University of Pittsburgh

CS 1502 Spring 2004 Exam 1

There are a total of 100 points. This is a closed book exam.

You may ask a question if you think there is an error of some kind.

We can't answer questions like *What do you want for this question?* or *I don't understand the question.* That would be too disruptive, and also wouldn't be fair, because some people would get more information than others.

Good luck!!

1. (6 points) Determine whether the following is a **tautology** and/or whether it is a **logical necessity**. Support your answers. Your answers must show that you know what **tautologies** and **logical necessities** are.

$$c = c \vee \mathrm{Tet}(a)$$

Tautology? (circle one) **YES** **NO*****

Logical Necessity? (circle one) **YES***** **NO**

SUPPORT YOUR ANSWERS HERE:

```
c=c  Tet(a) |  c=c v Tet(a)
T      T           T
T      F           T
F      T           T
F      F           F

We can see from the last row in the truth table
that the sentence is not always true. Hence,
the sentence is not a tautology.

However, the last two rows of the truth table are spurious
(since c=c cannot be false).  Since the sentence is true
in all the non-spurious rows, the sentence is a logical
necessity.
```

2. (16 points) Translate the following English sentences into first-order logic.

   (a) Neither **a** nor **b** is a cube, but both of them are large.

   ```
   ~(cube(a) v cube(b)) ^ large(a) ^ large(b)
   ```

   (b) **a** is a cube only if it's large.

   ```
   Cube(a) --> Large(a)
   ```

   (c) **a** is a cube unless it's large.

   ```
   ~Large(a) --> Cube(a)
   ```

   (d) If **a** is a cube, then if it isn't small then **b** is neither a cube nor a tet.

   ```
   Cube(a) --> (~Small(a) --> ~(Cube(b) v Tet(b)))
   ```

3. (6 points) Convert the following sentence into conjunctive normal form. Please show your work.

$$\neg(R \wedge (P \vee Q))$$

```
~(R ^ (P v Q))
~R v ~(P v Q)
~R v (~P ^ ~Q)
(~R v ~P) ^ (~R v ~Q)

    OR

~(R ^ (P v Q))
~((R ^ P) v (R ^ Q))
~(R ^ P) ^ ~(R ^ Q)
(~R v ~P) ^ (~R v ~Q)
```

4. (10 points) Give a resolution proof of the following argument. Please **show your work.**

1. P $\vee$ Q
2. $\neg$P

—

3. Q $\vee$ W

```
Negate goal: ~(Q v W) equiv ~Q ^ ~W

So, the clauses are {P,Q}, {~P}, {~Q}, {~W}

Resolution Proof:

   {P,Q}     {~P}
   --------------
      {Q}    {~Q}
     ------------
         {}
```

5. (10 points) For each of the following, state whether the argument is valid or invalid. If the argument is **invalid**, **show that it is invalid**. If it is **valid**, there is nothing else to do.

$\text{Tet(a)} \rightarrow \text{Large(a)}$
Large(a)
—
Tet(a)

Invalid. The premises are true and the conclusion is false if, for example, **a** is a large cube.

$\text{Tet(a)} \rightarrow \text{Large(a)}$
—
$(\text{Tet(a)} \vee \text{Cube(b)}) \rightarrow \text{Large(a)}$

Invalid. The premises are true and the conclusion is false if, for example, **b** is a cube and **a** is a small cube.

$\text{Tet(a)} \rightarrow \text{Large(a)}$
$\neg\text{Large(a)}$
—
$\neg\text{Tet(a)}$

Valid.

$\text{Tet(a)} \rightarrow \text{Large(a)}$
—
$(\text{Tet(a)} \wedge \text{Cube(b)}) \rightarrow \text{Large(a)}$

Valid.