Planning

Planning problem:
- find a sequence of actions that achieves some goal
- an instance of a search problem
- the state description is typically very complex and relies on a logic-based representation

Methods for modeling and solving a planning problem:
- Situation calculus based on FOL
- STRIPS – state-space search algorithm
- Partial-order planning algorithms
Situation calculus

Provides a framework for representing change, actions and for reasoning about them

• Situation calculus
  – based on the first-order logic,
  – a situation variable models possible states of the world
  – properties and relations depend on different world states (situations)
  – action objects model activities

• Inference:
  – inference methods developed for FOL to do the reasoning

Situation calculus. Blocks world example.

Initial state

\[
\begin{align*}
  On(A, Table, s_0) \\
  On(B, Table, s_0) \\
  On(C, Table, s_0) \\
  Clear(A, s_0) \\
  Clear(B, s_0) \\
  Clear(C, s_0) \\
  Clear(Table, s_0)
\end{align*}
\]

Goal

\[
\begin{align*}
  On(A, B, s) \\
  On(B, C, s) \\
  On(C, Table, s)
\end{align*}
\]

Find a state (situation) \( s \), such that
Knowledge base: Axioms

Knowledge base is needed to support the reasoning:
• Must represent changes in the world due to actions.

Two types of axioms:
• **Effect axioms**
  – changes in situations that result from actions
• **Frame axioms**
  – things preserved from the previous situation

**Example:**
• blocks world with *On, Clear* predicates
• *Move* actions

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Blocks world example. Effect axioms.

**Effect axioms:**
• represent the changes after the action is executed

Moving x from y to z.  \(MOVE(x, y, z)\)

Effect of move changes on *On* relations
\[On(x, y, s) \land Clear(x, s) \land Clear(z, s) \rightarrow On(x, z, DO(MOVE(x, y, z), s))\]
\[On(x, y, s) \land Clear(x, s) \land Clear(z, s) \rightarrow \neg On(x, y, DO(MOVE(x, y, z), s))\]

Effect of move changes on *Clear* relations
\[On(x, y, s) \land Clear(x, s) \land Clear(z, s) \rightarrow Clear(y, DO(MOVE(x, y, z), s))\]
\[On(x, y, s) \land Clear(x, s) \land Clear(z, s) \land (z \neq Table) \rightarrow \neg Clear(z, DO(MOVE(x, y, z), s))\]
Blocks world example. Frame axioms.

• **Frame axioms.**
  – Represent relations/properties that remain unchanged by the executed action
  – Explicitly move the relations to the next situation (after the action)

**On relations:**
\[ On(u, v, s) \land (u \neq x) \land (v \neq y) \rightarrow On(u, v, DO(MOVE(x, y, z), s)) \]

**Clear relations:**
\[ Clear(u, s) \land (u \neq z) \rightarrow Clear(u, DO(MOVE(x, y, z), s)) \]

Planning in situation calculus

**Planning problem:**
• find a sequence of actions that lead to the goal

**Planning in situation calculus is converted to the theorem proving problem**

**Goal state:**
\[ \exists s \ On(A, B, s) \land On(B, C, s) \land On(C, Table, s) \]

• Possible inference approaches:
  – **Inference rule approach**
  – **Conversion to SAT**

• **Plan** (solution) is a byproduct of theorem proving
• **Example:** blocks world
Planning in the blocks world.

Initial state (s0)

\[
\begin{align*}
A & \quad B & \quad C \\
\end{align*}
\]

\[s_0 = \]
\[
\text{On}(A, \text{Table}, s_0) & \quad \text{Clear}(A, s_0) & \quad \text{Clear}(\text{Table}, s_0) \\
\text{On}(B, \text{Table}, s_0) & \quad \text{Clear}(B, s_0) \\
\text{On}(C, \text{Table}, s_0) & \quad \text{Clear}(C, s_0)
\]

**Action:** \( \text{MOVE}(B, \text{Table}, C) \)

\[
\begin{align*}
A & \quad B & \quad C  \\
\end{align*}
\]

\[s_1 = \text{DO}(\text{MOVE}(B, \text{Table}, C), s_0) \]
\[
\begin{align*}
\text{On}(A, \text{Table}, s_1) & \quad \text{Clear}(A, s_1) & \quad \text{Clear}(\text{Table}, s_1) \\
\text{On}(B, C, s_1) & \quad \text{Clear}(B, s_1) \\
\neg \text{On}(B, \text{Table}, s_1) & \quad \neg \text{Clear}(C, s_1)
\end{align*}
\]

Planning in the blocks world.

