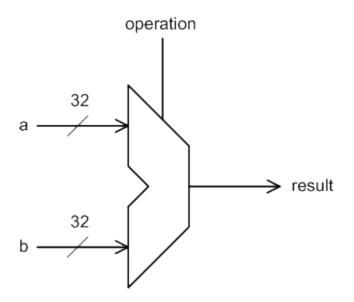
Binary arithmetic

- (Sounds scary)
- So far we studied
 - Instruction set architecture basic
 - MIPS architecture & assembly language
- We will review binary arithmetic algorithms and their implementations
- Binary arithmetic will form the basis for CPU's datapath design



Binary number representations

- We looked at how to represent a number (in fact the value represented by a number) in binary
 - Unsigned numbers everything is positive
- We will deal with more complicated cases
 - Negative numbers
 - Time permitting: Real numbers (a.k.a. floating-point numbers)

Unsigned Binary Numbers

- Limited number of binary numbers (patterns of 0s and 1s)
 - 8-bit number: 256 patterns, 00000000 to 11111111
 - in general, there are 2^N bit patterns, where N is bit width

16 bit:
$$2^{16} = 65,536$$
 bit patterns

32 bit:
$$2^{32} = 4,294,967,296$$
 bit patterns

- Unsigned numbers use patterns for θ and positive numbers
 - 8-bit number range [0..255] corresponds to

- 32-bit number range [0..4294,967,295]
- in general, the range is $[0..2^{N}-1]$

Addition / Subtraction Rules

Binary addition

```
0 + 0 = 0, carry = 0 (no carry)
1 + 0 = 1, carry = 0
```

•
$$0 + 1 = 1$$
, carry = 0

•
$$1 + 1 = 0$$
, carry = 1

Binary subtraction

```
• 0 - 0 = 0, borrow = 0 (no borrow)
```

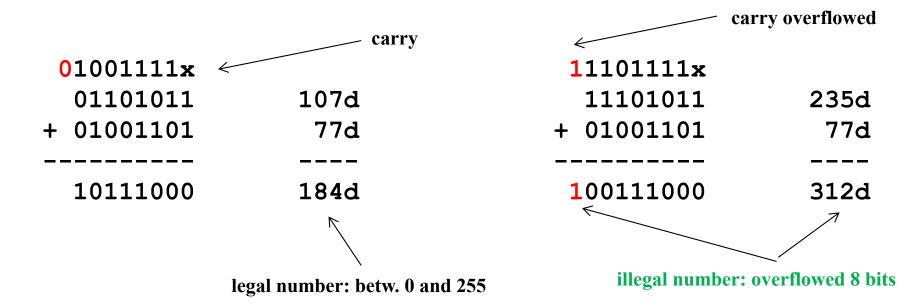
•
$$1 - 0 = 1$$
, borrow = 0

•
$$0 - 1 = 1$$
, borrow = 1

•
$$1 - 1 = 0$$
, borrow = 0

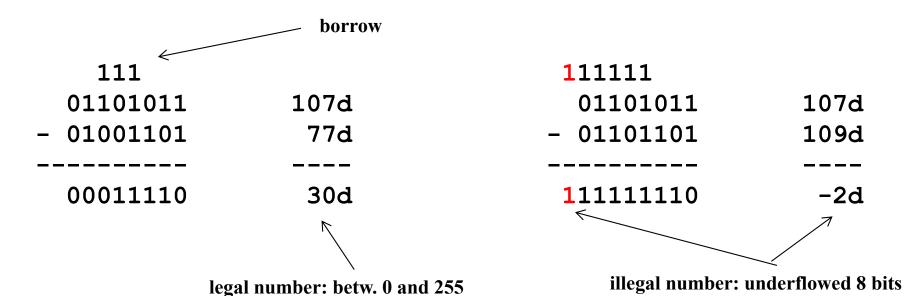
Unsigned Binary Numbers

- Binary arithmetic is straightforward
- Addition: Just add numbers and carry as necessary
- Consider adding 8-bit numbers:



Unsigned Binary Numbers

- Binary arithmetic is straightforward
- Subtraction: Just subtract and borrow as necessary
- Consider subtracting 8-bit numbers:



(i.e., "borrow overflow")

Unsigned Binary to Decimal

- How to convert binary number?
 - First, each digit is position *i*, numbered right to left
 - e.g., for 8-bit number: $b_7b_6b_5b_4$ $b_3b_2b_1b_0$
- Now, we just add up powers of 2
 - $b_0 \times 2^0 + b_1 \times 2^1 + b_2 \times 2^2 + \dots + b_7 \times 2^7$
- An example

1011 0111
=
$$1 \times 2^{0} + 1 \times 2^{1} + 1 \times 2^{2} + 0 \times 2^{3} + 1 \times 2^{4} + 1 \times 2^{5} + 0 \times 2^{6} + 1 \times 2^{7}$$

