CS 1674: Intro to Computer Vision

Support Vector Machines: Exercise

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Example

Simplified case: Support vectors are given

Example adapted from Dan Ventura
Solving for the alphas

- We know for the support vectors, \( f(x) = 1 \) or \(-1\) exactly
- Add a 1 in the feature representation for the bias
- The support vectors have coordinates and labels:
  - \( x_1 = [? \ ? 1], y_1 = -1 \)
  - \( x_2 = [? \ ? 1], y_2 = +1 \)
  - \( x_3 = [? \ ? 1], y_3 = +1 \)

\[
\mathbf{w}^T \mathbf{x} + b \quad \text{vs} \quad \mathbf{w'}^T \mathbf{x'} \quad \text{where} \quad \mathbf{w'} = [\mathbf{w}, \mathbf{x'}, b] \quad \text{and} \quad \mathbf{x'} = [\mathbf{x}, 1]
\]
Solving for the alphas

• For support vectors, $w^T x_j = y_j$ so $\Sigma_i \alpha_i y_i \text{dot}(x_i, x_j) = y_j$
• Thus we can form the following system of linear equations (one for each of three $j$’s), with $\alpha_1, \alpha_2, \alpha_3$ as the unknowns:
• $\alpha_1 = \_\_\_, \alpha_2 = \_\_\_, \alpha_3 = \_\_\_\_ $
Solving for $w$, $b$

We know $w = \alpha_1 y_1 x_1 + \ldots + \alpha_N y_N x_N$ where $N = \#$ SVs

Thus $w = ?$, $b = ?$
For SVMs, we used this eq for a line: \( ax + cy + b = 0 \)
where \( w = [a \ c] \)
Thus \( ax + b = -cy \)  
\[ y = \left( -\frac{a}{c} \right) x + \left( -\frac{b}{c} \right) \]

Thus \( y \)-intercept is ?
Slope of decision boundary?
Effect of margin size vs miscl. cost \( (c) \)

Training set

Misclassification ok, want large margin (low cost)
Misclassification not ok (high cost)

Image: Kent Munthe Caspersen
Effect of margin size vs miscl. cost (c)

Find a test set A which is better classified using a small / low cost $c$

Find a test set B which is better classified using a large / high cost $c$