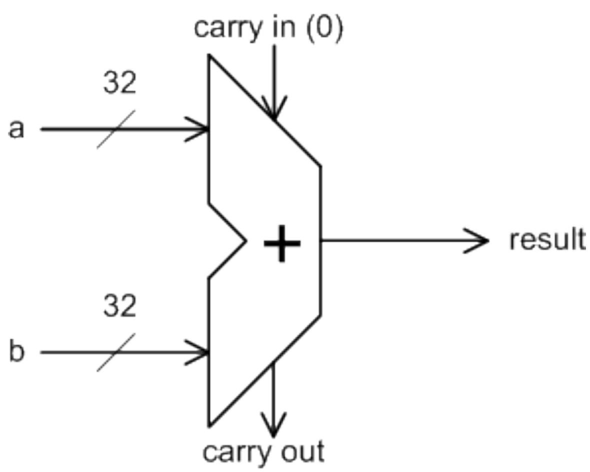
Addition

- We are quite familiar with adding two numbers in decimal
  - What about adding two binary numbers?
- If we use the two's complement method to represent binary numbers, addition can be done in a straightforward way



Suppose:
N=8
a=20
b=30

What is result and carry out?

Addition

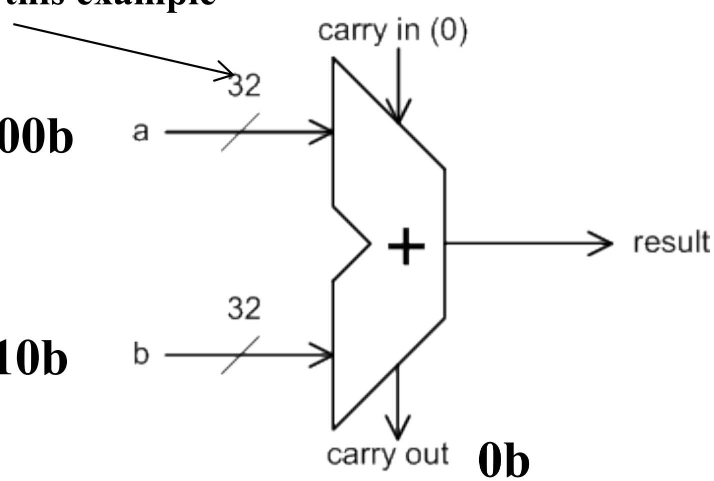
- N=8, a=20, b=30
- Do binary addition to get result and carryout

- Convert A and B to binary? How?
  - $a=20=4+16=2^2+2^4$ => a is 0001 0100b
  - $b=30=16+8+4+2=2^4+2^3+2^2+2^1$ => b is 0001 1110b

NOTE: N=8 in this example

0001 0100b

0001 1110b



0011 0010b

0b

0001 0100b
+ 0001 1110b
----
0 0011 0010b

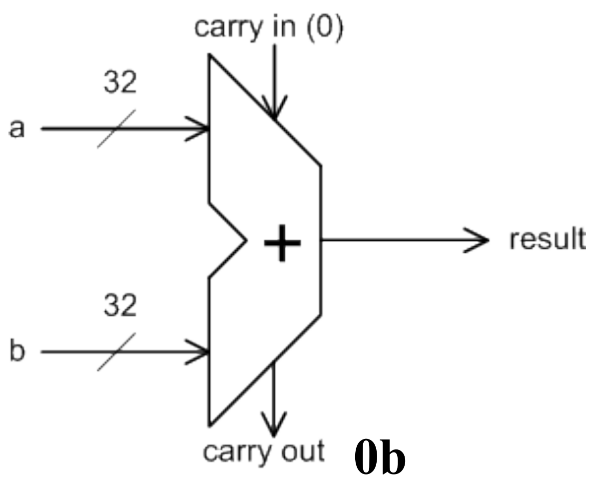
Addition

- N=8, a=80, b=50
- Do binary addition to get result and carryout

- Convert A and B to binary? How?
  - $A=80=64+16=2^6+2^4$ => a is 0101 0000b
  - $b=50=32+16+2=2^5+4^3+2^1$ => b is 0011 0010b

0101 0000b

0011 0010b



1000 0010b

0b

0101 0000b
+ 0011 0010b
----
0 1000 0010b

Result is NEGATIVE!

