CS 1674: Intro to Computer Vision Grouping: Edges, Lines, Circles, Segments

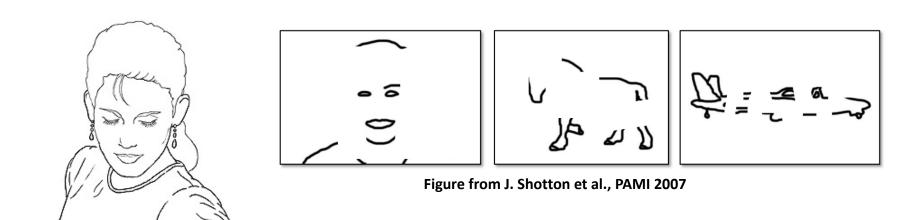
Prof. Adriana Kovashka
University of Pittsburgh
February 10, 2022

Plan for this lecture

- Edges
 - Extract gradients and threshold
- Lines and circles
 - Find which edge points are collinear or belong to another shape e.g. circle
 - Automatically detect and ignore outliers
- Segments
 - Find which pixels form a consistent region
 - Clustering (e.g. K-means)

Edge detection

- Goal: map image from 2d array of pixels to a set of curves or line segments or contours.
- Why?

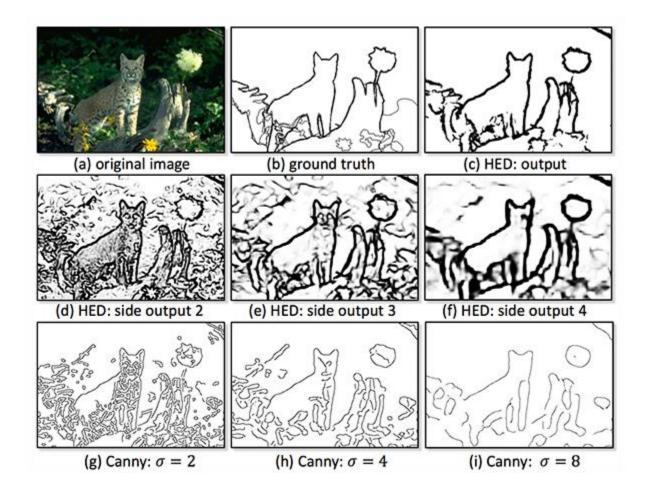


Main idea: look for strong gradients, post-process

Designing an edge detector

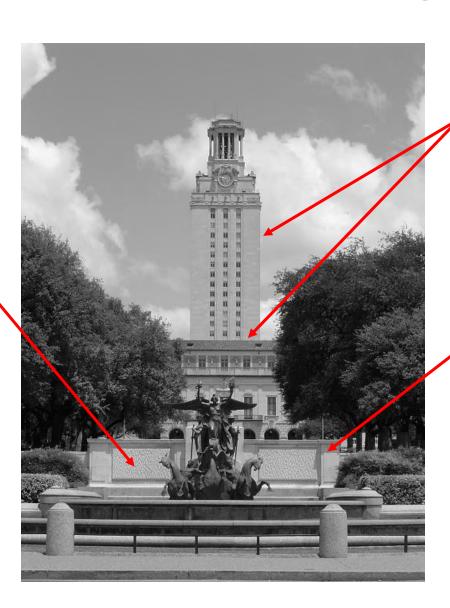
- Criteria for a good edge detector
 - Good categorization (edge vs not edge)
 - find all real edges, ignoring noise or other artifacts
 - Good localization
 - detect edges as close as possible to the true edges
 - return one point only for each true edge point (true edges = the edges humans drew on an image)
- Cues of edge detection
 - Bottom-up: Differences in color, intensity, or texture across the boundary
 - Top-down: Continuity and closure, high-level knowledge

Examples of edge detection results



What causes an edge?

Reflectance change: appearance information, texture

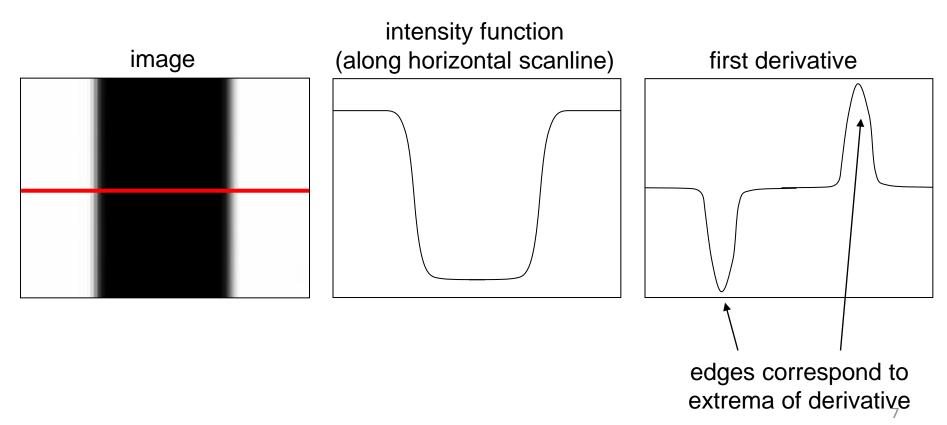


Depth discontinuity: object boundary

Cast shadows

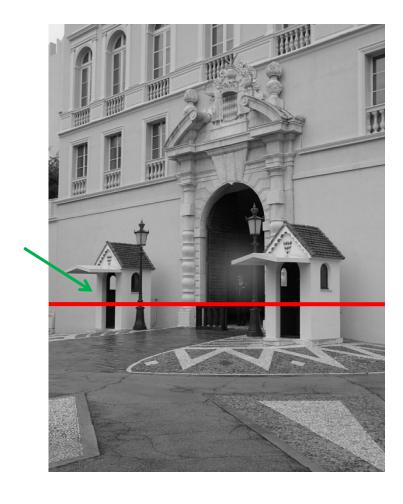
Characterizing edges

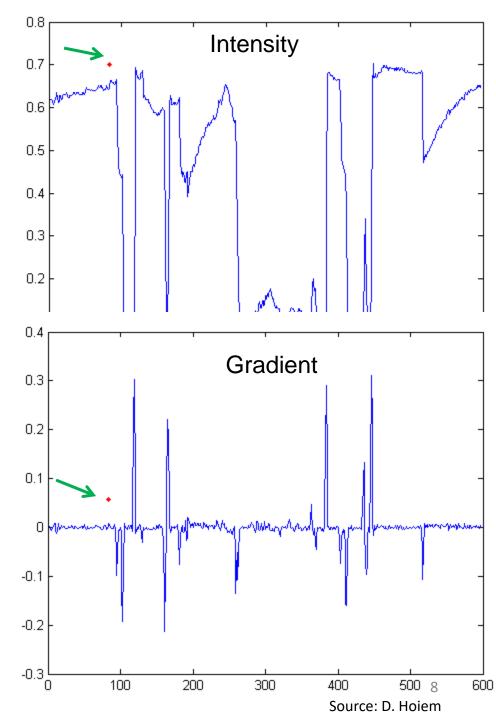
An edge is a place of rapid change in the image intensity function



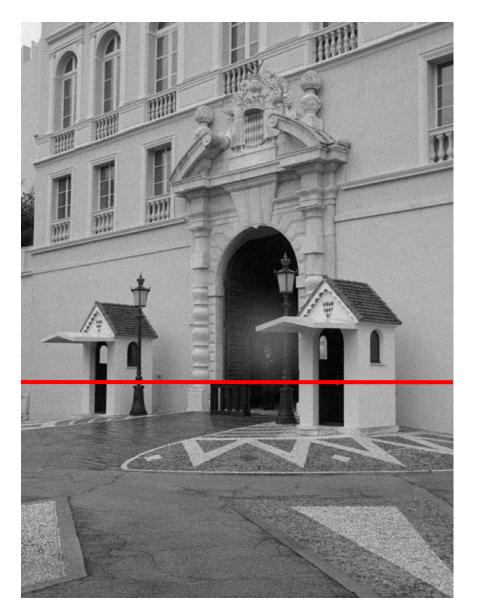
Source: L. Lazebnik

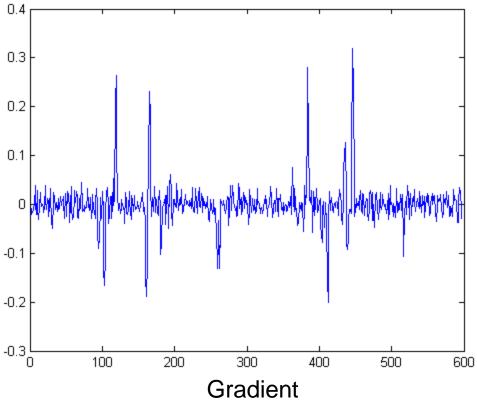
Intensity profile





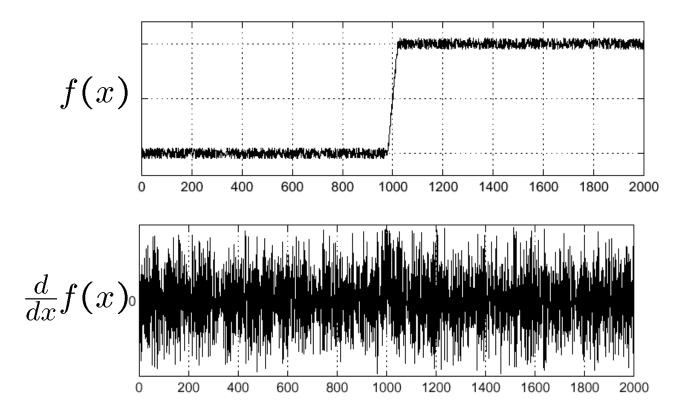
With a little Gaussian noise





Effects of noise

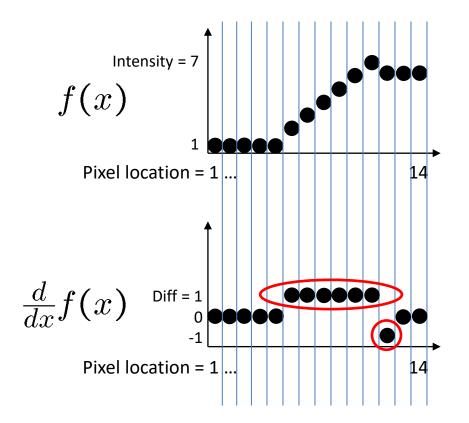
- Consider a single row or column of the image
 - Plotting intensity as a function of position gives a signal



Where is the edge?