= $1 + 2 + 4 + 0 + 16 + 32 + 0 + 128$
= 183d

• $v = \sum (b_i \times 2^i)$, where $0 \le i \le K-1$, where K=# bits, i is bit posn

Important 7-bit Unsigned Numbers

- American Standard Code for Information Interchange (ASCII)
 - Developed in early 60s, rooted in telecomm
 - Maps 128 bit patterns (2⁷) into control, alphabet, numbers, graphics
 - Provides control values present in other important codes (at the time)
 - 8th bit might be present and used for error detection (parity)
- Control: Null (0), Bell (7), BS (8), LF (0A), CR (0D), DEL (7F)
- Numbers: (30-39)
- Alphabet: Uppercase (41-5A), Lowercase (61-7A)
- Other (punctuation, etc): 20-2F, 3A-40, 5E-60, 7B-7E
- Unicode: A larger (8,16,32 bit) encoding; backward compatible with ASCII

																		-	and the latest terminal to the latest terminal t	-				
000d	00h		(nul)	016d	10h	(dle)	032d	20h	sp	048d	30h	0	064d	40h	@	080d	50h	P	096d	60h	`	112d	70h	p
001d	01h	0	(soh)	017d	11h	(dc1)	033d	21h	!	049d	31h	1	065d	41h	A	081d	51h	Q	097d	61h	a	113d	71h	q
002d	02h	•	(stx)	018d	12h	‡ (de2)	034d	22h		050d	32h	2	066d	42h	В	082d	52h	R	098d	62h	b	114d	72h	\mathbf{r}
003d	03h	*	(etx)	019d	13h	‼ (de3)	035d	23h	#	051d	33h	3	067d	43h	C	083d	53h	S	099d	63h	с	115d	73h	s
004d	04h	٠	(eot)	020d	14h	¶ (dc4)	036d	24h	\$	052d	34h	4	068d	44h	D	084d	54h	Т	100d	64h	d	116d	74h	t
005d	05h	٠	(enq)	021d	15h	§ (nak)	037d	25h	%	053d	35h	5	069d	45h	Е	085d	55h	U	101d	65h	е	117d	75h	u
006d	06h	٠	(ack)	022d	16h	(syn)	038d	26h	&	054d	36h	6	070d	46h	F	086d	56h	v	102d	66h	f	118d	76h	v
007d	07h	•	(bel)	023d	17h	(etb)	039d	27h		055d	37h	7	071d	47h	G	087d	57h	W	103d	67h	g	119d	77h	w
008d	08h		(bs)	024d	18h	↑ (can)	040d	28h	(056d	38h	8	072d	48h	н	088d	58h	x	104d	68h	h	120d	78h	x
009d	09h		(tab)	025d	19h	↓ (enn)	041d	29h)	057d	39h	9	073d	49h	1	089d	59h	Y	105d	69h	i	121d	79h	у
010d	OAh		(1f)	026d	1Ah	(eof)	042d	2Ah	*	058d	3Ah	:	074d	4Ah	J	090d	5Ah	Z	106d	6Ah	j	122d	7Ah	z
011d	OBh	ੋ	(vt)	027d	1Bh	← (esc)	043d	2Bh	+	059d	3Bh	;	075d	4Bh	К	091d	5Bh	[]	107d	6Bh	k	123d	7Bh	- {
012d	0Ch	₽	(np)	028d	1Ch	- (fs)	044d	2Ch	,	060d	3Ch	<	076d	4Ch	L	092d	5Ch	١	108d	6Ch	1	124d	7Ch	- 1
013d	ODh		(cr)	029d	1Dh	↔ (gs)	045d	2Dh	_	061d	3Dh	-	077d	4Dh	M	093d	5Dh	- 1	109d	6Dh	m	125d	7Dh	}
014d	0Eh	Ą)	(so)	030d	1Eh	* (rs)	046d	2Eh		062d	3Eh	>	078d	4Eh	N	094d	5Eh	^	110d	6Eh	n	126d	7Eh	~
015d	OFh	0	(si)	031d	1Fh	* (us)	047d	2Fh	/	063d	3Fh	?	079d	4Fh	0	095d	5Fh	_	111d	6Fh	О	127d	7Fh	
011d 012d 013d 014d	OBh OCh ODh OEh	\$	(vt) (np) (cr) (so)	027d 028d 029d 030d	1Bh 1Ch 1Dh 1Eh	← (esc) ← (fs) ↔ (gs) ▲ (rs)	043d 044d 045d 046d	2Bh 2Ch 2Dh 2Eh	* - /	059d 060d 061d 062d	3Bh 3Ch 3Dh 3Eh	; < = > ?	075d 076d 077d 078d	4Bh 4Ch 4Dh 4Eh	K L M	091d 092d 093d 094d	5Bh 5Ch 5Dh 5Eh	z [\]	107d 108d 109d 110d	6Bh 6Ch 6Dh 6Eh	1 m n	123d 124d 125d 126d	7Bh 7Ch 7Dh 7Eh	

