Knowledge Representation.
Propositional logic

Knowledge-based agent

- Knowledge base (KB):
  - Knowledge that describe facts about the world in some formal (representational) language
  - Domain specific
- Inference engine:
  - A set of procedures that use the representational language to infer new facts from known ones or answer a variety of KB queries. Inferences typically require search.
  - Domain independent
Example: MYCIN

- MYCIN: an expert system for diagnosis of bacterial infections
- **Knowledge base** represents
  - Facts about a specific patient case
  - Rules describing relations between entities in the bacterial infection domain

<table>
<thead>
<tr>
<th>If</th>
<th>Then</th>
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<tbody>
<tr>
<td>1. The stain of the organism is gram-positive, and</td>
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<tr>
<td>2. The morphology of the organism is coccus, and</td>
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<td>3. The growth conformation of the organism is chains</td>
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<tr>
<td>the identity of the organism is streptococcus</td>
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</table>

- **Inference engine:**
  - manipulates the facts and known relations to answer diagnostic queries (consistent with findings and rules)

Knowledge representation

- **Objective:** express the knowledge about the world in a computer-tractable form
- **Knowledge representation languages (KRLs)**
  Key aspects:
  - **Syntax:** describes how sentences in KRL are formed in the language
  - **Semantics:** describes the meaning of sentences, what is it the sentence refers to in the real world
  - **Computational aspect:** describes how sentences and objects in KRL are manipulated in concordance with semantic conventions

Many KB systems rely on and implement some variant of logic
Logic

A formal language for expressing knowledge and for making logical inferences

**Defined by:**

- **A set of sentences:** A sentence is constructed from a set of primitives according to **syntactic rules**
- **A set of interpretations:** An interpretation I gives a semantic to primitives. It associates primitives with objects or values
  - I: primitives $\rightarrow$ objects/values
- **The valuation (meaning) function** $V$:
  - Assigns a value (typically the truth value) to a given sentence under some interpretation
  $$V : \text{sentence} \times \text{interpretation} \rightarrow \{\text{True, False}\}$$

Propositional logic

- The simplest logic

- **Definition:**
  - A proposition is a statement that is either true or false.

- **Examples:**
  - Pitt is located in the Oakland section of Pittsburgh.
    - (T)
Propositional logic

• The simplest logic

• **Definition:**
  – A **proposition** is a statement that is either true or false.

• Examples:
  – Pitt is located in the Oakland section of Pittsburgh.
    • (T)
  – $5 + 2 = 8$.
    • (?)
Propositional logic

• The simplest logic

• **Definition:**
  – A proposition is a statement that is either true or false.

• Examples:
  – Pitt is located in the Oakland section of Pittsburgh.
    • (T)
  – $5 + 2 = 8$.
    • (F)
  – It is raining today.
    • (either T or F)

• Examples (cont.):
  – How are you?
    • ?
Propositional logic

- Examples (cont.):
  - How are you?
    - a question is not a proposition
  - $x + 5 = 3$
    - ?

- $2$ is a prime number.
  - ?
Propositional logic

- Examples (cont.):
  - How are you?
    - a question is not a proposition
  - $x + 5 = 3$
    - since $x$ is not specified, neither true nor false
  - 2 is a prime number.
    - (T)

Propositional logic. Syntax

- Formally propositional logic P:
  - Is defined by Syntax+interpretation+semantics of P

Syntax:
- Symbols (alphabet) in P:
  - Constants: True, False
  - Propositional symbols
    - Examples:
      - $P$
      - Pitt is located in the Oakland section of Pittsburgh,
      - It rains outside, etc.
    - A set of connectives:
      $\neg, \land, \lor, \Rightarrow, \Leftrightarrow$
Propositional logic. Syntax

Sentences in the propositional logic:
- **Atomic sentences:**
  - Constructed from constants and propositional symbols
  - True, False are (atomic) sentences
  - \( P, Q \) or Light in the room is on, It rains outside are (atomic) sentences
- **Composite sentences:**
  - Constructed from valid sentences via logical connectives
  - If \( A, B \) are sentences then
  \[
  \neg A \quad (A \land B) \quad (A \lor B) \quad (A \Rightarrow B) \quad (A \Leftrightarrow B)
  \]
  or
  \[
  (A \lor B) \land (A \lor \neg B)
  \]
  are sentences

Propositional logic. Semantics.