Without noise

- Consider a single row or column of the image
 - Plotting intensity as a function of position gives a signal

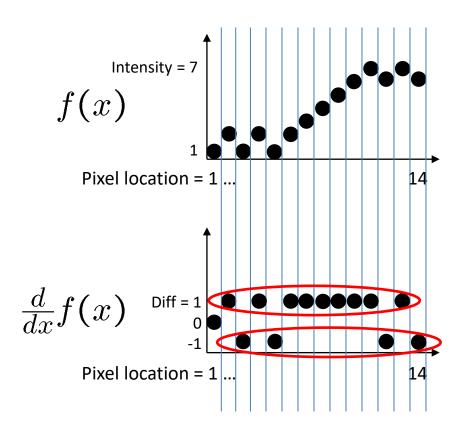


$$=\frac{\Delta f(a)}{\Delta a}=\frac{f(a+h)-f(a)}{(a+h)-(a)}=\frac{f(a+h)-f(a)}{h}$$
$$=f(x+1)-f(x)$$

Where is the edge?

With noise

- Consider a single row or column of the image
 - Plotting intensity as a function of position gives a signal



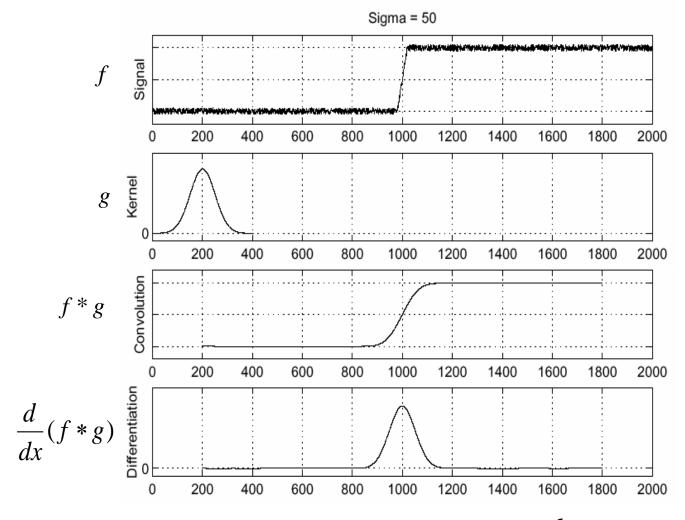
$$=\frac{\Delta f(a)}{\Delta a} = \frac{f(a+h) - f(a)}{(a+h) - (a)} = \frac{f(a+h) - f(a)}{h}$$
$$= f(x+1) - f(x)$$

Where is the edge?

Effects of noise

- Difference filters respond strongly to noise
 - Image noise results in pixels that look very different from their neighbors
 - Generally, the larger the noise the stronger the response
- What can we do about it?

Solution: smooth first



To find edges, look for peaks in

$$\frac{d}{dx}(f*g)$$

Derivative theorem of convolution

• Differentiation is convolution, and convolution is associative: $d_{(f, h, a)} = d$

$$\frac{d}{dx}(f*g) = f*\frac{d}{dx}g$$

This saves us one operation:

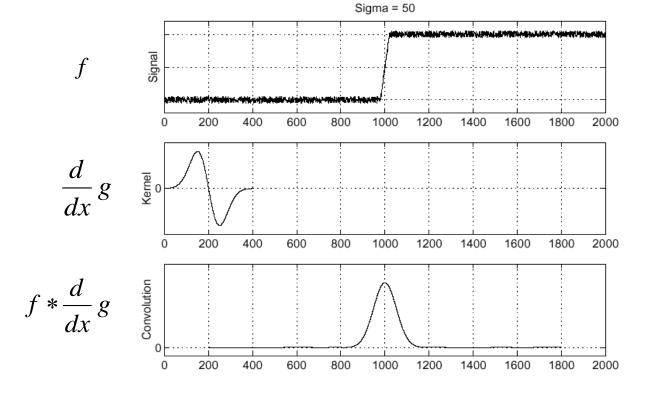


Image with edge

Derivative of Gaussian

Edge = max of derivative

15

Source: S. Seitz

Canny edge detector

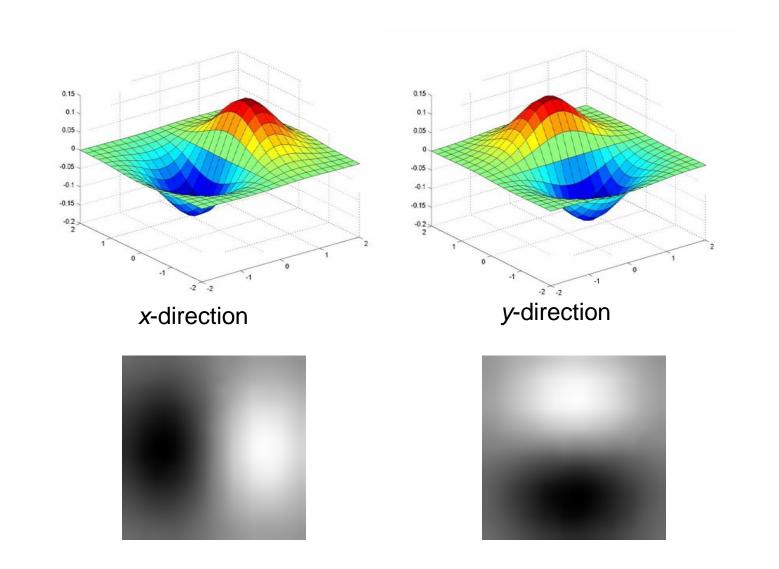
- Filter image with derivative of Gaussian
- Find magnitude and orientation of gradient
- Threshold: Determine which local maxima from filter output are actually edges
- Non-maximum suppression:
 - Thin wide "ridges" down to single pixel width
- Linking and thresholding (hysteresis):
 - Define two thresholds: low and high
 - Use the high threshold to start edge curves and the low threshold to continue them

Example



input image ("Lena")

Derivative of Gaussian filter



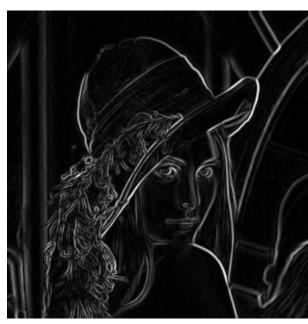
Compute Gradients



X-Derivative of Gaussian



Y-Derivative of Gaussian



Gradient Magnitude

Thresholding

- Choose a threshold value t
- Set any pixels less than t to 0 (off)
- Set any pixels greater than or equal to t to 1 (on)

The Canny edge detector



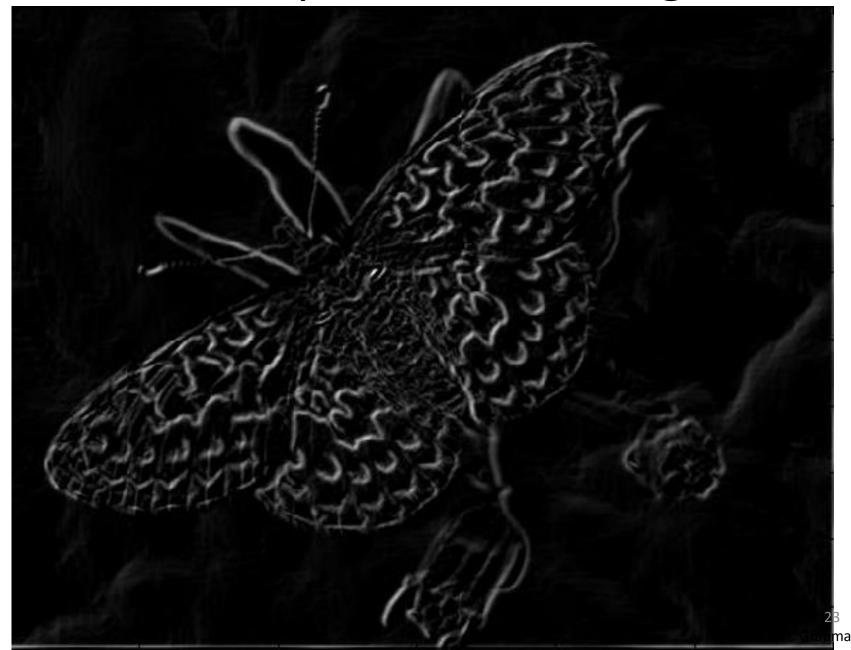
norm of the gradient (magnitude)

The Canny edge detector



thresholding

Another example: Gradient magnitudes



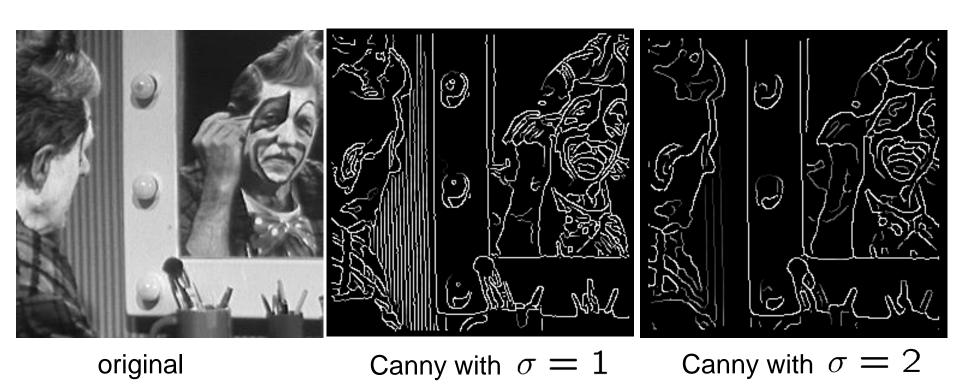
Thresholding gradient with a lower threshold



Thresholding gradient with a higher threshold



Effect of σ of Gaussian kernel



The choice of σ depends on desired behavior

- large σ detects large scale edges
- small σ detects fine edges

26

Source: S. Seitz

Plan for this lecture

- Edges
 - Extract gradients and threshold
- Lines and circles
 - Find which edge points are collinear or belong to another shape e.g. circle
 - Automatically detect and ignore outliers
- Segments
 - Find which pixels form a consistent region
 - Clustering (e.g. K-means)

Line detection (fitting)

Why fit lines?
 Many objects characterized by presence of straight lines

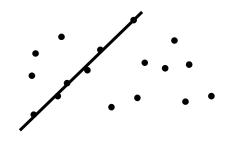


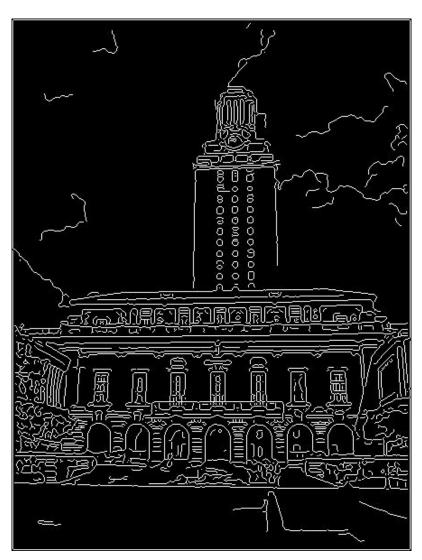




Why aren't we done just by running edge detection?