Extended ASCII Chart (character codes 128 - 255; Codepage 850)

80h	Ç	144d	90h	É	160d	AOh	á	176d	BOh	- 1	192d	COh	L	208d	DOh	D 224d	E0h	Ó	240d	FOh	-
81h	ü	145d	91h	æ	161d	A1h	í	177d	B1h	20	193d	C1h	1	209d	D1h	Đ 225d	E1h	ß	241d	F1h	±
82h	é	146d	92h	Æ	162d	A2h	ó	178d	B2h		194d	C2h	т	210d	D2h	È 226d	E2h	Ô	242d	F2h	-
83h	â	147d	93h	ô	163d	A3h	ú	179d	B3h	- 1	195d	C3h	H	211d	D3h	Ë 227d	E3h	Ò	243d	F3h	34
84h	ā	148d	94h	ö	164d	A4h	ñ	180d	B4h	+	196d	C4h	-	212d	D4h	È 228d	E4h	ő	244d	F4h	- 1
85h	à	149d	95h	δ	165d	A5h	Ñ	181d	B5h	Á	197d	C5h	+	213d	D5h	1 229d	E5h	Ő	245d	F5h	§
86h	å	150d	96h	û	166d	A6h	a	182d	B6h	Â	198d	C6h	ã	214d	D6h	Ì 230d	E6h	μ	246d	F6h	÷
87h	ç	151d	97h	ù	167d	A7h	ō	183d	B7h	À	199d	C7h	Ã	215d	D7h	Î 231d	E7h	þ	247d	F7h	
88h	ê	152d	98h	ÿ	168d	A8h	i	184d	BSh	0	200d	C8h	L	216d	D8h	Ï 232d	E8h	Þ	248d	F8h	6
89h	ë	153d	99h	Ö	169d	A9h	®	185d	B9h	4	201d	C9h	F	217d	D9h	J 233d	E9h	Ú	249d	F9h	-
8Ah	è	154d	9Ah	Ü	170d	AAh	_	186d	BAh	- 1	202d	CAh	T	218d	DAh	г 234d	EAh	Û	250d	FAh	
8Bh	ï	155d	9Bh	ø	171d	ABh	1/2	187d	BBh	9	203d	CBh	Ŧ	219d	DBh	235d	EBh	Ù	251d	FBh	1
8Ch	î	156d	9Ch	£	172d	ACh	1/4	188d	BCh	a	204d	CCh	ŀ	220d	DCh	■ 236d	ECh	ý	252d	FCh	2
8Dh	ì	157d	9Dh	Ø	173d	ADh	i	189d	BDh	c	205d	CDh	-	221d	DDh	237d	EDh	Ý	253d	FDh	8
8Eh	Ä	158d	9Eh	×	174d	AEh	**	190d	BEh	¥	206d	CEh	÷	222d	DEh	Ì 238d	EEh	-	254d	FEh	•
-8Fh	Å	159d	9Fh	f	175d	AFh	39	191d	BFh	٦	207d	CFh	Ħ	223d	DFh	■ 239d	EFh	-	255d	FFh	
	81h 82h 83h 84h 85h 86h 87h 88h 89h 8Ah 8Bh 8Ch 8Dh 8Eh	81h ü 82h é 83h â 84h ã 85h à 86h å 87h ç 88h è 89h ë 88h ï 8Ch î 8Dh ì 8Eh Ä	81h ü 145d 82h é 146d 83h â 147d 84h ã 148d 85h à 149d 86h â 150d 87h ç 151d 88h è 152d 89h ë 153d 8Ah è 154d 8Bh ï 155d 8Ch î 156d 8Dh ì 157d 8Eh Ä 158d	81h ü 145d 91h 82h é 146d 92h 83h â 147d 93h 84h ă 148d 94h 85h à 149d 95h 86h â 150d 96h 87h ç 151d 97h 88h è 152d 98h 89h ë 153d 99h 8Ah è 154d 9Ah 8Bh ï 155d 9Bh 8Ch î 156d 9Ch 8Dh ì 157d 9Dh 8Eh Ä 158d 9Eh	81h ü 145d 91h æ 82h é 146d 92h æ 83h â 147d 93h ô 84h ã 148d 94h ö 85h à 149d 95h ò 86h â 150d 96h û 87h ç 151d 97h ù 88h ê 152d 98h ÿ 89h ë 153d 99h Ö 8Ah è 154d 9Ah Ü 8Bh ï 155d 9Bh ø 8Ch î 156d 9Ch £ 8Dh ì 157d 9Dh Ø 8Eh Ä 158d 9Eh ×	81h ü 145d 91h æ 161d 82h é 146d 92h æ 162d 83h â 147d 93h ô 163d 84h ã 148d 94h ö 164d 85h à 149d 95h ò 165d 86h â 150d 96h û 166d 87h ç 151d 97h ù 167d 88h è 152d 98h ÿ 168d 89h ë 153d 99h ö 169d 8Ah è 154d 9Ah Ü 170d 8Bh ï 155d 9Bh ø 171d 8Ch î 156d 9Ch £ 172d 8Dh ì 157d 9Dh Ø 173d 8Eh Ä 158d 9Eh × 174d	81h ü 145d 91h æ 161d A1h 82h é 146d 92h Æ 162d A2h 83h â 147d 93h ô 163d A3h 84h ā 148d 94h ö 164d A4h 85h à 149d 95h ò 165d A5h 86h â 150d 96h û 166d A6h 87h ç 151d 97h ù 167d A7h 88h è 152d 98h ÿ 168d A8h 89h ë 153d 99h Ö 169d A9h 8Ah è 154d 9Ah Ü 170d AAh 8Bh ï 155d 9Bh ø 171d ABh 8Ch î 156d 9Ch £ 172d ACh 8Dh ì 157d	81h ü 145d 91h æ 161d A1h í 82h é 146d 92h Æ 162d A2h ó 83h â 147d 93h ô 163d A3h ú 84h ã 148d 94h ö 