The semantic gives the meaning to sentences.

the semantics in the propositional logic is defined by:

1. **Interpretation of propositional symbols and constants**
   - Semantics of atomic sentences

2. **Through the meaning of logical connectives**
   - Meaning (semantics) of composite sentences
Semantic: propositional symbols

A **propositional symbol**
- a statement about the world that is either true or false

Examples:
- Pitt is located in the Oakland section of Pittsburgh
- It rains outside
- Light in the room is on

- An **interpretation** maps symbols to one of the two values: **True** ($T$), or **False** ($F$), depending on whether the symbol is satisfied in the world

$I$: Light in the room is on -> **True**, It rains outside -> **False**

$I'$: Light in the room is on -> **False**, It rains outside -> **False**

The **meaning (value)** of the propositional symbol for a specific interpretation is given by its interpretation

$I$: Light in the room is on -> **True**, It rains outside -> **False**

\[ V(\text{Light in the room is on}, I) = \text{True} \]

\[ V(\text{It rains outside}, I) = \text{False} \]

$I'$: Light in the room is on -> **False**, It rains outside -> **False**

\[ V(\text{Light in the room is on}, I') = \text{False} \]

<table>
<thead>
<tr>
<th>Interpretations</th>
<th>Meanings (values)</th>
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<tbody>
<tr>
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<td>It rains outside</td>
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M. Hauskrecht
Semantics: constants

- The meaning (truth) of constants:
  - True and False constants are always (under any interpretation) assigned the corresponding True, False value

\[
V(\text{True}, I) = \text{True} \quad \text{For any interpretation } I \\
V(\text{False}, I) = \text{False}
\]

Semantics: composite sentences.

- The meaning (truth value) of complex propositional sentences.
  - Determined using the standard rules of logic:

<table>
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<tr>
<th></th>
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<th>¬P</th>
<th>P \land Q</th>
<th>P \lor Q</th>
<th>P \Rightarrow Q</th>
<th>P \Leftrightarrow Q</th>
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Translation

Translation of English sentences to propositional logic:
(1) identify atomic sentences that are propositions
(2) Use logical connectives to translate more complex composite sentences that consist of many atomic sentences

Assume the following sentence:
• It is not sunny this afternoon and it is colder than yesterday.

Atomic sentences:
• \( p = \) It is sunny this afternoon
• \( q = \) it is colder than yesterday

Translation: \( \neg p \land q \)

Translation

Assume the following sentences:
• It is not sunny this afternoon and it is colder than yesterday.
• We will go swimming only if it is sunny.
• If we do not go swimming then we will take a canoe trip.
• If we take a canoe trip, then we will be home by sunset.

Denote:
• \( p = \) It is sunny this afternoon
• \( q = \) it is colder than yesterday
• \( r = \) We will go swimming
• \( s = \) we will take a canoe trip
• \( t = \) We will be home by sunset
Translation

Assume the following sentences:

- It is not sunny this afternoon and it is colder than yesterday. \( \neg p \land q \)
- We will go swimming only if it is sunny.
- If we do not go swimming then we will take a canoe trip.
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- \( p = \) It is sunny this afternoon
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Translation

Assume the following sentences:

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- We will go swimming only if it is sunny. \( r \rightarrow p \)
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Translation

Assume the following sentences:

- It is not sunny this afternoon and it is colder than yesterday. \( \neg p \land q \)
- We will go swimming only if it is sunny. \( r \rightarrow p \)
- If we do not go swimming then we will take a canoe trip. \( \neg r \rightarrow s \)
- If we take a canoe trip, then we will be home by sunset. \( s \rightarrow t \)

