CS 1510 Midterm 3
Fall 2013

1. (20 points) The input to the Hamiltonian Cycle Problem is an undirected graph G. The problem is to find a Hamiltonian cycle, if one exists. A Hamiltonian cycle C is a simple cycle that spans G. So each vertex is visited exactly once in C. Show that the Hamiltonian cycle problem is self-reducible. That is, show that if there is a polynomial time algorithm that determines whether a graph has a Hamiltonian cycle, then there is a polynomial time algorithm to find Hamiltonian cycles.

2. (20 points) The input to the Subset Sum problem is positive integers $x_1, \ldots, x_n, L$. The output should be 1 if there is a subset of the x_i's that sums to to L. Show that the subset sum problem is NP-hard using a reduction from the 3SAT problem. The input to the 3SAT problem is a Boolean formula in conjunctive normal form (an AND of clauses which are an OR of variables or negation of variables) with exactly three literals (from 3 distinct variables) per clause. The output should be 1 if the formula is satisfiable, that is, there is an assignment to the variables that makes the formula true.

   The start of your writeup must explain how one generally shows that a problem is NP-hard. That is, you need to explain what sort of object you need to provide, and what properties that you must show the object has.

3. (20 points) Show that if one of the following three problems has a polynomial time algorithm then they all do.

   - The input is two undirected graphs G and H. The problem is to determine if the graphs are isomorphic.
   - The input is two directed graphs G and H. The problem is to determine if the graphs are isomorphic.
   - The input is two undirected graphs G and H, and an integer k. The problem is to determine if the graphs are isomorphic and all the vertices in each graph have degree k.

   Intuitively, two graphs are isomorphic if on can name/label the vertices so that the graphs are identical. More formally, two undirected graphs G and H are isomorphic if there is a bijection f from the vertices of G to the vertices of H such that (v, w) is an edge in G if and only if $(f(v), f(w))$ is an edge in H. More formally, two directed graphs G and H are isomorphic if there is a bijection f from the vertices of G to the vertices of H such that (v, w) is a directed edge in G if and only if $(f(v), f(w))$ is a directed edge in H. The degree of a vertex is the number of edges incident to that vertex.

   You must start your write-up by explaining your general strategy for accomplishing this. That is, you must state which reductions you will show. Then have a separate paragraph for each reduction, making it clear which reduction is handled in each paragraphs. For each reduction, you need only explain how you will transform the input. You should give some intuitive explanation why the transformation is correct. You need not give a complete formal proof, but your explanation needs to be convincing. Some of the reductions are much easier than others; The harder ones are worth more points.