Difficulty of line fitting





- Noise in measured edge points, orientations:
 - e.g. edges not collinear where they should be
 - how to detect true underlying parameters?
- Extra edge points (clutter):
 - which points go with which line, if any?
- Only some parts of each line detected, and some parts are missing:
 - how to find a line that bridges missing evidence?

Least squares line fitting

•Data: $(x_1, y_1), ..., (x_n, y_n)$

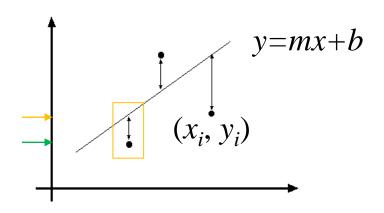
•Line equation: $y_i = mx_i + b$

•Find (m, b) to minimize

$$E = \sum_{i=1}^{n} (mx_i + b - y_i)^2$$

where line you found tells you point is along y axis

where point really is along y axis

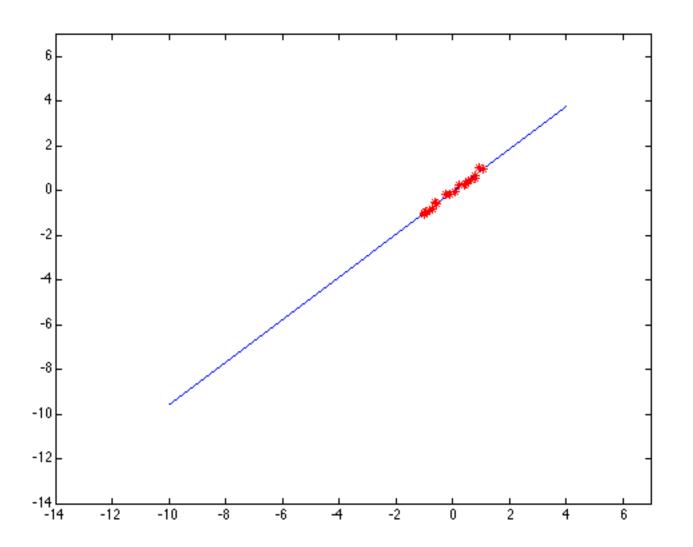


You want to find a single line that "explains" all of the points in your data, but data may be noisy!

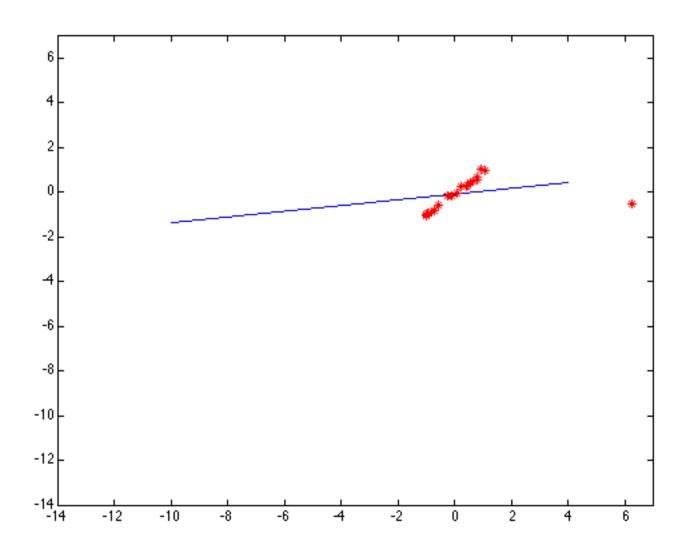
$$E = \sum_{i=1}^{n} \left(\begin{bmatrix} x_i & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - y_i \right)^2 = \begin{bmatrix} x_1 & 1 \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ y_n \end{bmatrix} \begin{bmatrix} y_1 \\ y_n \end{bmatrix} = \|\mathbf{A}\mathbf{p} - \mathbf{y}\|^2$$

Matlab: $p = A \setminus y$; or p = pinv(A) *y;

Outliers affect least squares fit



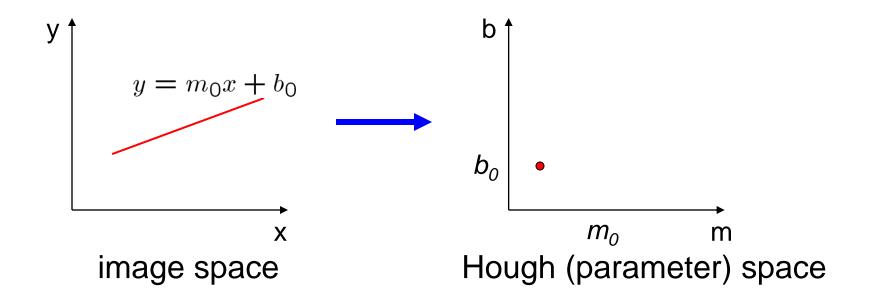
Outliers affect least squares fit



Dealing with outliers: Voting

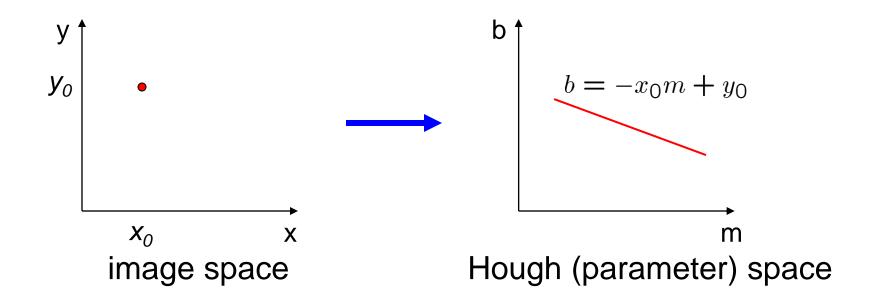
- Voting is a general technique where we let the features vote for all models that are compatible with it.
 - Cycle through features, cast votes for model parameters.
 - Look for model parameters that receive a lot of votes.
- Noise & clutter features?
 - They will cast votes too, but typically their votes should be inconsistent with the majority of "good" features.
- Common techniques
 - Hough transform
 - RANSAC

Finding lines in an image: Hough space



Connection between image (x,y) and Hough (m,b) spaces $y = m_0x + b_0$ • A line in the image corresponds to a point in Hough space

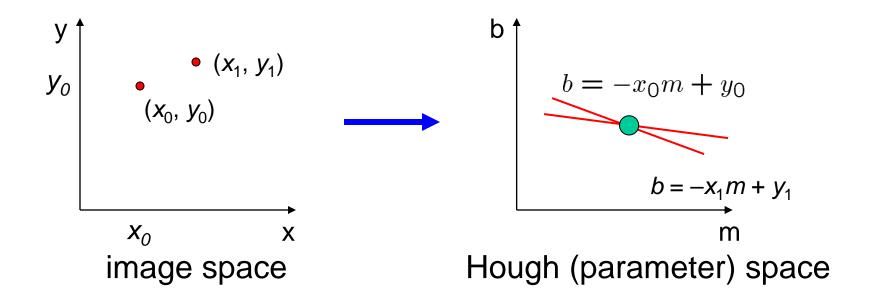
Finding lines in an image: Hough space



Connection between image (x,y) and Hough (m,b) spaces

- $y = m_0 x + b_0$ A line in the image corresponds to a point in Hough space
 - What does a point (x₀, y₀) in the image space map to?
 - Answer: the solutions of $b = -x_0 m + y_0$
 - This is a line in Hough space
 - Given a pair of points (x,y), find all (m,b) such that y = mx + b

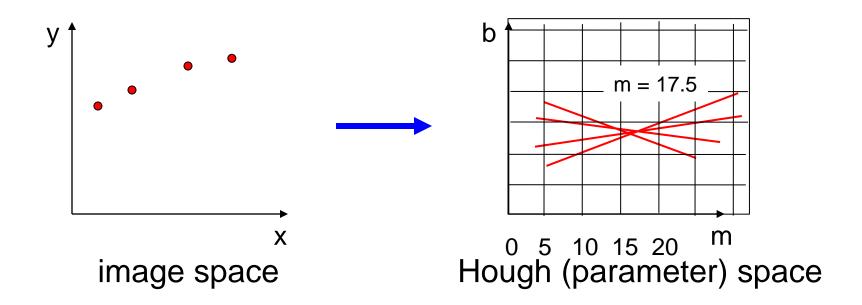
Finding lines in an image: Hough space



What are the line parameters for the line that contains both (x_0, y_0) and (x_1, y_1) ?

• It is the intersection of the lines $b = -x_0m + y_0$ and $b = -x_1m + y_1$

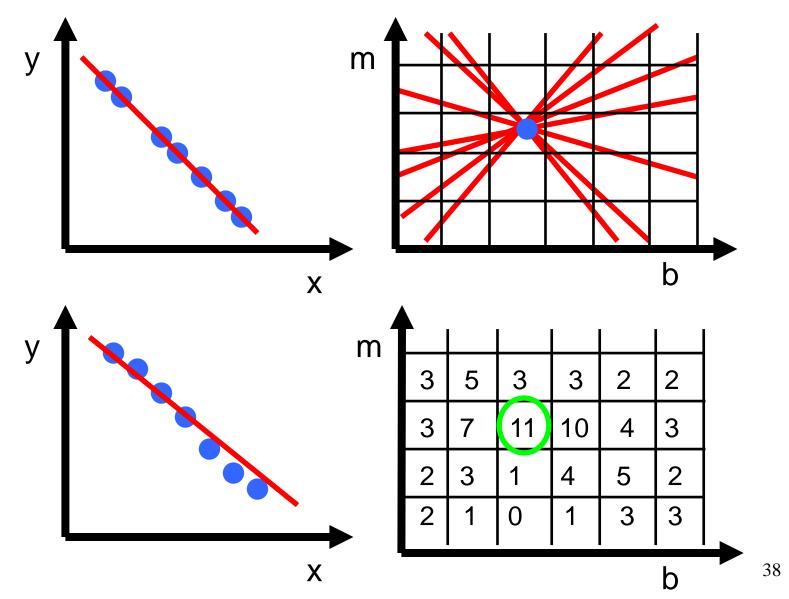
Finding lines in an image: Hough space



How can we use this to find the most likely parameters (m,b) for the most prominent line in the image space?

