CS 1657
Privacy in the Electronic Society

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08: Classic side-channel attacks

Continuing: Why isn't crypto enough?

Modular exponentiation and its timing variance

- How are these done in practice,

Paul Kocher's timing attack (1996)

- Recover keys by observing timing variance
- Works even with seemingly constant-time multiplication
  - Montgomery multiplication

Protections from timing attacks

- Noise
- Extra care to achieve "constant-time"
- Homomorphic blinding

Other "classical" side-channel attacks

- Differential power analysis
- Cache timing

First, a basic modular exponentiation algorithm

To compute $y^x \bmod n$:

Let $r = 1$

For $k = 0$ to $m - 1$ (i.e., left to right):

$r = r^2 \bmod n$

If bit k of x is 1:

$r = (r \cdot y) \bmod n$

Return r

$5^{20} \bmod 131$: $y = 5, n = 131$

$x = 20 = 0\text{b}10100$

$r = 1$

- $k = 0$, bit $k = 1$

$r = 1^2 \bmod 131 = 1$

$r = (1 \cdot 5) \bmod 131 = 5$

- $k = 1$, bit $k = 0$

$r = 5^2 \bmod 131 = 25$

- $k = 2$, bit $k = 1$

$r = 25^2 \bmod 131 = 101$

$r = (101 \cdot 5) \bmod 131 = 112$

- $k = 3$, bit $k = 0$

$r = 112^2 \bmod 131 = 99$

- $k = 4$, bit $k = 0$

$r = 99^2 \bmod 131 = 107$

We can make this more consistent by eliminating *branching* (if statements)

To compute $y^x \bmod n$:

Let $r_0 = 1, r_1 = 1$

For $k = 0$ to $m - 1$ (i.e., left to right):

$b =$ bit k of x

$r_0 = r_0^2 \bmod n$

$r_b = (r_b \cdot y) \bmod n$

Return r_0

r_1 is not really needed, but helps us keep the number of operations more constant

Montgomery multiplications, briefly

Especially good for repeated multiplications!

When n is large, modular multiplications can be expensive due to division

- To compute $a \cdot b \bmod n$, compute $a \cdot b$, then divide by n and take remainder

Montgomery form: Replace each number a with $\bar{a} = aR \bmod n$, where R is easy to divide out afterward

- Imagine $R = 100$; by hand, easy to divide out by dropping trailing 0s
- R must be greater than, and coprime with, n
- Usually $R = 2^k$, where k is the bit length of n
- R is easy to multiply in (and divide out via shift, if it divides evenly)

Instead of computing $a \cdot b \bmod n$, compute $\bar{a} \cdot \bar{b} \bmod n = abRR \bmod n$

- Need to eliminate extra R by dividing
- What if R doesn't divide it evenly? Find a value equivalent $\bmod n$ that it does!
- $abRR + kn \equiv abRR \pmod n$ for any k, find a k that results in multiple of R

Even with more "constant-time" algorithms, timing information leaks key info

Idea: Want to know the exponent (think RSA, Diffie-Hellman)

- Observe timings for a decryption/signing
- Use statistics to approximate likelihood that a "real" multiplication step was completed for the first bit
  - Even though the same operations occurred, their inputs change
- Predict this bit, use this to determine next bit
- …

What do I need to know to carry this out?

- Need y, n to determine x in $y^x \bmod n$
- What does this information correspond to in, say, RSA?
- What type of attack is this? How could one achieve this?

Some mathematical details of the attack

Let:

- t_i be the time required for multiplication and squaring for bit i
- e be the loop overhead, measurement error, etc.
- Thus, $T = e + \sum_{i=0}^{w-1} t_i$ (total time, as recorded/expected)
- Note: These are not constant values, but random variables!

If we can guess x_b (first b bits of x), we can approximate $\mathrm{T_b} = \sum_{i=0}^{b-1} t_i$

- e.g., recompute locally while measuring timing, assuming known algorithm
- Compute $T - T_b = e + \sum_{i=0}^{w-1} t_i - \sum_{i=0}^{b-1} t_i = e + \sum_{i=b}^{w-1} t_i$
- Recall summation rules!

If we can observe the variance of $T - T_b$ for a large sample, we can predict further bits!

- (assuming we are correct about x_b)

Multiple rounds are independent! This means that variance is linear

Assume x_b is correct:

- $V(T - T_b) = V\left(e + \sum_{i=b}^{w-1} t_i\right) = V(e) + (w - b)V(t)$

However, if x_b is not correct (say the last d bits are incorrect):

- $V(T - T_b) = V(e) + (w - b + 2d)V(t)$
  - Our variance measurements for last d bits we guessed are different from reality
  - $2d$ because the variance adds for these bits, rather than cancelling

Thus, a correct guess for the next bit reduces $V(T - T_b)$, incorrect guess increases!