Overflow

- Because we use a limited number of digits to represent a number, the result of an operation may not fit
- No overflow when result remains in expected range
  - We ***add two numbers with different signs***
  - We ***subtract a number from another number with the same sign***
- When can overflow happen?

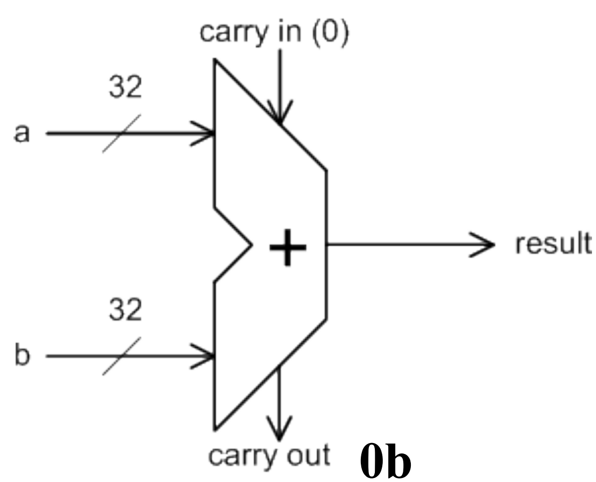
a	**b**	**overflow possible?**
+	+	**yes**
+	-	**no**
-	+	**no**
-	-	**yes**

Overflow

- What is special about the cases where overflow happened?
  - ***The input values signs are the same; so, can go outside range***
- Overflow detection
  - Adding two positive numbers yields a negative number
  - Adding two negative numbers yields a positive number

0101 0000b

0011 0010b



1000 0010b

0b

Check signs
a is positive
b is positive
result isn't!

What happens on overflow?

- The CPU can
  - Generate an exception (what is an exception?)
  - Set a flag in the status register (what is the status register?)
  - Do nothing

- Languages may have different notions about overflow

- Do we have overflows in the case of unsigned, always positive numbers?
  - Example: addu, addiu, subu

Unsigned Binary Numbers in MIPS

- MIPS instruction set provides support
  - addu $1,$2,$3 - adds two unsigned numbers ($2,$3)
  - addiu $1,$2,10 - adds unsigned number with **signed** immediate
  - subu $1,$2,$3 - subtracts two unsigned numbers
  - etc.
- Primary issue: **The carry/borrow out is ignored**
  - Overflow is possible, but it is ignored
  - Signed versions take special action on overflow (we'll see shortly!)
- Unsigned memory accesses: lbu, lhu
  - Loaded value is treated as unsigned number
  - Convert from smaller bit width (8 or 16) to a 32-bit number
  - **Upper bits in the 32-bit destination register are set to 0s**

MIPS example

- I looked at the MIPS32 instruction set manual
- ADD, ADDI instructions generate an ***exception*** on overflow
- ADDU, ADDIU are ***silent***

```
li     $t0,0x40000000
add    $t1,$t0,$t0
```

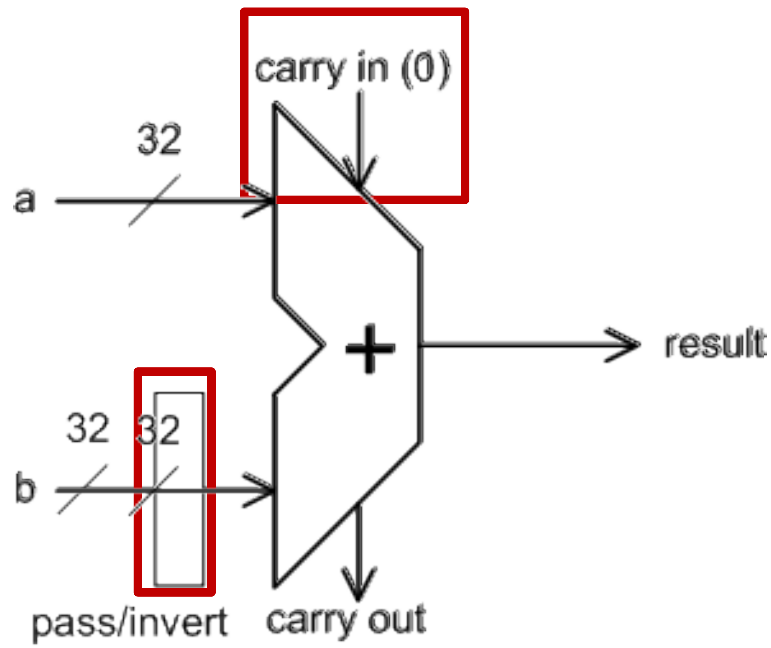
MARS give error

```
li     $t0,0x40000000
addu   $t1,$t0,$t0
```

