1. Build the DFA

1. **match transitions:** if in state \( j \) and next char \( c \) == \( \text{pat.charAt}(j) \), goto \( j + 1 \)

2. **mismatch transitions:** if in state \( j \) and next char \( c \) != \( \text{pat.charAt}(j) \), then the last \( j - 1 \) characters of input are \( \text{pat}[1..j-1] \)

To compute \( \text{dfa}[c][j] \): simulate \( \text{pat}[1..j-1] \) on DFA and take transition \( c \).

Running time: takes only constant time if we maintain state \( X \)

★ Algorithm (pseudo code) - Try understand the process:

```java
for each state \( j \)
    copy \( \text{dfa}[][][x] \) to \( \text{dfa}[][j] \) for mismatch case
    \( \text{set} \ \text{dfa}[\text{pat.charAt}(j)][j] \) to \( j + 1 \) for match case
    \( X = \text{dfa}[\text{pat.charAt}(j)][x] \) // update restart state \( x \)
```

Algorithm (Java code):

```java
public KMP(String pat) {
    this.pat = pat;
    M = pat.length();
    dfa = new int[R][M];
    dfa[pat.charAt(0)][0] = 1;
    for (int X=0, j=1; j < M; ++j) {
        for (int c = 0; c < R; ++c)
            dfa[c][j] = dfa[x][X]; // copy mismatch cases
        dfa[pat.charAt(j)][j] = 1 + j; // set match case
        X = dfa[pat.charAt(j)][x]; // update restart state x
    }
}
```

What is interpretation of DFA after reading \( \text{txt}[i] \)?

State = number of characters in pattern that have been matched

2. Apply string matching:

```java
public int search(String txt) {
    int i, j, N = txt.length();
    for (i = 0, j = 0; i < N && j < M; ++i)
        j = dfa[txt.charAt(i)][j];
    if (j == M) return i - M;
    else return N;
}
```
3. Example of Creating the DFA

Build DFA for pattern ABABAC

a. Initial DFA

x denotes the tracking state

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6(AC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Initial DFA Diagram]

b. Transitions from state 1

\( x = 0 \)

copy \( DFA[0] \) to \( DFA[1] \)

\( DFA[B][1] = 2 \)

\( x = DFA[B][x] = 0 \)

<table>
<thead>
<tr>
<th></th>
<th>0(x)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6(AC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

![Transitions from state 1 Diagram]
c. Transitions from state 2

\[ x = 0 \]

Copy DFA[0] to DFA[2]

\[ DFA[A][2] = 3 \]

\[ x = DFA[A][x] = 1 \]

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>A</th>
<th>B</th>
<th>A</th>
<th>C</th>
<th>6(AC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


\[ x = 1 \]

Copy DFA[1] to DFA[3]

\[ DFA[B][3] = 4 \]

\[ x = DFA[B][x] = 2 \]

\[   | A  | B  | A  | B  | A  | C  | 6(AC) |
<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tbody>
</table>
e. Transitions from state 4

\[ x = 2 \]

copy DFA[4][2] to DFA[4][4]

\[ DFA[A][4] = 5 \]

\[ x = DFA[A][x] = 3 \]

\[
\begin{array}{cccccc}
A & B & 2(x) & A & B & C \\
\hline
A & 1 & 1 & 3 & 1 & 5 \\
B & 0 & 2 & 0 & 4 & 0 \\
C & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

f. Transitions from state 5

\[ x = 3 \]

copy DFA[4][3] to DFA[4][5]

\[ DFA[C][5] = 6 \]

\[ x = DFA[C][x] = 0 \]

\[
\begin{array}{cccccc}
A & B & 2(x) & A & B & C \\
\hline
A & 1 & 1 & 3 & 1 & 5 \\
B & 0 & 2 & 0 & 4 & 0 \\
C & 0 & 0 & 0 & 0 & 6 \\
\end{array}
\]