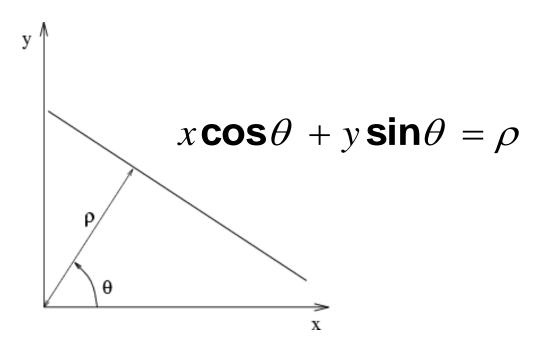
- Let each edge point in image space vote for a set of possible parameters in Hough space
- Accumulate votes in discrete set of bins; parameters with the most votes indicate line in image space. 37

Finding lines in an image: Hough space



Parameter space representation

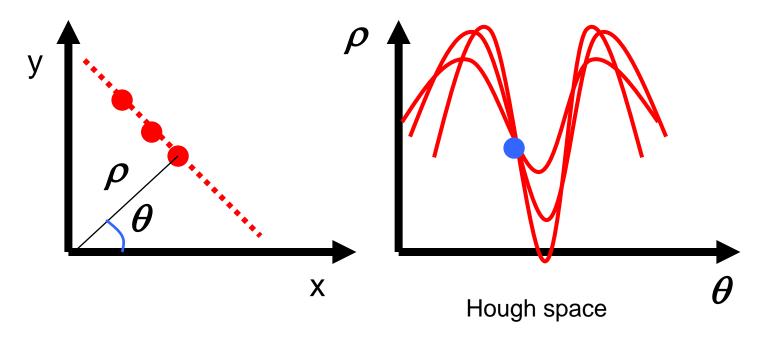
- Problems with the (m, b) space:
 - Unbounded parameter domains
 - Vertical lines require infinite m
- Alternative: polar representation



Each point (x,y) will add a sinusoid in the (θ,ρ) parameter space

Parameter space representation

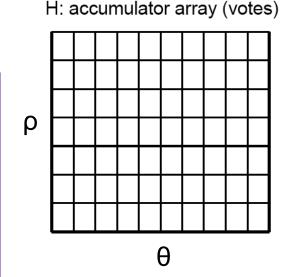
- Problems with the (m,b) space:
 - Unbounded parameter domains
 - Vertical lines require infinite m
- Alternative: polar representation



Each point (x,y) will add a sinusoid in the (θ,ρ) parameter space

Algorithm outline: Hough transform

- Initialize accumulator H to all zeros
- For each edge point (x,y) in the image
 For $\theta = 0$ to 180 $\rho = x \cos \theta + y \sin \theta$ $H(\theta, \rho) = H(\theta, \rho) + 1$ end

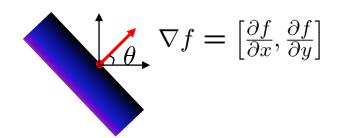


- Find the value(s) of (θ*, ρ*) where H(θ, ρ) is a local maximum
 - The detected line in the image is given by
 ρ* = x cos θ* + y sin θ*

end

Incorporating image gradients

- Recall: when we detect an edge point, we also know its gradient direction
- But this means that the line is uniquely determined!

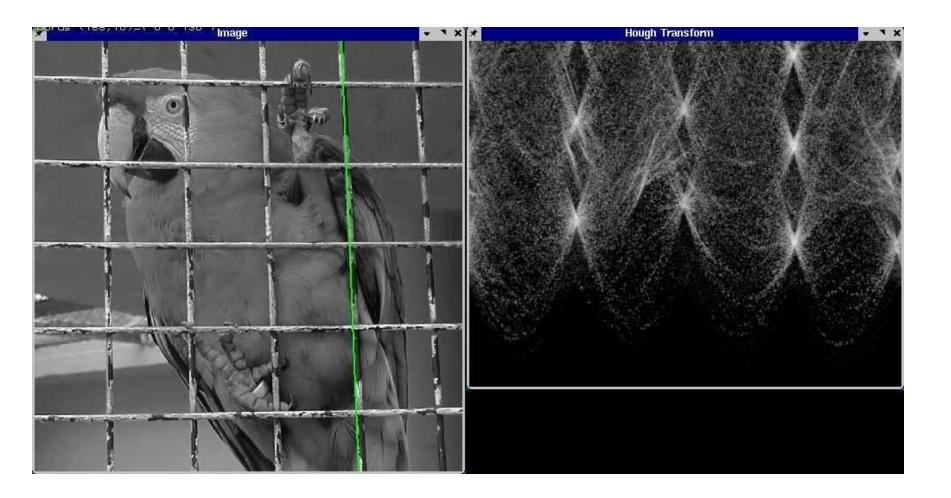


$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

Modified Hough transform:

```
For each edge point (x,y) in the image \theta = gradient orientation at (x,y) \rho = x \cos \theta + y \sin \theta H(\theta, \rho) = H(\theta, \rho) + 1 end
```

Hough transform example



Impact of noise on Hough

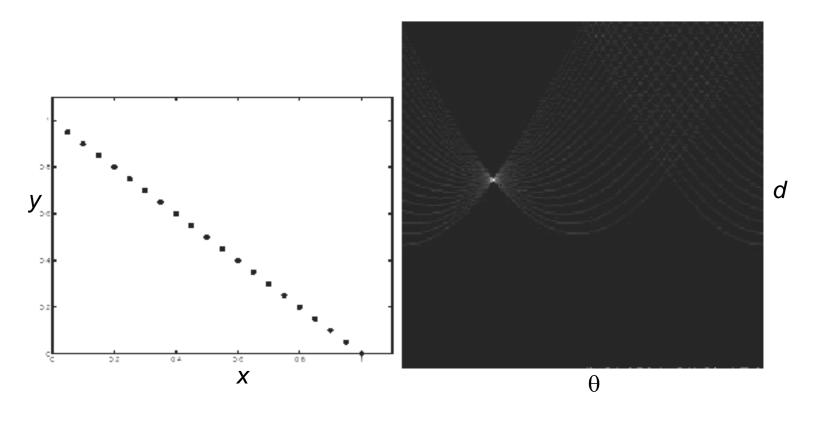


Image space edge coordinates

Votes

Impact of noise on Hough

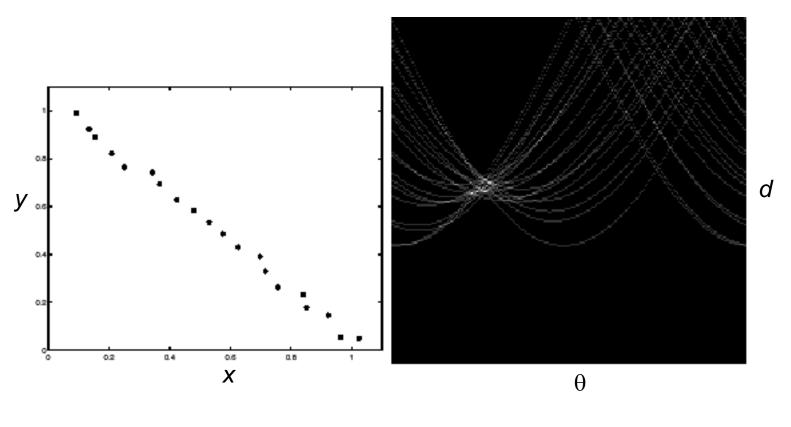


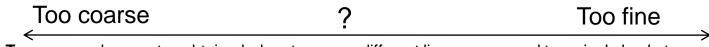
Image space edge coordinates

Votes

What difficulty does this present for an implementation?⁴⁵

Voting: practical tips

- Minimize irrelevant tokens first (reduce noise)
- Choose a good grid / discretization

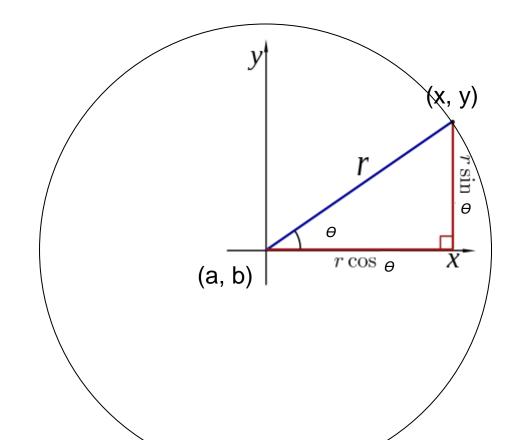


- Too coarse: large votes obtained when too many different lines correspond to a single bucket
- Too fine: miss lines because points that are not exactly collinear cast votes for different buckets
- Vote for neighbors (smoothing in accumulator array)
- Use direction of edge to reduce parameters by 1
- To read back which points voted for "winning" peaks, keep tags on the votes

 A circle with radius r and center (a, b) can be described as:

$$x = a + r \cos(\theta)$$

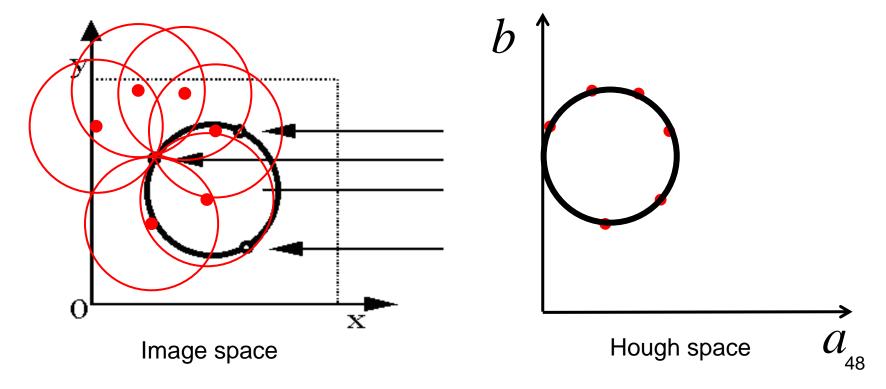
 $y = b + r \sin(\theta)$



Circle: center (a, b) and radius r

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

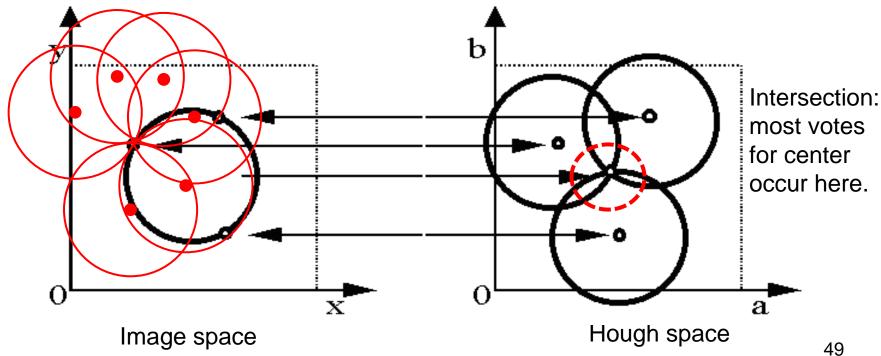
For a fixed radius r, unknown gradient direction



Circle: center (a, b) and radius r

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

For a fixed radius r, unknown gradient direction



For every edge pixel (x,y):

$$x = a + r \cos(\theta)$$

 $y = b + r \sin(\theta)$

For each possible radius value r.

