A Mathematical Language for the Modeling of Geospatial Static Rules

Robert Laurini\textsuperscript{a,}\textsuperscript{*}, Robert Laurini\textsuperscript{a} and Robert Laurini\textsuperscript{a}

\textsuperscript{a}Knowledge Systems Institute, USA, and University of Lyon, France

\begin{abstract}
More and more in information technologies, rules are considered as first-class citizens; and many applications in business intelligence are built on rules. But in territorial intelligence and in smart city planning, few works have been done in this direction. The role of this paper is to show the importance of geographic rules and to propose, beyond the modeling in natural language, a mathematical language to model them. This language is primary based on logic and set theory in which some relations and operators coming from topology, computational geometry and operation research are integrated, and will cover only static rules. By mathematical language, one means that it is independent from any software product or application, and its formal grammar is presented by means of syntactic diagrams. Several examples are provided especially in urban planning.
\end{abstract}

1. Introduction

According to Graham [6] and Morgan [14], rules must be considered as first-class citizens in information technologies, meaning first that several computer-based activities must be revisited. By definition, a rule is a sequence antecedents-implication-consequents which can be noted by \((A) \Rightarrow (B)\), in which \(A\) is a conjunction of conditions. Instead of antecedents, sometimes the expression 'premise' is used. In logic, \(B\) can be either a disjunction of conditions or a set of assertions. They come from the so-called Horn clauses which are also the basis of logic programming, where it is common to write definite clauses in the form of an implication: \((p \land q \land ... \land l) \Rightarrow u\).

However, in urban planning, rules are essentially coming from laws and by-laws. Moreover, some experts can use other rules in their daily practice, sometimes called best practices. In addition to that, more and more specialists in spatial data mining analytics can discover what they call associative rules.

In the knowledge society, in smart city management, it is important not only to identify rules, but also to combine them to automate reasoning. Facing this objective, the scope of this paper is to propose a language in order to model rules. Indeed, rules are often made explicit in natural language; two main sources will be used, namely rules written in natural language and associative rules as extracted from data mining.

This language will be based on mathematics without taking into account practical implementation. In other words, this is not a computer language (code). Voluntarily, this paper will not deal with 3D issues, neither with temporal issues: only geographic static rules will be considered.

This paper is organized as follows. First, some elements will be given to explain the role of rules in computing, and especially in geoprocessing. Then the formal grammar of this mathematical language will be detailed. Finally, examples especially in urban planning, will clarify the expressive power of this language.

1.1 About Geographic Rules

In Laurini et al. [10], several examples of geographic rules were given, and some important semantical aspects were extracted. Anyhow, geographic rules are a way to model what Shoorcheh [21] has called spatial causality.
1.2 Generalities About the Model

Based on the previous descriptions, a general diagram for rules can be designed regrouping all aspects, their origin, components, temporal dimension, mathematical tools, their management, their usage and the various modes of implication (Figure 1).

- For physical rules, the implication is mandatory;
- For legal rules, the implication is also mandatory, but the sanctions may or may not exist; in this case, a second rule with the negative conditions will lead to the sanctions;
- For best practices, the implication is more or less a kind of recommendation; in other words, nobody is obliged to follow this kind of rules; perhaps some additional conditions could be considered; at a first approximation, random variates can help to select best practices if any.
- For rules coming from data mining, it is necessary to provide support and confidence. See Shekar et al. [20] for details.

Figure 1 explains the main characteristics of geospatial rules, namely, origin, mathematical tools, temporal dimension, semantics, modality, usage and management. Remark that an existing rule can be superseded by another novel rule, sometimes for a period or a narrower place.

2. Example in Urban Planning

Building permit delivery is a very complex task for local authorities. Indeed, a projected building must be compliant with two types of constraints, national laws and local rules. National laws which are valid everywhere in the country, generally consider structural aspects, such as electricity, insulation, access for handicapped people, etc. – issues which are outside the goal of this paper – whereas local rules consider the location of the building and its conformance to local planning rules, such as height, distance to neighboring parcels, floorspace ratios, etc. Figure 11 depicts the map with the extension of planning zones, and Table 1 gives some limit parameters like maximum height, maximum floorspace ratio and maximum footprint of the building.

<table>
<thead>
<tr>
<th>Zone ID</th>
<th>Max Height (in m)</th>
<th>Max Floorspace ratio</th>
<th>Max Footprint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downtown</td>
<td>12</td>
<td>3</td>
<td>80 %</td>
</tr>
<tr>
<td>Suburban area</td>
<td>15</td>
<td>4</td>
<td>70 %</td>
</tr>
<tr>
<td>Rural area</td>
<td>12</td>
<td>0.5</td>
<td>30 %</td>
</tr>
<tr>
<td>Near airport (Bowie)</td>
<td>8</td>
<td>2</td>
<td>50 %</td>
</tr>
<tr>
<td>Airstrip</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

2.1 Rules for Zones

First let’s suppose we have rules for dividing the territory and creating zones.

And so, for ZoneB, "Suburban area"; ZoneC, "Rural area"; ZoneD, "Near airport" and ZoneE, "Airstrip", with the appropriate coordinates, respectively numbered Rule 19, Rule 20 and Rule 21. For a projected building supposedly located in the Suburban Area, the rule will be as follows to be accepted.

Figure 2: Planning zones.

Suburban area and rural area. However, historical monuments are located in downtown, and it is forbidden to construct new buildings round them (for instance, within 200 meters). In the rural area, there is an airport for which some limitations exist (Figure 11). See Rule 22.

But for Downtown and Rural Area, the situation is more complex. For Downtown, we have to take historical monuments into account, for instance within a 200 m buffer around its centroid. See Rule 23.

3. Conclusion

The goal of this paper was to present the first version of a mathematical language for geographic static rule
modeling, independent from any computer language and able to integrate all semantics. Thus, this language is based on several mathematical domains such as logic, set theory, computational geometry, topology, etc.

References