164d A4h ñ 85h à 149d 95h ò 165d A5h Ñ 86h â 150d 96h û 166d A6h ª 87h ç 151d 97h ù 167d A7h 9 88h è 152d 98h ÿ 168d A8h è 89h ë 153d 99h ö 169d A9h ® 8Ah è 154d 9Ah Ü 170d AAh ¬ 8Bh ï 155d 9Bh ø 171d ABh ½ 8Ch	81h ü 145d 91h æ 161d A1h í 177d 82h é 146d 92h Æ 162d A2h ó 178d 83h â 147d 93h ô 163d A3h ú 179d 84h ã 148d 94h ö 164d A4h ñ 180d 85h à 149d 95h ò 165d A5h Ñ 181d 86h â 150d 96h û 166d A6h a 182d 87h ç 151d 97h ù 167d A7h 9 183d 88h è 152d 98h ÿ 168d A8h ù 184d 89h ë 153d 99h Ö 169d A9h ® 185d 8Ah è 154d 9Ah Ü 170d AAh 186d 8Bh ï 155d 9Bh ø 171d ABh ½ 187d 8Ch î 156d 9Ch £ 172d ACh ¼ 188d 8Dh ì 157d 9Dh <	81h ü 145d 91h æ 161d A1h i 177d B1h 82h 6 146d 92h æ 162d A2h 6 178d B2h 83h â 147d 93h ô 163d A3h ú 179d B3h 84h ã 148d 94h ö 164d A4h ñ 180d B4h 85h à 149d 95h ò 165d A5h Ñ 181d B5h 86h â 150d 96h û 166d A6h * 182d B6h 87h ç 151d 97h ù 167d A7h ° 183d B7h 88h è 152d 98h ÿ 168d A8h è 184d B8h 89h ë 153d 99h Ö 169d A9h ® 185d BAh	81h ü 145d 91h æ 161d A1h í 177d B1h 82h 6 146d 92h æ 162d A2h 6 178d B2h 83h â 147d 93h ô 163d A3h ú 179d B3h 84h ã 148d 94h ô 164d A4h ñ 180d B4h 85h â 149d 95h ò 165d A5h Ñ 181d B5h Á 86h â 150d 96h û 166d A6h a 182d B6h Â 87h ç 151d 97h ù 167d A7h 9 183d B7h À 88h è 152d 98h ÿ 168d A8h i 184d B8h © 89h ë 153d 99h ö 169d A9h ® 185d B9h 8Bh ï 155d </td <td>81h ü 145d 91h æ 161d A1h i 177d B1h ll 193d 82h é 146d 92h æ 162d A2h ó 178d B2h ll 194d 83h â 147d 93h ô 163d A3h ú 179d B3h l 195d 84h ã 148d 94h ö 164d A4h ñ 180d B4h d 195d 85h à 149d 95h ò 165d A5h Ñ 181d B5h Á 197d 86h â 150d 96h û 166d A6h a 182d B6h Â 198d 87h ç 151d 97h ù 167d A7h g 183d B7h À 199d 88h è 152d 98h ÿ 168d A8h i 184d B8h g 200d 89h ë 153d 99h</td> <td>81h ü 145d 91h æ 161d A1h í 177d B1h 193d C1h 82h 6 146d 92h æ 162d A2h 6 178d B2h 194d C2h 83h â 147d 93h ô 163d A3h ú 179d B3h 195d C3h 84h ã 148d 94h ô 164d A4h ñ 180d B4h 195d C3h 85h â 149d 95h ò 165d A5h Ñ 181d B5h Á 197d C5h 86h â 150d 96h û 166d A6h a 182d B6h Â 197d C5h 86h â 150d 96h û 166d A6h a 182d B6h Â 199d C7h 88h ê 152d 98h ÿ 168d A8h i 184d B8h © 200d</td> <td>81h ü 145d 91h æ 161d A1h i 177d B1h II 193d C1h 1 82h é 146d 92h Æ 162d A2h ó 178d B2h II 194d C2h T 83h â 147d 93h ô 163d A3h ú 179d B3h I 195d C3h F 84h ā 148d 94h ô 164d A4h ñ 180d B4h I 196d C4h — 85h à 149d 95h ò 165d A5h Ñ 181d B5h Á 197d C5h † 86h â 150d 96h û 166d A6h a 182d B6h Â 198d C6h â 87h ç 151d 97h ù 167d A7h 9 183d B7h À 199d C7h Â 88h è 152d 98h<</td> <td>81h ü 145d 91h æ 161d A1h í 177d B1h 193d C1h 1 209d 82h 6 146d 92h Æ 162d A2h 6 178d B2h 194d C2h T 210d 83h â 147d 93h ô 163d A3h ú 179d B3h 195d C3h † 211d 84h ã 148d 94h ô 164d A4h ñ 180d B4h † 196d C4h - 212d 85h â 149d 95h ò 165d A5h Ñ 181d B5h Á 197d C5h † 213d 86h â 150d 96h û 166d A6h 182d B6h Â 198d C6h â 214d 87h ç 151d 97h ù 167d A7h 9 183d B7h À 199d C7h Å 215d</td> <td>81h ü 145d 91h æ 161d A1h i 177d B1h