So, guess the first b bits, then measure to guess the next

That means that this method is error-correcting!

If we get in a position where both bit values increase the variance, an earlier bit is probably wrong

How does this help us?

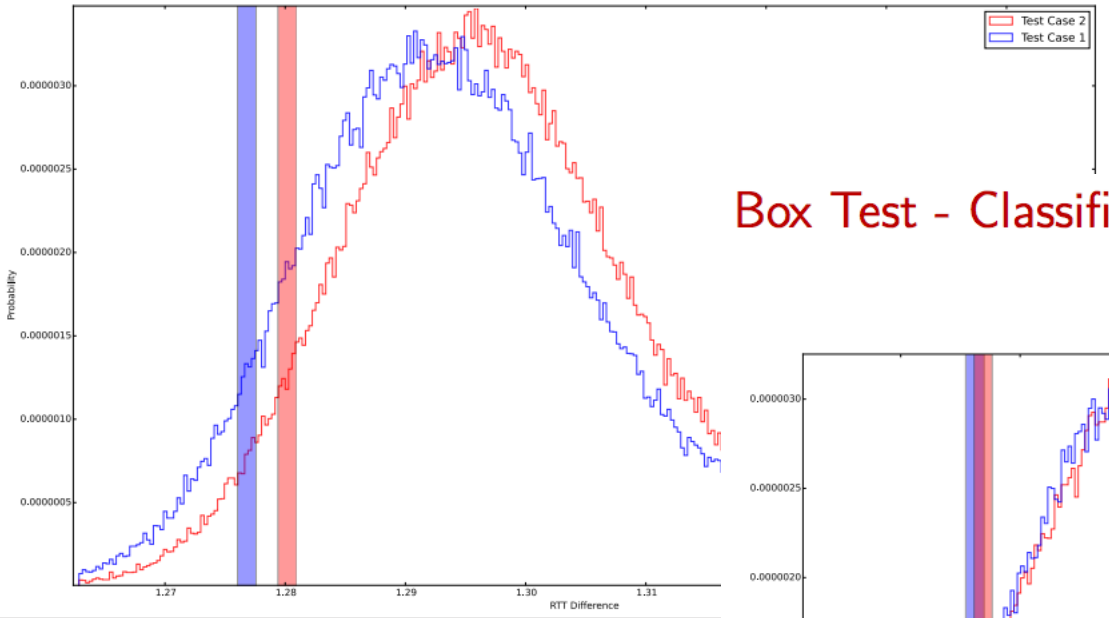
- How do we take advantage to improve the attack?
- Can move on to next bit even with moderate confidence, knowing we can backtrack if confidence drops later
- Keep several guesses and levels of confidence, work on whichever is highest likelihood

Is this really practical over the internet?

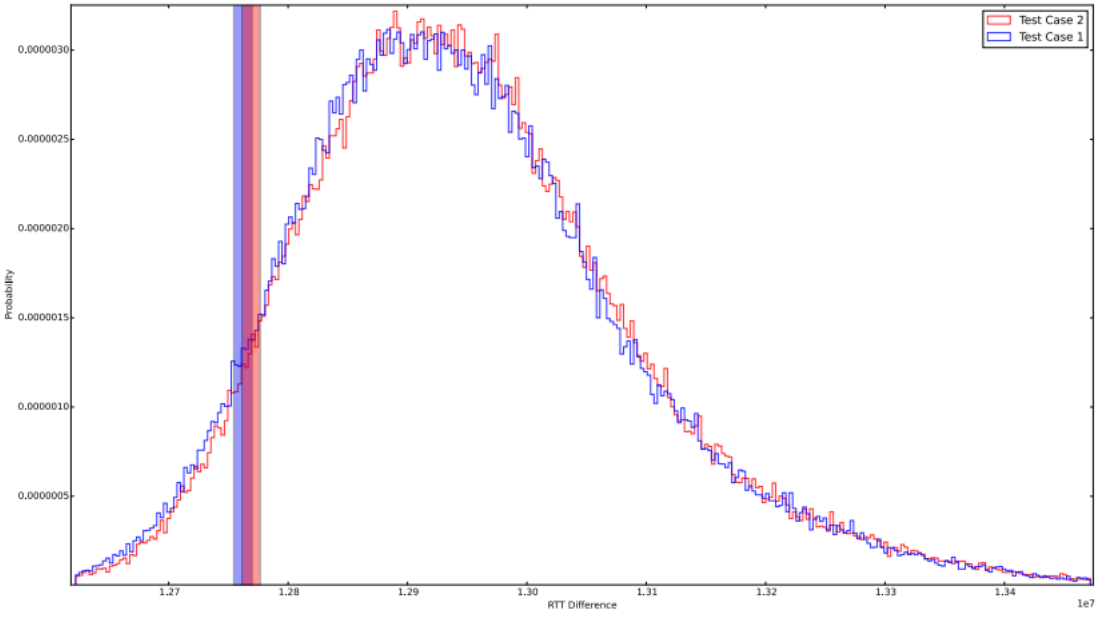
Timothy and Jason Morgan (2015) made progress in achieving this attack in practice

