*CS 1674: Intro to Computer Vision*

# Support Vector Machines: Exercise

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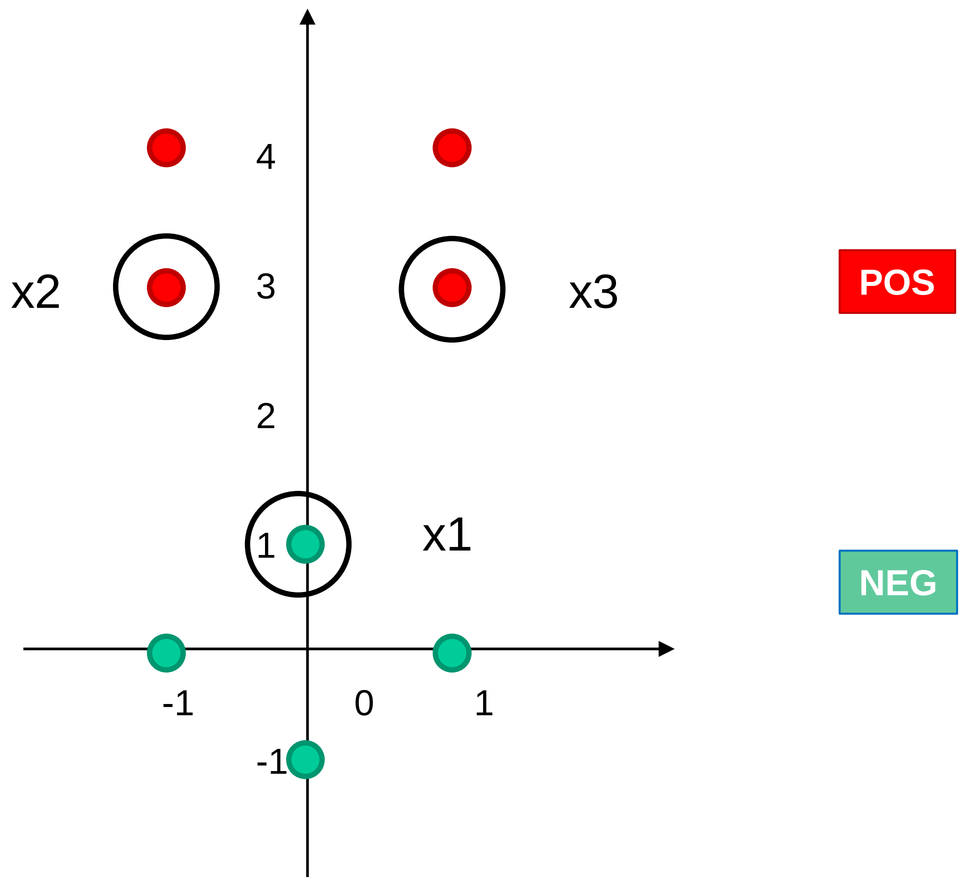
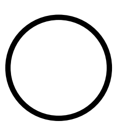
# Example

## Simplified case: Support vectors are given




 = support vectors

Example adapted from Dan Ventura

# Solving for the alphas

- We know for the support vectors, f(x) = 1 or -1 exactly
- Add a 1 in the feature representation for the bias
- The support vectors have coordinates and labels:
  - x1 = [? ? 1], y1 = -1
  - x2 = [? ? 1], y2 = +1
  - x3 = [? ? 1], y3 = +1

$$\mathbf{w}^T\mathbf{x} + b \quad \text{vs} \quad \mathbf{w'}^T\mathbf{x'} \text{ where } \mathbf{w'} = \begin{bmatrix} \mathbf{w}, \\ b \end{bmatrix} \quad \mathbf{x'} = \begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix}$$

# Solving for the alphas

- For support vectors, $w^T x_j = y_j$ so $\Sigma_i\, \alpha_i\, y_i\, \mathrm{dot}(x_i, x_j) = y_j$
- Thus we can form the following system of linear equations (one for each of three j's), with α1, α2, α3 as the unknowns:
- α1 = ?, α2 = ?, α3 = ?

# Solving for w, b

We know $w = \alpha_1 y_1 x_1 + \ldots + \alpha_N y_N x_N$ where N = # SVs

Thus w = ?, b = ?