For each possible gradient direction θ :

// or use estimated gradient at (x,y)

$$a = x - r \cos(\theta) // \text{column}$$

$$b = y - r \sin(\theta) // \text{row}$$

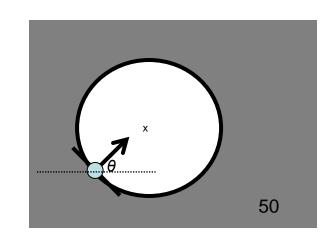
$$H[a,b,r] += 1$$

end

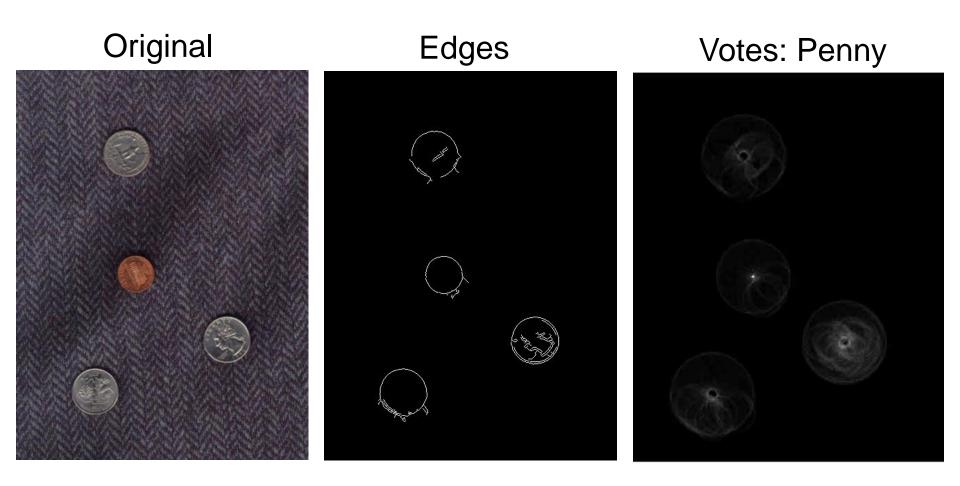
end

end

Your homework!



Example: detecting circles with Hough



Note: a different Hough transform (with separate accumulators) was used for each circle radius (quarters vs. penny).

Example: detecting circles with Hough

Comb@rigindeltections Edges Votes: Quarter

Note: a different Hough transform (with separate accumulators) was used for each circle radius (quarters vs. penny).

Hough transform: pros and cons

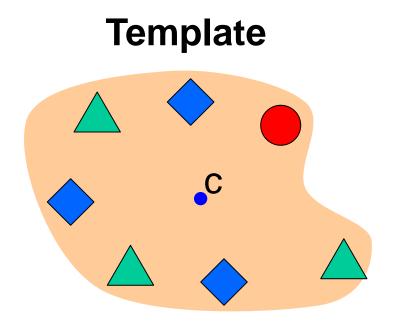
Pros

- All points are processed independently, so can cope with occlusion, gaps
- Some robustness to noise: noise points unlikely to contribute consistently to any single bin
- Can detect multiple instances of a model in a single pass

<u>Cons</u>

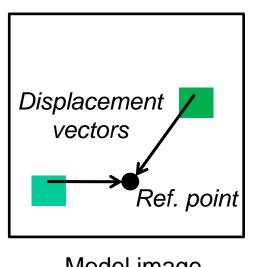
- Complexity of search time for maxima increases exponentially with the number of model parameters
 - If 3 parameters and 10 choices for each, search is O(10³)
- Quantization: can be tricky to pick a good grid size

 We want to find a template defined by its reference point (center) and several distinct types of landmark points in stable spatial configuration

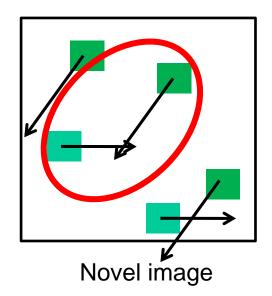


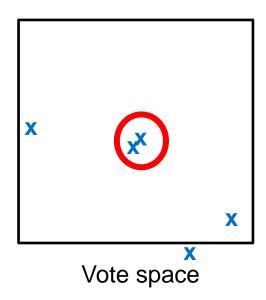
Triangle, circle, diamond: some *type* of visual token, e.g. feature or edge point

Intuition:



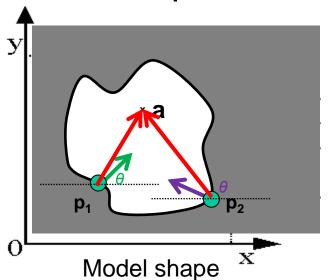


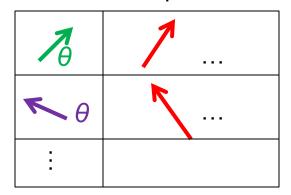




Now suppose those colors encode gradient directions...

Define a model shape by its boundary points and a reference point.





Offline procedure:

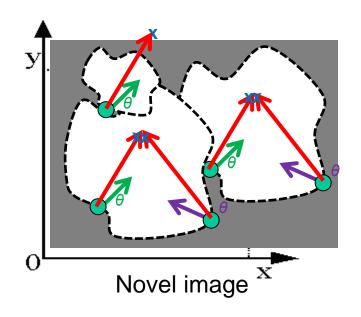
At each boundary point, compute displacement vector: $\mathbf{r} = \mathbf{a} - \mathbf{p_i}$.

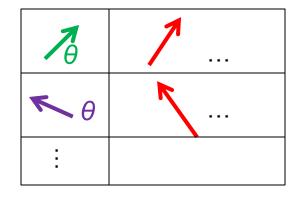
Store these vectors in a table indexed by gradient orientation θ .

Detection procedure:

For each edge point:

- Use its gradient orientation θ to index into stored table
- Use retrieved r vectors to vote for reference point



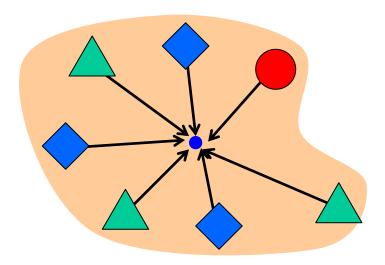


Assuming translation is the only transformation here, i.e., orientation and scale are fixed.

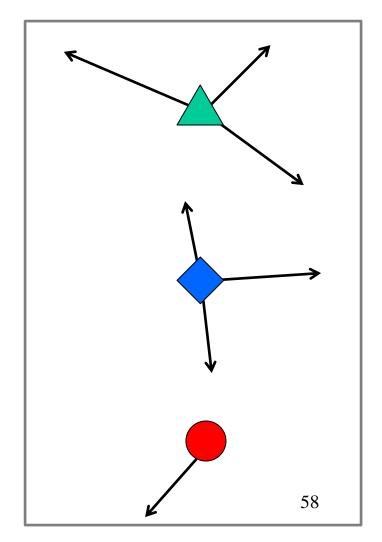
Template representation

 For each type of landmark point, store all possible displacement vectors towards the center

Template



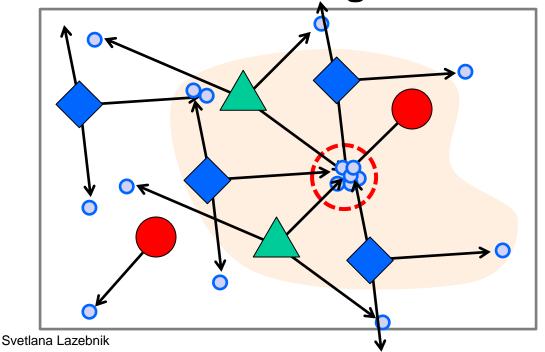
Model



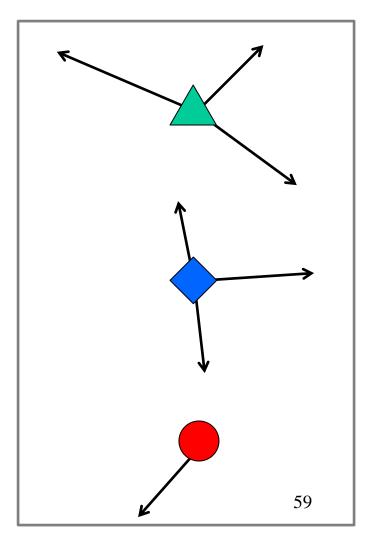
Detecting the template

 For each feature in a new image, look up that feature type in the model and vote for the possible center locations associated with that type in the model

Test image

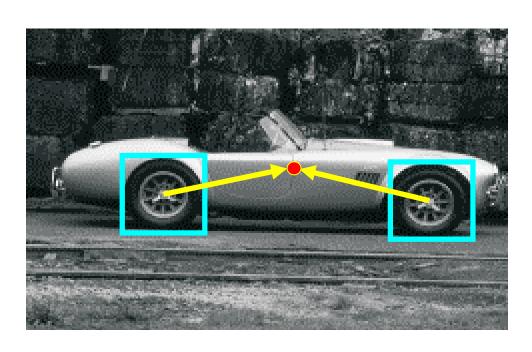


Model



Hough for object detection

Index displacements by "visual codeword"





"visual codeword" with displacement vectors

training image

B. Leibe, A. Leonardis, and B. Schiele, <u>Combined Object Categorization and Segmentation with an Implicit Shape Model</u>, ECCV Workshop on Statistical Learning in Computer Vision 2004

RANdom Sample Consensus

- Approach: we want to avoid the impact of outliers, so let's look for "inliers", and use those only.
- Intuition: if an outlier is chosen to compute the current fit, then the resulting line won't have much support from rest of the points.

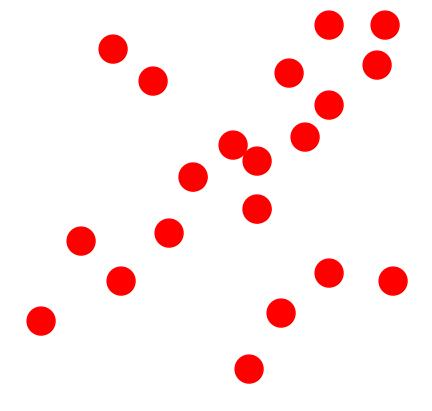
RANSAC: General form

- RANSAC loop:
- Randomly select a seed group of s points on which to base model estimate (e.g. s=2 for a line)
- 2. Fit model to these **s** points
- 3. Find *inliers* to this model (i.e., points whose distance from the line is less than *t*)
- 4. Repeat N times
- Keep the model with the largest number of inliers

(RANdom SAmple Consensus):

Fischler & Bolles in '81.