II 193d C1h I 209d D1h 82h 6 146d 92h Æ 162d A2h 6 178d B2h II 194d C2h T 210d D2h 83h â 147d 93h ô 163d A3h ú 179d B3h I 195d C3h F 211d D3h 84h ã 148d 94h ô 164d A4h ñ 180d B4h I 196d C4h — 212d D4h 85h â 149d 95h ò 165d A5h Ñ 181d B5h Á 197d C5h + 212d D4h 86h â 150d 96h û 166d A6h a 182d B6h Â 198d C6h â 214d D6h 87h ç 151d 97h ù 167d</td> <td>81h ü 145d 91h æ 161d A1h i 177d B1h l 193d C1h l 209d D1h D 225d 82h 6 146d 92h l 162d A2h 6 178d B2h l 194d C2h T 210d D2h E 226d 83h â 147d 93h ô 163d A3h ú 179d B3h l 195d C2h T 210d D2h E 226d 84h â 148d 94h ô 164d A4h ñ 180d B4h l 196d C4h - 212d D4h E 228d 85h à 149d 95h ò 165d A5h Ñ 181d B5h Á 197d C5h † 213d D5h 1 229d 86h â 150d 96h û 166d A6h l 182d B6h Â 199d C7h <</td> <td>81h ü 145d 91h æ 161d A1h i 177d B1h ll 193d C1h l 209d D1h D 225d E1h 82h 6 146d 92h E 162d A2h 6 178d B2h l 194d C2h T 210d D2h E 226d E2h 83h â 147d 93h ô 163d A3h ú 179d B3h l 195d C3h l 211d D3h E 227d E3h 84h â 148d 94h ô 164d A4h ñ 180d B4h l 196d C4h - 212d D4h E 227d E3h 85h â 149d 95h ò 165d A5h Ñ 181d B5h Å 197d C5h + 213d D5h 1 229d E5h 86h â 151d 97h ù 167d A7h 9</td> <td>81h</td> <td>81h û 145d 91h æ 161d A1h i 177d B1h I 193d C1h I 209d D1h D 225d E1h B 241d 82h 6 146d 92h E 162d A2h 6 178d B2h I 194d C2h T 210d D2h E 226d E2h Ô 242d 83h â 147d 93h ô 163d A3h ú 179d B3h I 195d C3h F 211d D3h E 227d E3h Ò 242d 84h â 148d 94h ô 164d A4h ñ 180d B4h I 196d C4h — 212d D4h E 227d E3h Ò 243d 85h â 149d 95h ò 165d A5h Ñ 181d B5h Á 197d C5h † 212d D4h E 228d E4h ò <td< td=""><td>81h ü 145d 91h æ 161d Alh i 177d Blh II 193d Clh I 209d Dlh D 225d Elh B 241d Flh 82h 6 146d 92h E 162d A2h 6 178d B2h II 194d C2h T 210d D2h E 226d E2h 0 242d F2h 83h â 147d 93h 6 163d A3h û 179d B3h 1 195d C3h F 211d D3h E 227d E3h 0 242d F2h 84h â 148d 94h 6 164d A4h ñ 180d B4h 1 195d C3h F 211d D3h E 227d E3h 0 244d F4h 85h â 149d 95h ô 165d A5h Ñ 181d B5h Å 197d C5h 19 213d D5h</td></td<></td>	81h ü 145d 91h æ 161d A1h i 177d B1h ll 193d 82h é 146d 92h æ 162d A2h ó 178d B2h ll 194d 83h â 147d 93h ô 163d A3h ú 179d B3h l 195d 84h ã 148d 94h ö 164d A4h ñ 180d B4h d 195d 85h à 149d 95h ò 165d A5h Ñ 181d B5h Á 197d 86h â 150d 96h û 166d A6h a 182d B6h Â 198d 87h ç 151d 97h ù 167d A7h g 183d B7h À 199d 88h è 152d 98h ÿ 168d A8h i 184d B8h g 200d 89h ë 153d 99h	81h ü 145d 91h æ 161d A1h í 177d B1h 193d C1h 82h 6 146d 92h æ 162d A2h 6 178d B2h 194d C2h 83h â 147d 93h ô 163d A3h ú 179d B3h 195d C3h 84h ã 148d 94h ô 164d A4h ñ 180d B4h 195d C3h 85h â 149d 95h ò 165d A5h Ñ 181d B5h Á 197d C5h 86h â 150d 96h û 166d A6h a 182d B6h Â 197d C5h 86h â 150d 96h û 166d A6h a 182d B6h Â 199d C7h 88h ê 152d 98h ÿ 168d A8h i 184d B8h © 200d	81h ü 145d 91h æ 161d A1h i 177d B1h II 193d C1h 1 82h é 146d 92h Æ 162d A2h ó 178d B2h II 194d C2h T 83h â 147d 93h ô 163d A3h ú 179d B3h I 195d C3h F 84h ā 148d 94h ô 164d A4h ñ 180d B4h I 