MARS doesn't give error
$t1=0x80000000

Subtraction

- We know how to add
- We know how to negate a number
- We will use the above two known operations to perform subtraction
- A – B = A + (-B)
- The hardware used for addition can be extended to handle subtraction!



Subtraction

- N=8, a=90, b=20
- Do binary subtraction (A+(-B)) to get result and carryout
- Convert A and B to binary? How?
  - a=90 is 0101 1010b
  - b=20 is 0001 0100b

find –b

invert 0001 0100b
= 1110 1011b
+ 0000 0001b

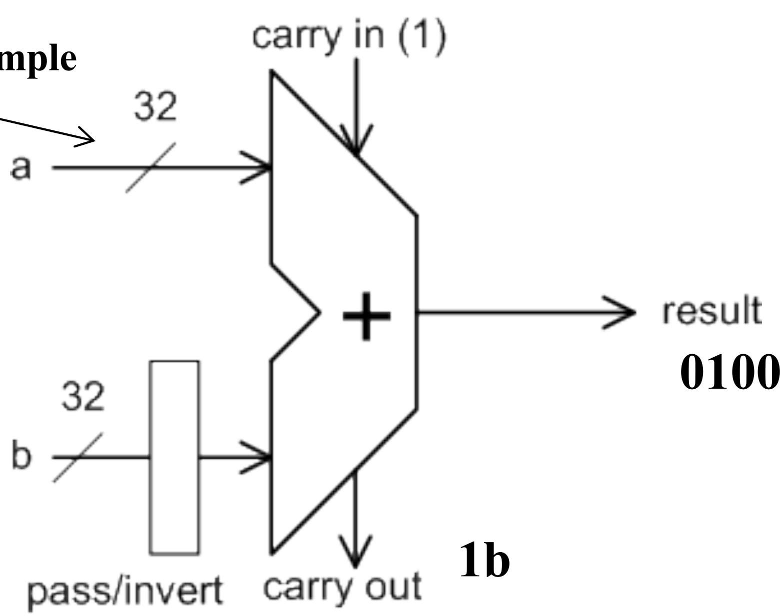
1110 1100b

Now, add a
0101 1010b
+ 1110 1100b
----
1 0100 0110b

NOTE: N=8 in this example

0101 1010b

0001 0100b



0100 0110b

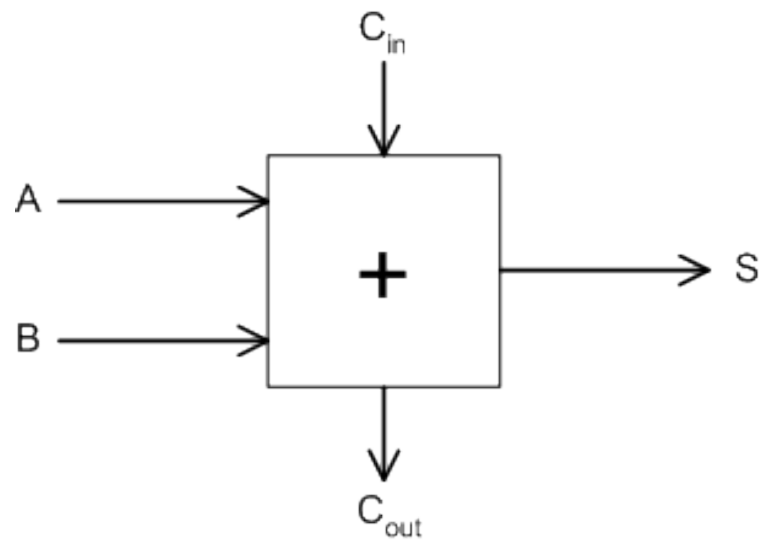
1b

1-bit adder

- We will look at a single-bit adder
  - Will build on this adder to design a 32-bit adder