Denote:

- \( p \) = It is sunny this afternoon
- \( q \) = it is colder than yesterday
- \( r \) = We will go swimming
- \( s \) = we will take a canoe trip
- \( t \) = We will be home by sunset
Contradiction and Tautology

Some composite sentences may always (under any interpretation) evaluate to a single truth value:

- **Contradiction** (always *False*)
  \[ P \land \neg P \]

- **Tautology** (always *True*)
  \[ P \lor \neg P \]

\[ \neg(P \lor Q) \iff (\neg P \land \neg Q) \]
\[ \neg(P \land Q) \iff (\neg P \lor \neg Q) \]

DeMorgan’s Laws

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Model, validity and satisfiability

- An interpretation is a model for a set of sentences if it assigns true to each sentence in the set.

- **Example 1:**
  - **Primitives:** \( P, Q \)
  - **Sentence:** \( P \lor Q \)
  - **Interpretations:**
    - \( P \Rightarrow True, Q \Rightarrow True \)

<table>
<thead>
<tr>
<th>( P )</th>
<th>( Q )</th>
<th>( P \lor Q )</th>
<th>( (P \lor Q) \land \neg Q )</th>
<th>( ((P \lor Q) \land \neg Q) \Rightarrow P )</th>
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Model, validity and satisfiability

• An interpretation **is a model for a set of sentences** if it assigns true to each sentence in the set.

• **Example 1:**
  • Primitives: $P, Q$
  • Sentence: $P ∨ Q$
  • Interpretations: Model ?
    - $P → True$, $Q → True$ Yes
    - $P → True$, $Q → False$ ?

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<th>$P$</th>
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Model, validity and satisfiability

• An interpretation \textbf{is a model for a set of sentences} if it assigns true to each sentence in the set.

• \textbf{Example 1:}
  
  – Primitives: $P, Q$
  
  – Sentence: $P \lor Q$
  
  – Interpretations: Model ?
  
  \begin{itemize}
  \item $P \rightarrow \text{True, } Q \rightarrow \text{True} \quad \text{Yes}
  \item $P \rightarrow \text{True, } Q \rightarrow \text{False} \quad \text{Yes}
  \item $P \rightarrow \text{False, } Q \rightarrow \text{False} \quad \text{No}
  \end{itemize}

\begin{tabular}{|c|c|c|c|c|}
\hline
$P$ & $Q$ & $P \lor Q$ & $(P \lor Q) \land \neg Q$ & $((P \lor Q) \land \neg Q) \Rightarrow P$ \\
\hline
True & True & True & False & True \\
True & False & True & True & True \\
False & True & True & False & True \\
False & False & False & False & True \\
\hline
\end{tabular}

\textbf{Example 2:}

– Primitives: $P, Q$

– Sentences: $P \lor Q$

\begin{itemize}
\item $(P \lor Q) \land \neg Q$
\item $((P \lor Q) \land \neg Q) \Rightarrow P$
\end{itemize}

– Is there a model?

\begin{tabular}{|c|c|c|c|c|}
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$P$ & $Q$ & $P \lor Q$ & $(P \lor Q) \land \neg Q$ & $((P \lor Q) \land \neg Q) \Rightarrow P$ \\
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Model, validity and satisfiability

• An interpretation is a model for a set of sentences if it assigns true to each sentence in the set.

• Example 2:
  – Primitives: $P, Q$
  – Sentences:
    $P \lor Q$
    $\neg Q$
    $((P \lor Q) \land \neg Q) \Rightarrow P$

  – Is there a model? Yes $P \Rightarrow True$ $Q \Rightarrow False$

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Model, validity and satisfiability

• An interpretation is a model for a set of sentences if it assigns true to each sentence in the set.

• A sentence is satisfiable if it has a model;
  – There is at least one interpretation under which the sentence can evaluate to True

• Example:
  – Sentence: $((P \lor Q) \land \neg Q)$
  – Satisfiable?