Line fitting example



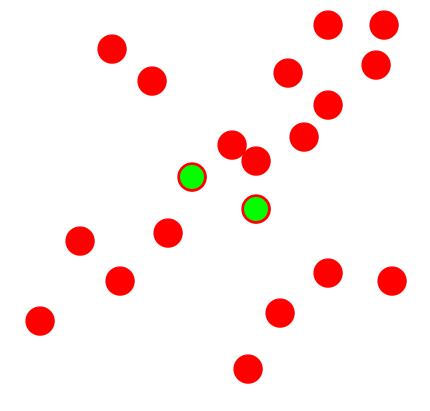
Algorithm:

- 1. **Sample** (randomly) the number of points required to fit the model
- 2. **Solve** for model parameters using samples
- 3. **Score** by the fraction of inliers within a preset threshold of the model

(RANdom SAmple Consensus):

Fischler & Bolles in '81.

Line fitting example



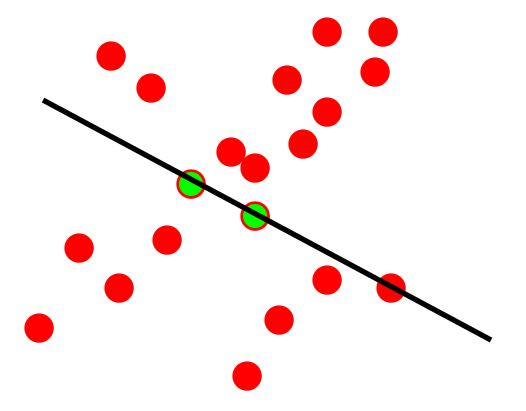
Algorithm:

- Sample (randomly) the number of points required to fit the model (#=2)
- 2. **Solve** for model parameters using samples
- 3. **Score** by the fraction of inliers within a preset threshold of the model

(RANdom SAmple Consensus):

Fischler & Bolles in '81.

Line fitting example



Algorithm:

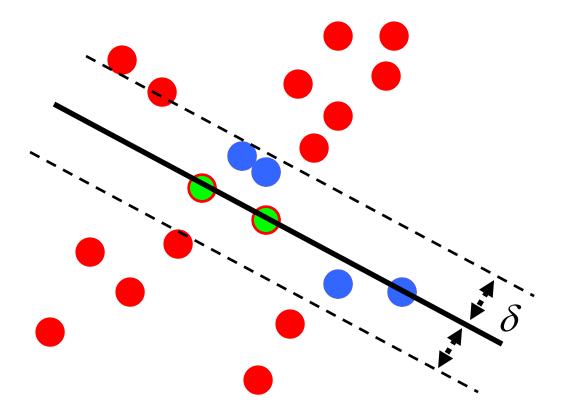
- 1. Sample (randomly) the number of points required to fit the model (#=2)
- 2. Solve for model parameters using samples
- 3. **Score** by the fraction of inliers within a preset threshold of the model

(RANdom SAmple Consensus):

Fischler & Bolles in '81.

Line fitting example

$$N_I = 6$$



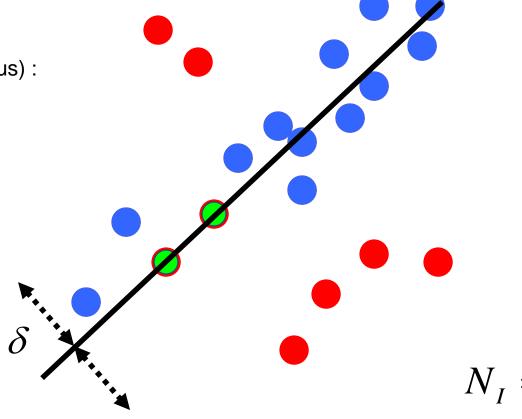
Algorithm:

- 1. **Sample** (randomly) the number of points required to fit the model (#=2)
- 2. **Solve** for model parameters using samples
- 3. **Score** by the fraction of inliers within a preset threshold of the model

(RANdom SAmple Consensus):

Fischler & Bolles in '81.

Line fitting example



 $N_{I} = 14$

Algorithm:

- 1. **Sample** (randomly) the number of points required to fit the model (#=2)
- 2. **Solve** for model parameters using samples
- 3. **Score** by the fraction of inliers within a preset threshold of the model

How to choose parameters?

- Number of samples N
 - Choose N so that, with probability p, at least one random sample is free from outliers (e.g. p=0.99) (outlier ratio: e)
- Number of sampled points s
 - Minimum number needed to fit the model
- Distance threshold δ
 - Choose δ so that a good point with noise is likely (e.g., prob=0.95) within threshold
 - E.g. for zero-mean Gaussian noise with std. dev. σ: δ^2 = 3.84 σ^2

$$N = \log(1-p)/\log(1-(1-e)^s)$$

Explanation in Szeliski 8.1.4

	proportion of outliers e						
S	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	161877

RANSAC pros and cons

Pros

- Applicable to many different problems, e.g. image stitching, relating two views
- Often works well in practice

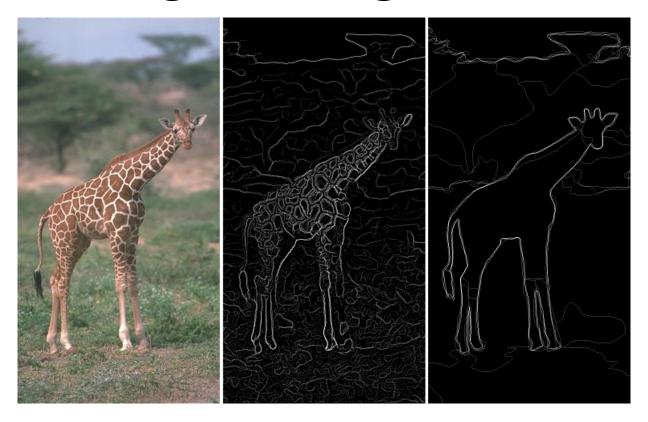
Cons

- Lots of parameters to tune (see previous slide)
- Doesn't work well for low inlier ratios (too many iterations, or can fail completely)

Plan for today

- Edges
 - Extract gradients and threshold
- Lines and circles
 - Find which edge points are collinear or belong to another shape e.g. circle
 - Automatically detect and ignore outliers
- Segments
 - Find which pixels form a consistent region
 - Clustering (e.g. K-means)

Edges vs Segments



- Edges: More low-level; don't need to be closed
- Segments: Ideally one segment for each semantic group/object; should include closed contours

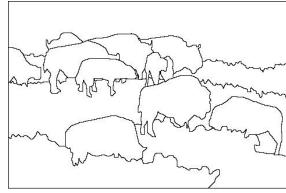
The goals of segmentation

Separate image into coherent "objects"

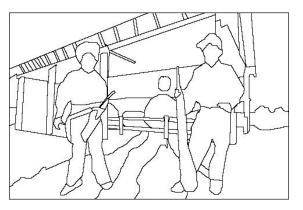
image

human segmentation





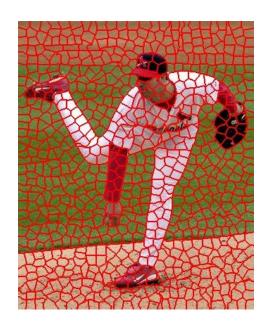


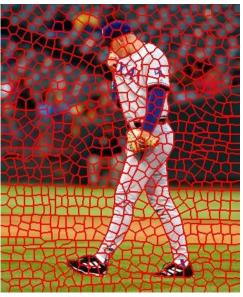


The goals of segmentation

- Separate image into coherent "objects"
- Group together similar-looking pixels for efficiency of further processing

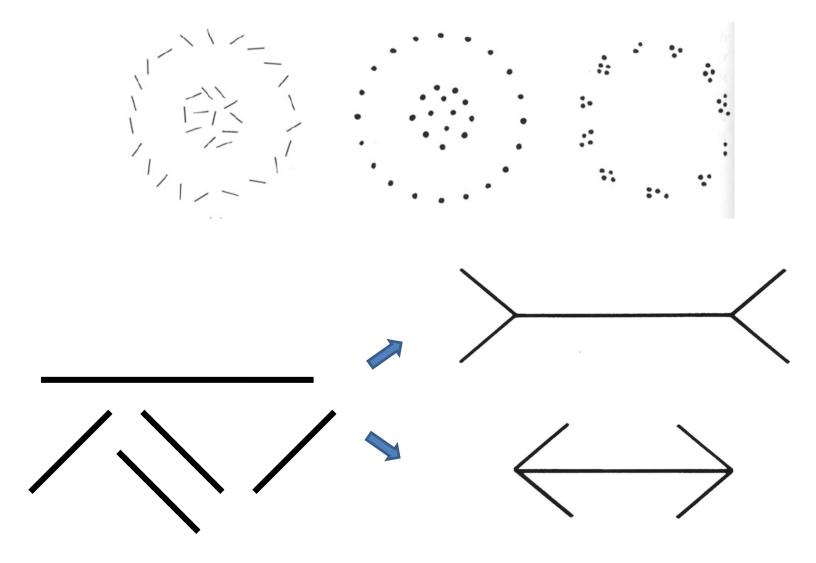
"superpixels"





X. Ren and J. Malik. Learning a classification model for segmentation. ICCV 2003.

We perceive the interpretation



Similarity









Slide: K. Grauman

Proximity



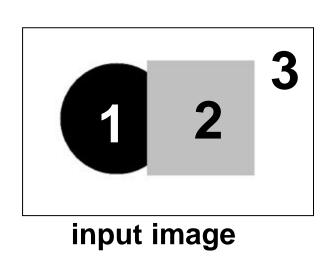


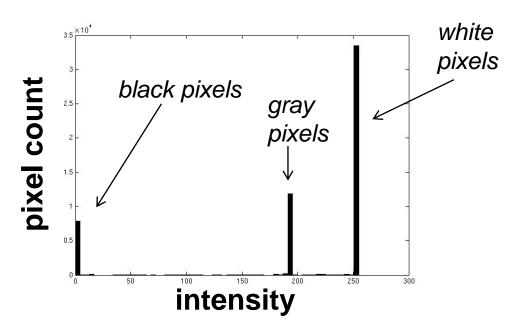
Common fate



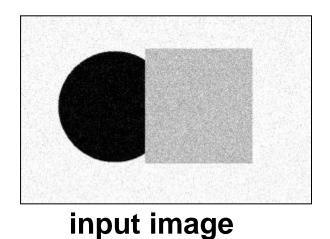


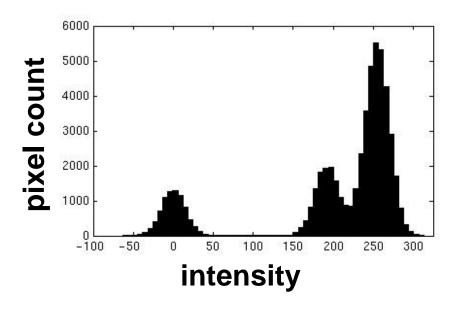
Image segmentation: toy example



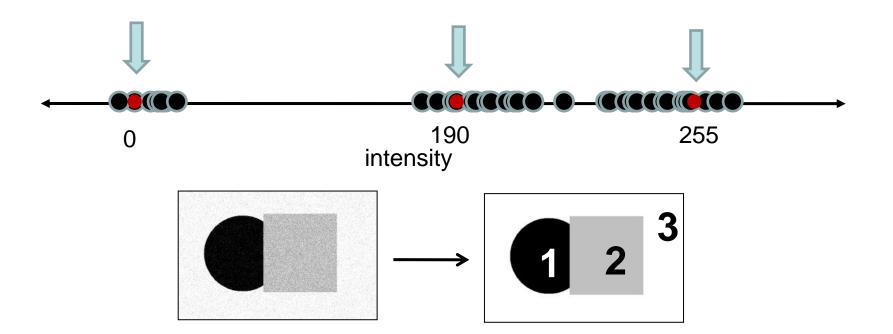


- These intensities define the three groups.
- We could label every pixel in the image according to which of these primary intensities it is.
 - i.e., segment the image based on the intensity feature.
- What if the image isn't quite so simple?