196d C4h — 85h à 149d 95h ò 165d A5h Ñ 181d B5h Á 197d C5h † 86h â 150d 96h û 166d A6h a 182d B6h Â 198d C6h â 87h ç 151d 97h ù 167d A7h 9 183d B7h À 199d C7h Â 88h è 152d 98h<	81h ü 145d 91h æ 161d A1h í 177d B1h 193d C1h 1 209d 82h 6 146d 92h Æ 162d A2h 6 178d B2h 194d C2h T 210d 83h â 147d 93h ô 163d A3h ú 179d B3h 195d C3h † 211d 84h ã 148d 94h ô 164d A4h ñ 180d B4h † 196d C4h - 212d 85h â 149d 95h ò 165d A5h Ñ 181d B5h Á 197d C5h † 213d 86h â 150d 96h û 166d A6h 182d B6h Â 198d C6h â 214d 87h ç 151d 97h ù 167d A7h 9 183d B7h À 199d C7h Å 215d	81h ü 145d 91h æ 161d A1h i 177d B1h II 193d C1h I 209d D1h 82h 6 146d 92h Æ 162d A2h 6 178d B2h II 194d C2h T 210d D2h 83h â 147d 93h ô 163d A3h ú 179d B3h I 195d C3h F 211d D3h 84h ã 148d 94h ô 164d A4h ñ 180d B4h I 196d C4h — 212d D4h 85h â 149d 95h ò 165d A5h Ñ 181d B5h Á 197d C5h + 212d D4h 86h â 150d 96h û 166d A6h a 182d B6h Â 198d C6h â 214d D6h 87h ç 151d 97h ù 167d	81h ü 145d 91h æ 161d A1h i 177d B1h l 193d C1h l 209d D1h D 225d 82h 6 146d 92h l 162d A2h 6 178d B2h l 194d C2h T 210d D2h E 226d 83h â 147d 93h ô 163d A3h ú 179d B3h l 195d C2h T 210d D2h E 226d 84h â 148d 94h ô 164d A4h ñ 180d B4h l 196d C4h - 212d D4h E 228d 85h à 149d 95h ò 165d A5h Ñ 181d B5h Á 197d C5h † 213d D5h 1 229d 86h â 150d 96h û 166d A6h l 182d B6h Â 199d C7h <	81h ü 145d 91h æ 161d A1h i 177d B1h ll 193d C1h l 209d D1h D 225d E1h 82h 6 146d 92h E 162d A2h 6 178d B2h l 194d C2h T 210d D2h E 226d E2h 83h â 147d 93h ô 163d A3h ú 179d B3h l 195d C3h l 211d D3h E 227d E3h 84h â 148d 94h ô 164d A4h ñ 180d B4h l 196d C4h - 212d D4h E 227d E3h 85h â 149d 95h ò 165d A5h Ñ 181d B5h Å 197d C5h + 213d D5h 1 229d E5h 86h â 151d 97h ù 167d A7h 9	81h	81h û 145d 91h æ 161d A1h i 177d B1h I 193d C1h I 209d D1h D 225d E1h B 241d 82h 6 146d 92h E 162d A2h 6 178d B2h I 194d C2h T 210d D2h E 226d E2h Ô 242d 83h â 147d 93h ô 163d A3h ú 179d B3h I 195d C3h F 211d D3h E 227d E3h Ò 242d 84h â 148d 94h ô 164d A4h ñ 180d B4h I 196d C4h — 212d D4h E 227d E3h Ò 243d 85h â 149d 95h ò 165d A5h Ñ 181d B5h Á 197d C5h † 212d D4h E 228d E4h ò <td< td=""><td>81h ü 145d 91h æ 161d Alh i 177d Blh II 193d Clh I 209d Dlh D 225d Elh B 241d Flh 82h 6 146d 92h E 162d A2h 6 178d B2h II 194d C2h T 210d D2h E 226d E2h 0 242d F2h 83h â 147d 93h 6 163d A3h û 179d B3h 1 195d C3h F 211d D3h E 227d E3h 0 242d F2h 84h â 148d 94h 6 164d A4h ñ 180d B4h 1 195d C3h F 211d D3h E 227d E3h 0 244d F4h 85h â 149d 95h ô 165d A5h Ñ 181d B5h Å 197d C5h 19 213d D5h</td></td<>	81h ü 145d 91h æ 161d Alh i 177d Blh II 193d Clh I 209d Dlh D 225d Elh B 241d Flh 82h 6 146d 92h E 162d A2h 6 178d B2h II 194d C2h T 210d D2h E 226d E2h 0 242d F2h 83h â 147d 93h 6 163d A3h û 179d B3h 1 195d C3h F 211d D3h E 227d E3h 0 242d F2h 84h â 148d 94h 6 164d A4h ñ 180d B4h 1 195d C3h F 211d D3h E 227d E3h 0 244d F4h 85h â 149d 95h ô 165d A5h Ñ 181d B5h Å 197d C5h 19 213d D5h