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- Example:
  - Sentence: \((P \lor Q) \land \neg Q\)
  - Satisfiable? Yes  True for \(P \Rightarrow True, Q \Rightarrow False\)

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- A sentence is valid if it is True in all interpretations
  - i.e., if its negation is not satisfiable (leads to contradiction)

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valid sentence ?

Entailment

- Entailment reflects the relation of one fact in the world following from the others

\[ KB | = \alpha \]

- Knowledge base KB entails sentence \( \alpha \) if and only if \( \alpha \) is true in all worlds where KB is true
Sound and complete inference

Inference is a process by which new sentences are derived from existing sentences in the KB
• the inference process is implemented on a computer

Assume an inference procedure \( i \) that
• derives a sentence \( \alpha \) from the KB: \( KB \vdash_i \alpha \)

Properties of the inference procedure in terms of entailment
• **Soundness:** An inference procedure is sound
  \[ \text{If} \quad KB \vdash_i \alpha \quad \text{then it is true that} \quad KB \models \alpha \]

• **Completeness:** An inference procedure is complete
  \[ \text{If} \quad KB \models \alpha \quad \text{then it is true that} \quad KB \vdash_i \alpha \]

Logical inference problem

Logical inference problem:
• **Given:**
  – a knowledge base KB (a set of sentences) and
  – a sentence \( \alpha \) (called a theorem),
• **Does a KB semantically entail \( \alpha \)? \( KB \models \alpha \)?
In other words: In all interpretations in which sentences in the KB are true, is also \( \alpha \) true?

**Question:** Is there a procedure (program) that can decide this problem in a finite number of steps?

**Answer:** Yes. Logical inference problem for the propositional logic is **decidable**.
Solving logical inference problem

In the following:

**How to design the procedure that answers:**

\[ KB \models \alpha ? \]

**Three approaches:**

- Truth-table approach
- Inference rules
- Conversion to the inverse SAT problem
  - Resolution-refutation

---

**Truth-table approach**

**Problem:** \[ KB \models \alpha ? \]

- We need to check all possible interpretations for which the KB is true (models of KB) whether \( \alpha \) is true for each of them

**Truth table:**

- enumerates truth values of sentences for all possible interpretations (assignments of True/False values to propositional symbols)

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**Example:**

<table>
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<tr>
<th></th>
<th>$KB$</th>
<th>$\alpha$</th>
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<tbody>
<tr>
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Truth-table approach

**Problem:** $KB |\models \alpha$ ?
- We need to check all possible interpretations for which the KB is true (models of KB) whether $\alpha$ is true for each of them

**Truth table:**
- enumerates truth values of sentences for all possible interpretations (assignments of True/False to propositional symbols)

**Example:**

<table>
<thead>
<tr>
<th></th>
<th>$KB$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>$Q$</td>
<td>$P \lor Q$</td>
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<tr>
<td>True</td>
<td>True</td>
<td>True</td>
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![✔️]
Truth-table approach

A two steps procedure:
1. Generate table for all possible interpretations
2. Check whether the sentence $\alpha$ evaluates to true whenever $KB$ evaluates to true

Example: $KB = (A \lor C) \land (B \lor \neg C)$ $\alpha = (A \lor B)$

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$A \lor C$</th>
<th>$(B \lor \neg C)$</th>
<th>$KB$</th>
<th>$\alpha$</th>
</tr>
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<tbody>
<tr>
<td>True</td>
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Truth-table approach

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1. Generate table for all possible interpretations
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• The truth-table approach is sound and complete for the propositional logic!!
Limitations of the truth table approach.

\[ KB \models \alpha \ ? \]

- What is the computational complexity of the truth table approach?
- ?

Exponential in the number of the propositional symbols

\[ 2^n \]

Rows in the table have to be filled
- the truth table is \textit{exponential} in the number of propositional symbols (we checked all assignments)