.

The following lemma surjective restrocomposes of this implies that $\mathcal{F}_{x_0} = \mathcal{F}_{x_0} = \mathcal{F}_{\mathcal{X},\dots,0}$.

Lemma 0.2. Let X be a locally Noetherian scheme over $S, E = \mathcal{F}_{X/S}$. Set $\mathcal{I} = \mathcal{J}_1 \subset \mathcal{I}'_n$. Since $\mathcal{I}^n \subset \mathcal{I}^n$ are nonzero over $i_0 \leq \mathfrak{p}$ is a subset of $\mathcal{J}_{n,0} \circ \overline{A}_2$ works.

Lemma 0.3. In Situation ??. Hence we may assume q' = 0.

Proof. We will use the property we see that \mathfrak{p} is the mext functor (??). On the other hand, by Lemma ?? we see that

$$D(\mathcal{O}_{X'}) = \mathcal{O}_X(D)$$

where K is an F-algebra where δ_{n+1} is a scheme over S.

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Proof. Omitted. This since $\mathcal{F} \in \mathcal{F}$ and $x \in \mathcal{G}$ the diagram **Lemma 0.1.** Let C be a set of the construction. Let C be a gerber covering. Let F be a guasi-coherent sheaves of O-modules. We have to show that Ox $\mathcal{O}_{\mathcal{O}_X} = \mathcal{O}_X(\mathcal{L})$ gor. *Proof.* This is an algebraic space with the composition of sheaves \mathcal{F} on $X_{\acute{e}tale}$ we have $\mathcal{O}_X(\mathcal{F}) = \{morph_1 \times_{\mathcal{O}_X} (\mathcal{G}, \mathcal{F})\}$ where \mathcal{G} defines an isomorphism $\mathcal{F} \to \mathcal{F}$ of \mathcal{O} -modules. **Lemma 0.2.** This is an integer Z is injective. Mor_{Sets} $d(\mathcal{O}_{\mathcal{X}_{\mathcal{X}/k}}, \mathcal{G})$ $\operatorname{Spec}(K_{\psi})$ Proof. See Spaces, Lemma ??. is a limit. Then \mathcal{G} is a finite type and assume S is a flat and \mathcal{F} and \mathcal{G} is a finite **Lemma 0.3.** Let S be a scheme. Let X be a scheme and X is an affine open type f_* . This is of finite type diagrams, and covering. Let $\mathcal{U} \subset \mathcal{X}$ be a canonical and locally of finite type. Let X be a scheme. • the composition of \mathcal{G} is a regular sequence, \$\mathcal{O}_{X'}\$ is a sheaf of rings. Let X be a scheme which is equal to the formal complex. The following to the construction of the lemma follows. *Proof.* We have see that $X = \operatorname{Spec}(R)$ and \mathcal{F} is a finite type representable by algebraic space. The property \mathcal{F} is a finite morphism of algebraic stacks. Then the Let X be a scheme. Let X be a scheme covering. Let cohomology of X is an open neighbourhood of U. $b: X \to Y' \to Y \to Y \to Y' \times_Y Y \to X.$ *Proof.* This is clear that \mathcal{G} is a finite presentation, see Lemmas ??. be a morphism of algebraic spaces over S and Y. A reduced above we conclude that U is an open covering of C. The functor \mathcal{F} is a "field $\mathcal{O}_{X,x} \longrightarrow \mathcal{F}_{\overline{x}} \quad -1(\mathcal{O}_{X_{\ell tale}}) \longrightarrow \mathcal{O}_{X_{\ell}}^{-1}\mathcal{O}_{X_{\lambda}}(\mathcal{O}_{X_{n}}^{\overline{v}})$ *Proof.* Let X be a nonzero scheme of X. Let X be an algebraic space. Let \mathcal{F} be a is an isomorphism of covering of \mathcal{O}_{X_i} . If \mathcal{F} is the unique element of \mathcal{F} such that Xquasi-coherent sheaf of \mathcal{O}_X -modules. The following are equivalent is an isomorphism. (1) \mathcal{F} is an algebraic space over S. The property \mathcal{F} is a disjoint union of Proposition ?? and we can filtered set of (2) If X is an affine open covering. presentations of a scheme \mathcal{O}_X -algebra with \mathcal{F} are opens of finite type over S. If \mathcal{F} is a scheme theoretic image points. Consider a common structure on X and X the functor $\mathcal{O}_X(U)$ which is locally of If \mathcal{F} is a finite direct sum $\mathcal{O}_{X_{\lambda}}$ is a closed immersion, see Lemma ??. This is a finite type. sequence of \mathcal{F} is a similar morphism.

Generating code with RNNs

```
static void do_command(struct seq_file *m, void *v)
{
  int column = 32 << (cmd[2] & 0x80);</pre>
  if (state)
    cmd = (int)(int_state ^ (in_8(&ch->ch_flags) & Cmd) ? 2 : 1);
  else
    seq = 1;
  for (i = 0; i < 16; i++) {
    if (k & (1 << 1))
     pipe = (in use & UMXTHREAD UNCCA) +
        ((count & 0x0000000fffffff8) & 0x000000f) << 8;
    if (count == 0)
      sub(pid, ppc md.kexec handle, 0x2000000);
    pipe set bytes(i, 0);
  }
  /* Free our user pages pointer to place camera if all dash */
  subsystem info = &of changes[PAGE SIZE];
  rek controls(offset, idx, &soffset);
  /* Now we want to deliberately put it to device */
  control check polarity(&context, val, 0);
  for (i = 0; i < COUNTER; i++)</pre>
    seq puts(s, "policy ");
}
```

Generated C code



CVPR 2015:

Deep Visual-Semantic Alignments for Generating Image Descriptions, Karpathy and Fei-Fei Show and Tell: A Neural Image Caption Generator, Vinyals et al.

Long-term Recurrent Convolutional Networks for Visual Recognition and Description, Donahue et al. Learning a Recurrent Visual Representation for Image Caption Generation, Chen and Zitnick

Adapted from Andrej Karpathy

Recurrent Neural Network



Convolutional Neural Network

Andrej Karpathy

test image







test image





test image



<START>





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"man in black shirt is playing guitar."



"a young boy is holding a baseball bat."



"construction worker in orange safety vest is working on road."



"a cat is sitting on a couch with a remote control."



"two young girls are playing with lego toy."



"a woman holding a teddy bear in front of a mirror."



"boy is doing backflip on wakeboard."



"a horse is standing in the middle of a road."