CS0441 Discrete Structures Recitation 1

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Question 1

- Which of these sentences are propositions? What are the truth values of those that are propositions?

  a. Boston is the capital of Massachusetts — True

  b. Miami is the capital of Florida — False

  c. 2 + 3 = 5. — True

  d. 5 + 7 = 10. — False

  e. x + 2 = 11. — Not proposition

  f. Answer this question — Not proposition

Question 2

- Let p and q be the propositions "The election is decided" and "The votes have been counted" respectively. Express each of these compound propositions as an English sentence.

$\neg p$	The election is not decided
$p \vee q$	The election is decided or the votes have been counted
$\neg p \wedge q$	The election is not decided, but the votes have been counted
$q \rightarrow p$	If the votes have been counted, then the election is decided

Question 2

- Let p and q be the propositions "The election is decided" and "The votes have been counted" respectively. Express each of these compound propositions as an English sentence.

$\neg q \rightarrow \neg p$

If the votes have not been counted, then the election is not decided

$\neg p \rightarrow \neg q$

If the election is not decided, then the votes have not been counted

$p \leftrightarrow q$

The election is not decided if and only if the votes have been counted

$\neg q \vee (\neg p \wedge q)$

The votes have not been counted, or the votes have been counted but the election is not decided

Question 3

- Let p and q be the propositions

  p: you drive over 65 miles per hour

  q: you get a speeding ticket

  a) You drive over 65 miles per hour, but you do not get a speeding ticket. $p \wedge \neg q$
  b) You will get a speeding ticket if you drive over 65 miles per hour. $p \rightarrow q$
  c) Driving over 65 miles per hour is sufficient for getting a speeding ticket $p \rightarrow q$

Question 3

- Let p and q be the propositions

  p: you drive over 65 miles per hour

  q: you get a speeding ticket

  d) You get a speeding ticket, but you do not drive over 65 miles per hour

$$q \wedge \neg p$$

  e) Whenever you get a speeding ticket, you are driving over 65 miles per hour

$$q \rightarrow p$$

Question 4

- Determine whether each of these conditional statements is true or false
  a. If 1+1 = 2, the 2 + 2 = 5 False
  b. If 1+1 = 3, then 2 + 2 = 4 True
  c. If monkeys can fly, then 1 + 1 = 3 True
  d. If 1 +1 = 2, then dogs can fly False

Question 5

- Write each of these statements in the form “if p, then q” in English

a. That the Pistons win the championship implies that they beat the Lakers

p implies q: p -> q
If the Pistons win the championship, then they beat the Lakers

b. It is necessary to walk 8 miles to get to the top of the Long’s peak

q is necessary to p: p -> q
If you get to the top of the Long’s peak, then you must have walked 8 miles

Question 5

- Write each of these statements in the form “if p, then q” in English

c. To get tenure as a professor, it is sufficient to be world-famous

p is sufficient for q: p -> q
If you are world-famous, then you will get tenure as a professor.

d. Your guarantee is good only if you bought your CD player less than 90 days ago

p only if q: p -> q
If your guarantee is good, then you must have bought your CD player less than 90 days ago

Question 5

- Write each of these statements in the form “if p, then q” in English

e. Jan will go swimming unless the water is too cold.

q unless not p: p -> q

If the water is not too cold, then Jan will swimming.

Question 6

- State the converse, contrapositive, and inverse of these conditional statements.

"If it snows today, I will ski tomorrow"

Converse: I will ski tomorrow only if it snows today

Contrapositive: If I do not ski tomorrow, then it will not have snowed today.

Inverse: If it does not snow today, then I will not ski tomorrow

Question 7

- Construct a truth table for each of these compound propositions

$$p \wedge \neg p$$

$$(p \vee \neg q) \rightarrow q$$

$$(p \rightarrow q) \rightarrow (q \rightarrow p)$$