- 3 inputs
  - A: 1st input
  - B: 2nd input
  - C_{in}: carry input

- 2 outputs
  - S: sum
  - C_{out}: carry out



1-bit adder

- What are the binary addition rules?
  - 0 + 0 = 0, Cout = 0
  - 0 + 1 = 1, Cout = 0
  - 1 + 0 = 1, Cout = 0
  - 1 + 1 = 0, Cout = 1

Input Values

Output Values

A	B	S	Cout
0	0	**0**	**0**
0	1	**1**	**0**
1	0	**1**	**0**
1	1	**0**	**1**

1-bit adder

- What about Cin?

A	B	Cin	S	Cout
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

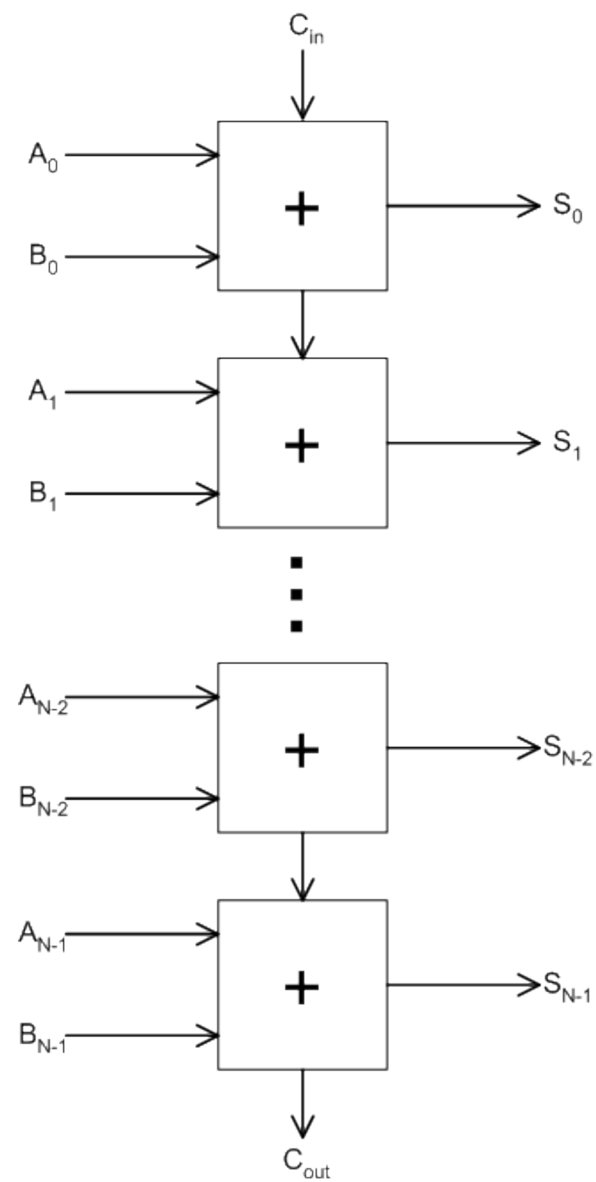
1-bit adder

- What about Cin?

A	B	Cin	S	Cout
0	0	0	**0**	**0**
0	0	1	**1**	**0**
0	1	0	**1**	**0**
0	1	1	**0**	**1**
1	0	0	**1**	**0**
1	0	1	**0**	**1**
1	1	0	**0**	**1**
1	1	1	**1**	**1**

N-bit adder

- An N-bit adder can be constructed with N single-bit adders
  - A carry out generated in a stage is propagated to the next ("ripple-carry adder")
- 3 inputs
  - A: N-bit, 1st input
  - B: N-bit, 2nd input
  - C_{in}: carry input
- 2 outputs
  - S: N-bit sum
  - C_{out}: carry out



N-bit ripple-carry adder