CS 1510 Midterm 2
Fall 2015

Instructions: Answer as many of problems 1, 2, 3, and 4 as you can. Answer at most one of problem 5 or 6. If you answer both problem 5 and problem 6, an arbitrary one will be graded.

1. (20 points) The input to the three coloring problem is a graph G, and the problem is to decide whether the vertices of G can be colored with three colors such that no pair of adjacent vertices are colored the same color. The three coloring problem is known to be NP-complete. Consider a new problem X, where the input is a graph H, and the problem is to determine whether H has some property P.

   (a) Explain how one would prove that the problem X is NP-hard.

   (b) Now consider the four coloring problem, where the input is a graph H, and the problem is to decide whether the vertices of H can be colored with four colors such that no pair of adjacent vertices are colored the same color. Show that the four coloring problem is NP-hard using the approach you outlined in the previous subproblem.

2. (20 points)

   (a) Explain how to solve the Parallel Prefix Problem on an EREW machine in time $O(\log n)$ on input of size n with n processors. Start with defining the Parallel Prefix Problem.

   (b) What is the efficiency of the algorithm in the previous subproblem? Start with a definition of efficiency. You need not have solved the previous subproblem to answer this question.

   (c) What would the Folding Principle say about the time for the algorithm in the first subproblem if there were only $n^{1/3}$ processors? Start with a definition of the Folding Principle. You need not have solved the first subproblem to answer this question.

   (d) Explain how to solve the Parallel Prefix Problem on an EREW machine in time $O(\log n)$ on input of size n with $n/\log n$ processors.

3. (20 points) Recall that the input for the Subset Sum Problem is positive integers $x_1, \ldots, x_n$, and L, and the problem is to determine whether there is a subset of the x_i's that sums to L. Also recall that the input to the 3SAT problem is a Boolean formula F in conjunctive normal form with exactly three distinct literals per clause, and the problem is to determine whether there is an assignment to the variables that makes F true.

   (a) Give an algorithm that takes as input an instance F of the 3SAT problem, and in polynomial time constructs an instance $x_1, \ldots, x_n$, and L of the Subset Sum Problem, with the property that F is satisfiable if and only if the instance to the Subset Sum Problem had a subset that sums to L.

   (b) Show the instance of the Subset Sum Problem that your algorithm creates from the 3SAT instance
   $$(x \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee \bar{y} \vee z) \wedge (x \vee y \vee z)$$

4. (20 points) Design a parallel algorithm that finds the maximum number in a sequence $x_1, \ldots, x_n$ of (not necessarily distinct) integers in the range 1 to n. Your algorithm should run in constant time on a CRCW Commmon PRAM with n processors.

Remember, answer at most one of problem 5 or 6. If you answer both problem 5 and problem 6, an arbitrary one will be graded.

5. (20 points) Show that if one of the following three problems has a polynomial time algorithm then they all do.

   - The input is two undirected graphs G and H. The problem is to determine if the graphs are isomorphic.
   - The input is two directed graphs G and H. The problem is to determine if the graphs are isomorphic.
   - The input is two undirected graphs G and H, and an integer k. The problem is to determine if the graphs are isomorphic and all the vertices in each graph have degree k.

   Intuitively, two graphs are isomorphic if on can name/label the vertices so that the graphs are identical. More formally, two undirected graphs G and H are isomorphic if there is a bijection f from the vertices of G to the vertices of H such that (v, w) is an edge in G if and only if $(f(v), f(w))$ is an edge in H. More formally, two directed graphs G and H are isomorphic if there is a bijection f from the vertices of G to the vertices of H such that (v, w) is a directed edge in G if and only if $(f(v), f(w))$ is a directed edge in H. The degree of a vertex is the number of edges incident to that vertex.

6. Explain how to solve the longest common subsequence problem in time $O(\log^2 n)$ using at most a polynomial number of processors on a CREW PRAM.