# Plotting the boundary

For SVMs, we used this eq for a line: $ax + cy + b = 0$
where $w = [a\ c]$

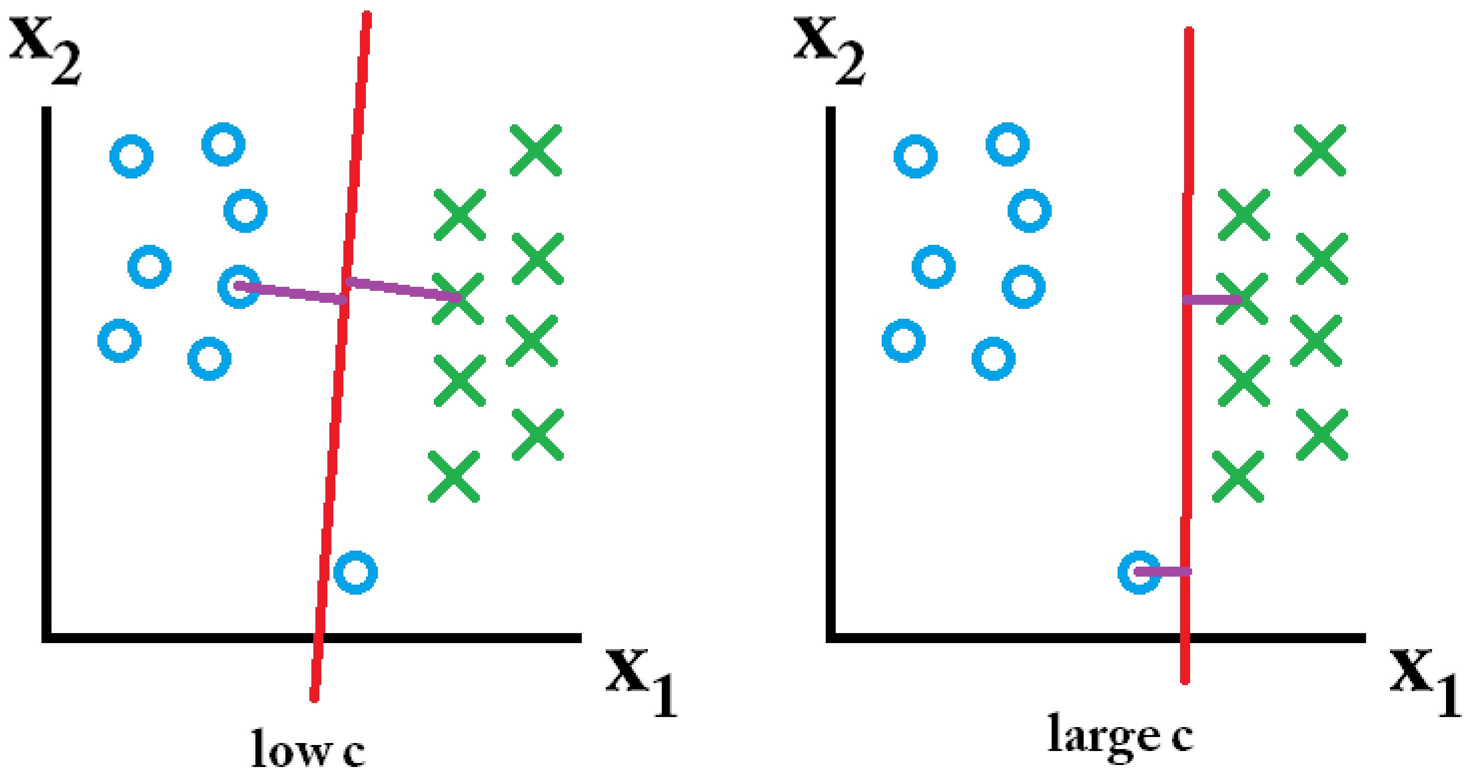
Thus $ax + b = -cy \Rightarrow y = (-a/c)\, x + (-b/c)$

Thus y-intercept is ?

Slope of decision boundary?

# Effect of margin size vs miscl. cost (c)

Training set




Misclassification ok, want large margin
(low cost)

Misclassification not ok
(high cost)

Image: Kent Munthe Caspersen

# Effect of margin size vs miscl. cost (c)

Find a test set A which is better classified using a small / low cost c

Find a test set B which is better classified using a large / high cost c