Hexadecimal to Binary

0	0000	8	1000
1	0001	9	1001
2	0010	A	1010
3	0011	В	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111

Groups of ASCII-Code in Binary

Bit 6	Bit 5	Group
0	0	Control Characters
0	1	Digits and Punctuation
1	0	Upper Case and Special
1	1	Lower Case and Special

Signed Numbers

- How to represent positive and *negative* numbers?
- We still have a limited number of bit patterns
 - 8-bit: 256 bit patterns (i.e., 00000000 ... 11111111)
 - 16 bit: $2^{16} = 65,536$ bit patterns
 - 32 bit: $2^{32} = 4,294,967,296$ bit patterns
- Re-assign bit patterns differently
 - Some patterns are assigned to negative numbers, some to positive
- How to assign available patterns? Three ways:
 - Sign magnitude, 1's complement, 2's complement

Method 1: sign-magnitude

- Same method we use for decimal numbers
- {sign bit, absolute value (magnitude)}
 - Sign bit (msb): 0 positive, 1 negative
 - Examples, assume 4-bit representation

```
• 0000 +0
• 0011 +3
• 1001 -1
• 1111 -7
• 1000 -0 (two 0's???)
```

Properties

- Two 0s a positive 0 and a negative 0?
- Equal # of positive and negative numbers
- A + (-A) does not give zero!
- Consider sign during arithmetic

Sign-magnitude

- Let's check A + (-A) is not zero
- Consider N = 5 bits number. Zero is 00000 or 10000.
- Try this: -4 + 4 = ?????

```
-4 is 10100
4 is 00100

so, let's add them together:
10100 -4d
+ 00100 4d
----- ---
11000 -8d YIKES!
```

Method 2: one's complement

- Negation of +X is $((2^N 1) X)$, where N is number of bits
 - $A + (-A) = 2^{N} 1$ (i.e., -0)
 - Given a number A, it's negation is done by (1111...1111 A)
 - In fact, simple bit-by-bit inversion will give the same-magnitude number with a different sign
 - Examples, assume 4-bit representation
 - 0000
 - 0011
 - 1001 [^]
 - 1111 -u
 - 1000
- Properties
 - There are two 0s
 - There are equal # of positive and negative numbers
 - A+(-A) = 0 (whew!) but... A+0=A only works for +0 (try it with -0!)
 - 2 step process for subtraction (accounts for "carry out")

One's Complement

- Negation of $X(2^N-1)-X$), positive are usual value
- Consider N=4

<u>Binary</u>	<u>One's</u>	<u>Binary</u>	<u>One's</u>
0000	0	1000	-7
0001	1	1001	-6
0010	2	1010	-5
0011	3	1011	-4
0100	4	1100	-3
0101	5	1101	-2
0110	6	1110	-1
0111	7	1111	-0

notice how the counting works: 1111 is -0... then -1... -2... etc.