- Now how to determine the three main intensities that define our groups?
- · We need to cluster.

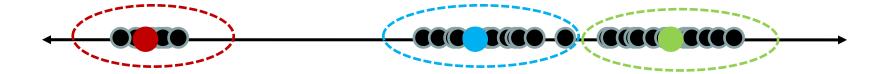


- Goal: choose three "centers" as the representative intensities, and label every pixel according to which of these centers it is nearest to.
- Best cluster centers are those that minimize sum of squared differences (SSD) between all points and their nearest cluster center ci:

$$\sum_{\text{clusters } i} \sum_{\text{points p in cluster } i} ||p - c_i||^2$$

Clustering

- With this objective, it is a "chicken and egg" problem:
 - If we knew the cluster centers, we could allocate points to groups by assigning each to its closest center.



 If we knew the group memberships, we could get the centers by computing the mean per group.



K-means clustering

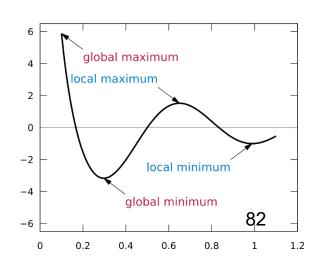
- Basic idea: randomly initialize the k cluster centers, and iterate between the two steps we just saw.
 - 1. Randomly initialize the cluster centers, c₁, ..., c_K
 - 2. Given cluster centers, determine points in each cluster
 - For each point p, find the closest c_i. Put p into cluster i
 - 3. Given points in each cluster, solve for ci
 - Set c_i to be the mean of points in cluster i
 - 4. If c_i have changed, repeat Step 2



Properties

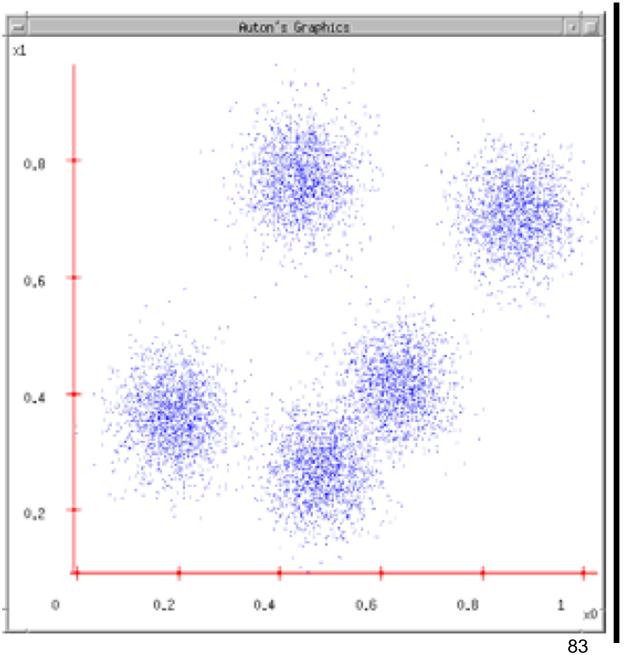
- Will always converge to some solution
- Can be a "local minimum" of objective:

$$\sum_{\text{clusters } i} \sum_{\text{points p in cluster } i} ||p - c_i||^2$$

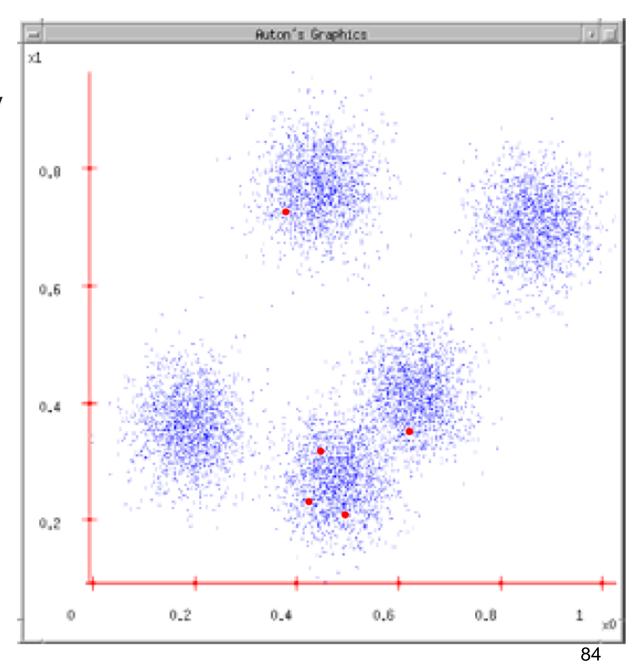


Slide: Steve Seitz, image: Wikipedia

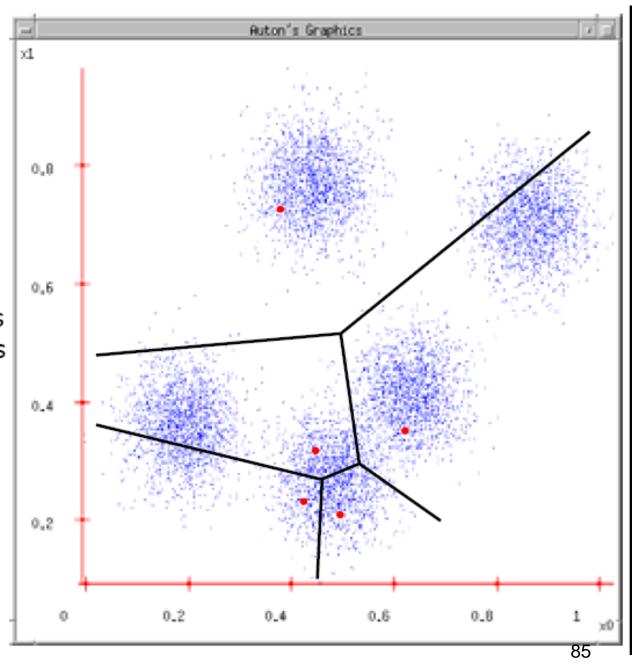
1. Ask user how many clusters they'd like. (e.g. k=5)



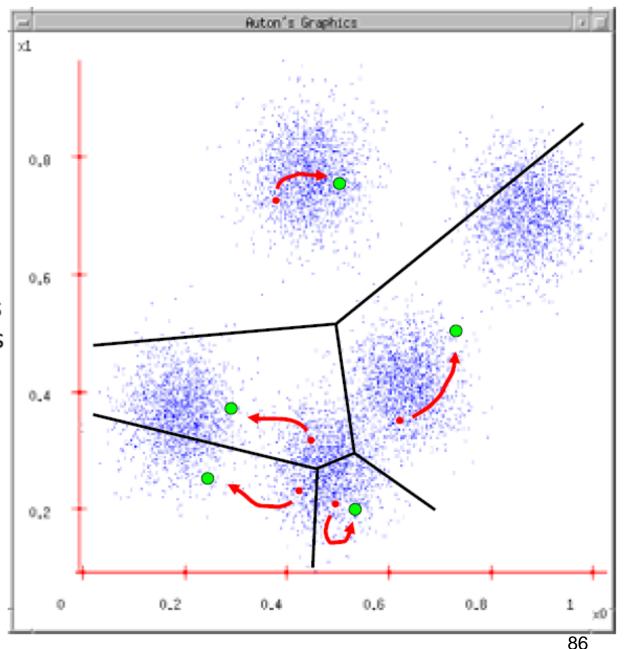
- 1. Ask user how many clusters they'd like. (e.g. k=5)
- 2. Randomly guess k cluster Center locations



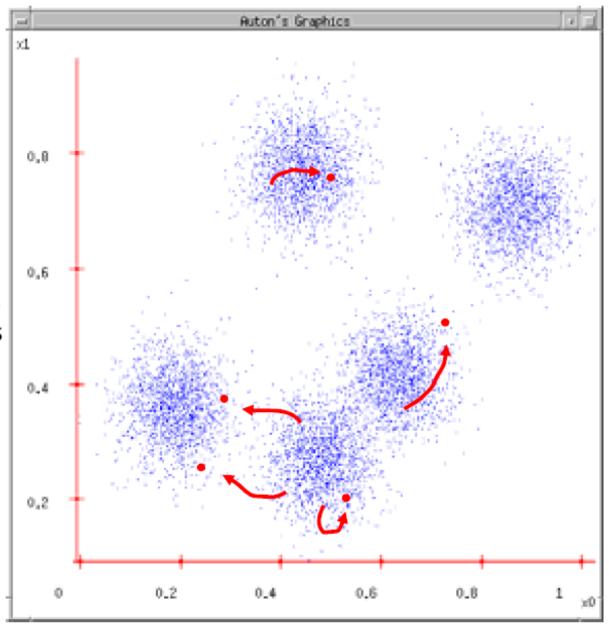
- Ask user how many clusters they'd like. (e.g. k=5)
- Randomly guess k cluster Center locations
- Each datapoint finds out which Center it's closest to. (Thus each Center "owns" a set of datapoints)



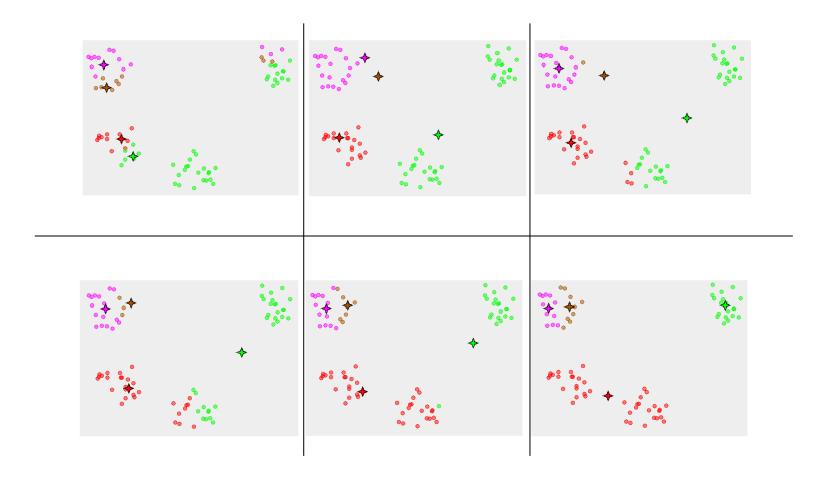
- Ask user how many clusters they'd like. (e.g. k=5)
- Randomly guess k cluster Center locations
- Each datapoint finds out which Center it's closest to.
- Each Center finds the centroid of the points it owns



- Ask user how many clusters they'd like. (e.g. k=5)
- Randomly guess k cluster Center locations
- Each datapoint finds out which Center it's closest to.
- Each Center finds the centroid of the points it owns...
- ...and jumps there
- 6. ...Repeat until terminated!