- Main problem: Noise
- Network delay noise drowns out signal in simple statistics
- Improvement: Use TCP timestamps added as RFC 1323
  - Added for better performance; estimate RTT to know when to resend
  - In timing attack, improves timing of RTT and reduces noise
- Improvement: Use Crosby's box test
  - Less susceptible to noise vs. variance
  - "Box in" a range of percentiles, determine if 2 distributions are the same by studying overlap

Box Test - Classified as Different



Box Test - Classified as the Same



So, how can we mitigate such attacks?

We could add a random delay to increase noise

- Downsides? How much noise is needed? Is it feasible?

Do better on constant-time operations

- Execute the union of statements for all cases, but only save those that are needed
- Downsides? Issues?

Consider a blinding approach utilizing RSA's multiplicative homomorphism

- Generate pair (u, v) where $u^d = v^{-1} \bmod n$ (i.e., $u^d v = 1 \bmod n$)
- Instead of $c^d \bmod n$, compute $v(uc)^d \bmod n = vu^d c^d \bmod n = c^d \bmod n$
- Timing modexp is really seeing $(uc)^d \bmod n$
- (u, v) should be secret and fresh; reuse can be revealed in timing
  - Kocher proposed $(u', v') = (u^2 \bmod n, v^2 \bmod n)$ as an update function

Similar attacks on power

Simple Power Analysis (SPA) inspects power graph for spikes corresponding to high-power operations

- Determine key based on when spikes occur

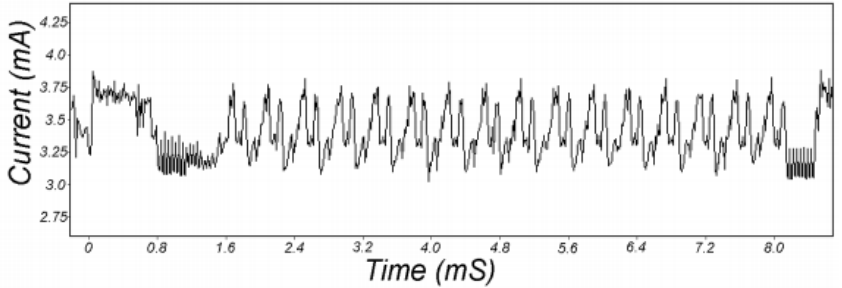


Figure 1: SPA trace showing an entire DES operation.

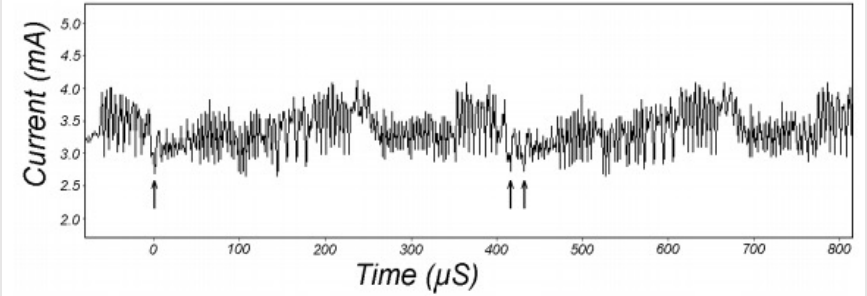


Figure 2: SPA trace showing DES rounds 2 and 3.

Differential Power Analysis (Kocher et al.) uses statistical analysis on the change in power

- Similar in the abstract to the timing attack we discussed today

Cache-timing attacks

High-level idea: Recently-accessed values are faster to access successively

- Recall S-boxes implementing non-linear transformation in AES
- If co-located, cache timing may reveal which S-box fields were used
- This, in turn, can leak key information
- See Bernstein 2005

Conclusions

Cryptography (still) isn't enough to protect our privacy alone

- Computers exist in, and interact with, the real world
- Timing reveals information about execution, and hiding it is hard
- Multiuser systems can still leak information on modern OSs
- Protect physical access, colocation with untrusted code
- Don't implement your own cryptography for production code

Next: Less traditional side-channel attacks