One's Complement

- Let's check the "0 property": A + (-A) = 0
- Suppose A = 5

```
5 is 0101 negation of 5 is (2^4-1)-5 = (16-1)-5 = 15-5 = 10 10 (unsigned) is 1010 check the table: 1010 is -5 in 1's complement now, let's try 5 + (-5) in 1's complement
```

0101	5	1010	
+ 1010	-5	+ 0000 (+0)	11 0)
1111	-0	1010 (-5)	(-6)

Method 3: two's complement

- Negation is $(2^N X)$
 - $A + (-A) = 2^{N}$
 - Given a number A, it's negation is done by (1111...1111 A) + 1
 - In fact, simple bit-by-bit inversion followed by adding 1 will give the same-magnitude number with a different sign
 - Examples, assume 4-bit representation
 - 0000
 - 0011
 - 1001
 - 1111
 - 1000 ?
- Properties
 - There is a single 0
 - There are unequal # of positive and negative numbers
 - Subtraction is simplified one step based on addition (we'll see! ©)

- Negation of $X(2^N X)$, positive are usual value
- Consider N=4

<u>Binary</u>	<u>One's</u>	<u>Binary</u>	<u>One's</u>
0000	0	1000	-8
0001	1	1001	-7
0010	2	1010	-6
0011	3	1011	-5
0100	4	1100	-4
0101	5	1101	-3
0110	6	1110	-2
0111	7	1111	-1

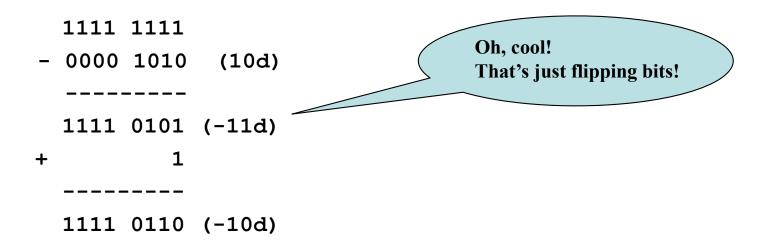
notice how the counting works: 1000 is -8... 1001 is -7... etc.

- Let's check the "0 property": A + (-A) = 0
- Suppose A = 5

```
5 is 0101 negation of 5 is 2^4 - 5 = 16 - 5 = 11 11(unsigned) is 1011 check the table: 1011 is -5 in 2's complement now, let's try 5 + (-5) in 2's complement
```

```
0101 5 1011 0111 (7)
+ 1011 -5 + 0000 (0) + 0001 (1)
----- 0 1011 (-5) 1000 (-8)
```

- Negation: $(2^8 X)$ vs. (1111111111 X) + 1
- Note 2⁸ needs 9 bits:
 - 2^8 is 256, from earlier conversion process: $1\ 0000\ 0000 = 1 * 2^8$
- Whereas the other form has only 8 bits. Let's try it!
 - Consider X = 10 and we want to find -10



- How to convert binary 2's complement number?
 - Same as before, except most significant bit is "sign"
- Consider an 8-bit 2's complement number
 - $b_0 \times 2^0 + b_1 \times 2^1 + b_2 \times 2^2 + \dots + b_7 \times (-2^7)$
- An example

1011 0111
=
$$1 \times 2^{0} + 1 \times 2^{1} + 1 \times 2^{2} + 0 \times 2^{3} + 1 \times 2^{4} + 1 \times 2^{5} + 0 \times 2^{6} + 1 \times (-2^{7})$$

= $1 + 2 + 4 + 0 + 16 + 32 + 0 + (-128)$
= $-73d$

- What is 73d in 2's complement binary number?
- $v = (\sum (b_i \times 2^i)) + b_{K-1} \times -2^{K-1}$, where $0 \le i \le K-1$, where K=# bits, i is bit posn

Summary

Code	Sign-Magnitude	1's Complement	2's Complement
000	+0	+0	+0
001	+1	+1	+1
010	+2	+2	+2
011	+3	+3	+3
100	-0	-3	-4
101	-1	-2	-3
110	-2	-1	-2
111	-3	-0	-1

Issues

- # of zeros
- Balance
- Arithmetic algorithm implementation