K-means converges to a local minimum



How can I try to fix this problem?

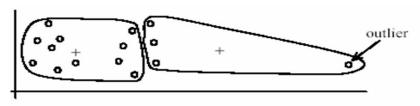
K-means: pros and cons

Pros

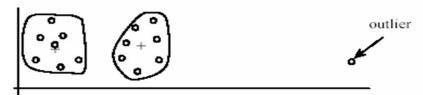
- Simple, fast to compute
- Converges to local minimum of within-cluster squared error

Cons/issues

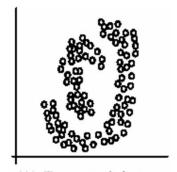
- Setting k?
 - One way: silhouette coefficient
- Sensitive to initial centers
 - Use heuristics or output of another method
- Sensitive to outliers
- Detects spherical clusters



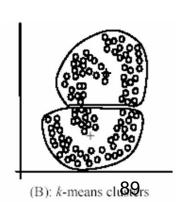
(A): Undesirable clusters



(B): Ideal clusters



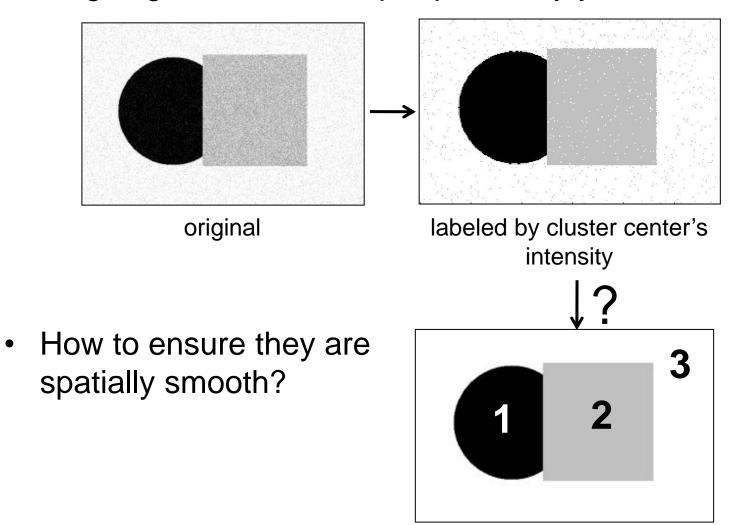
(A): Two natural clusters



Adapted from K. Grauman

Aside: Smoothing cluster assignments

Assigning a cluster label per pixel may yield outliers:



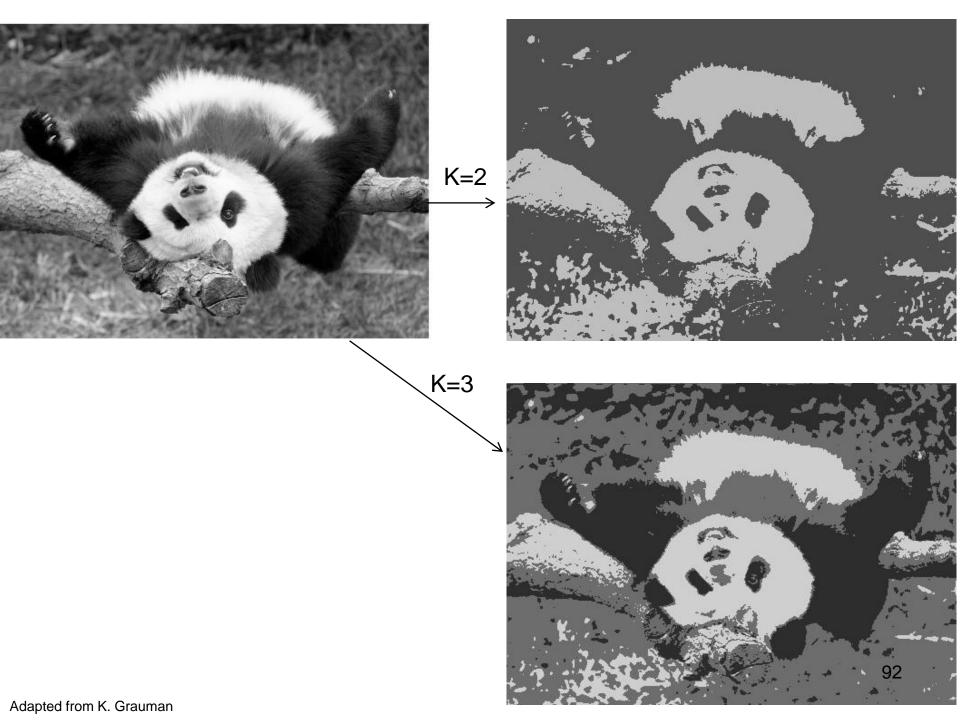
Depending on what we choose as the *feature space*, we can group pixels in different ways.

Grouping pixels based on **intensity** similarity





Feature space: intensity value (1-d)



Depending on what we choose as the *feature space*, we can group pixels in different ways.

Grouping pixels based on **intensity** similarity

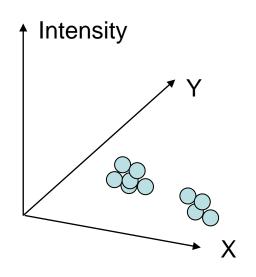


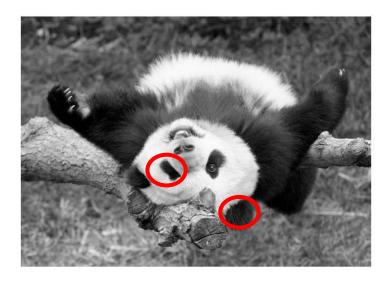
Clusters based on intensity similarity don't have to be spatially coherent.



Depending on what we choose as the *feature space*, we can group pixels in different ways.

Grouping pixels based on **intensity+position** similarity

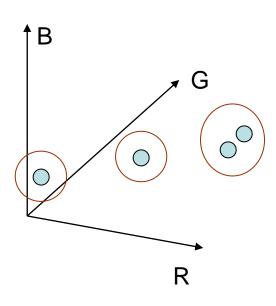


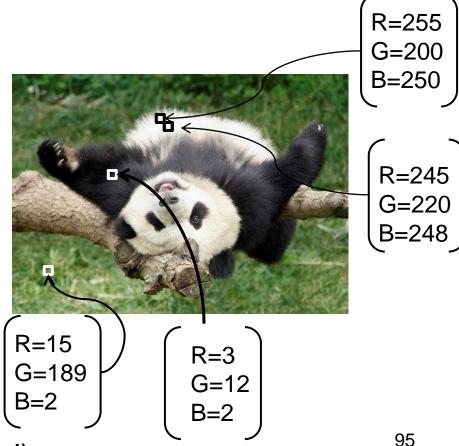


Both regions are black, but if we also include **position** (**x**,**y**), then we could group the two into distinct segments; way to encode both similarity & proximity.

Depending on what we choose as the feature space, we can group pixels in different ways.

Grouping pixels based on **color** similarity





Feature space: color value (3-d)

 Color, brightness, position alone are not enough to distinguish all regions...

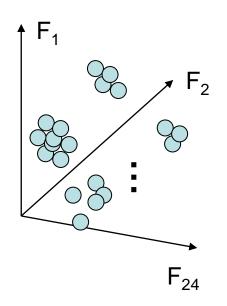




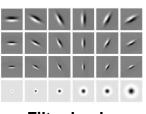


Depending on what we choose as the *feature space*, we can group pixels in different ways.

Grouping pixels based on **texture** similarity







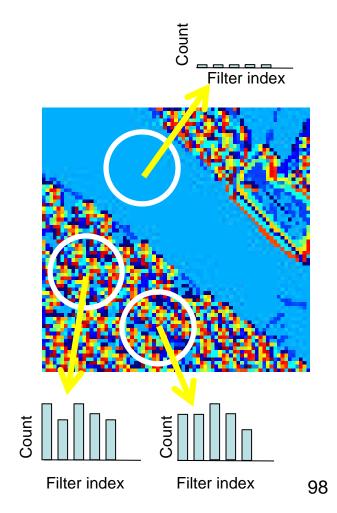
Filter bank of 24 filters

97

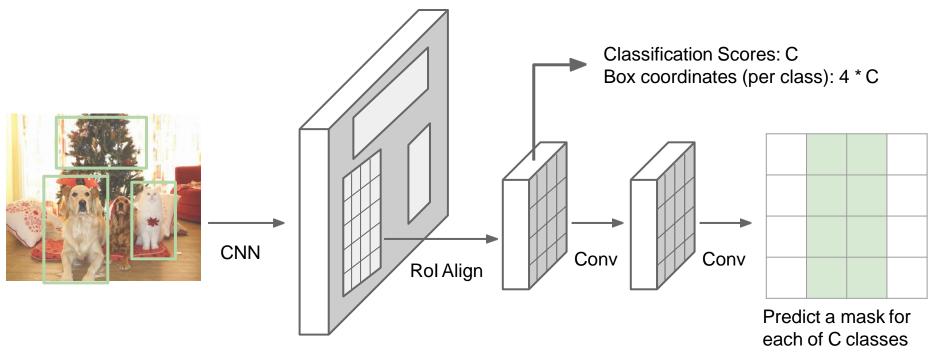
Segmentation w/ texture features

 Describe texture in a window as histogram over filter bank responses (simplified version, better: use "textons")

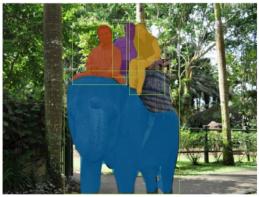




State-of-the-art (instance) segmentation: Mask R-CNN









Summary: classic approaches

- Edges: threshold gradient magnitude
- Lines: edge points vote for parameters of line, circle, etc. (works for general objects)
- Segments: use clustering (e.g. K-means) to group pixels by intensity, texture, etc.