Initial state (s0)

\[
\begin{align*}
A & \quad B & \quad C \\
\end{align*}
\]

\[s_1 = \text{DO}(\text{MOVE}(B, \text{Table}, C), s_0) \]
\[
\begin{align*}
\text{On}(A, \text{Table}, s_1) & \quad \text{Clear}(A, s_1) & \quad \text{Clear}(\text{Table}, s_1) \\
\text{On}(B, C, s_1) & \quad \text{Clear}(B, s_1) \\
\neg \text{On}(B, \text{Table}, s_1) & \quad \neg \text{Clear}(C, s_1)
\end{align*}
\]

**Action:** \( \text{MOVE}(A, \text{Table}, B) \)

\[
\begin{align*}
A & \quad B & \quad C  \\
\end{align*}
\]

\[s_2 = \text{DO}(\text{MOVE}(A, \text{Table}, B), s_1) = \text{DO}(\text{MOVE}(A, \text{Table}, B), \text{DO}(\text{MOVE}(B, \text{Table}, C), s_0)) \]
\[
\begin{align*}
\text{On}(A, B, s_2) & \quad \neg \text{On}(A, \text{Table}, s_2) & \quad \neg \text{Clear}(B, s_2) \\
\text{On}(B, C, s_2) & \quad \neg \text{On}(B, \text{Table}, s_2) & \quad \neg \text{Clear}(C, s_2) \\
\text{On}(C, \text{Table}, s_2) & \quad \text{Clear}(A, s_2) & \quad \text{Clear}(\text{Table}, s_2)
\end{align*}
\]
Planning in the blocks world.

Initial state ($s_0$)

$s_1 = DO(MOVE(B, Table, C), s_0)$

On(A, Table, $s_1$)

On(B, C, $s_1$)  Clear(A, $s_1$)

$\neg$On(B, Table, $s_1$)  Clear(B, $s_1$)

On(C, Table, $s_1$)  $\neg$Clear(C, $s_1$)

Action:  MOVE(A, Table, B)

$s_2 = DO(MOVE(A, Table, B), s_1)$  = $DO(MOVE(A, Table, B), DO(MOVE(B, Table, C), s_0))$

On(A, B, $s_2$)  $\neg$On(A, Table, $s_2$)  $\neg$Clear(B, $s_2$)

On(B, C, $s_2$)  $\neg$On(B, Table, $s_2$)  $\neg$Clear(C, $s_2$)

On(C, Table, $s_2$)  Clear(A, $s_2$)  Clear(Table, $s_2$)

Satisfies the goal

Planning in the blocks world.

Initial state ($s_0$)

$s_1 = DO(MOVE(B, Table, C), s_0)$

On(A, Table, $s_1$)

On(B, C, $s_1$)  Clear(A, $s_1$)

$\neg$On(B, Table, $s_1$)  Clear(B, $s_1$)

On(C, Table, $s_1$)  $\neg$Clear(C, $s_1$)

Action:  MOVE(A, Table, B)

$s_2 = DO(MOVE(A, Table, B), s_1)$  = $DO(MOVE(A, Table, B), DO(MOVE(B, Table, C), s_0))$

On(A, B, $s_2$)  $\neg$On(A, Table, $s_2$)  $\neg$Clear(B, $s_2$)

On(B, C, $s_2$)  $\neg$On(B, Table, $s_2$)  $\neg$Clear(C, $s_2$)

On(C, Table, $s_2$)  Clear(A, $s_2$)  Clear(Table, $s_2$)

DO functions capture the plan
**Situation calculus: problems**

**Frame problem** refers to:
- The need to represent a large number of frame axioms

**Solution:** combine positive and negative effects in one rule

\[ On(u, v, DO(MOVE(x, y, z), s)) \iff \neg((u = x) \land (v = y)) \lor On(u, v, s) \lor ((u = x) \land (v = z)) \lor On(x, y, s) \land Clear(x, s) \land Clear(z, s) \]

**Inferential frame problem:**
- We still need to derive properties that remain unchanged

**Other problems:**
- **Qualification problem** – enumeration of all possibilities under which an action holds
- **Ramification problem** – enumeration of all inferences that follow from some facts

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**Planning problems**

**Properties of many (real-world) planning problems:**
- The description of the state of the world is **very complex**
- **Many possible actions** to apply in any step
- Actions are typically **local**
  - they affect only a small portion of a state description
- Goals are defined as conditions referring only to a small portion of state
- Plans consists of a **long sequence of actions**

The situation calculus framework:
- too cumbersome and inefficient to represent and solve the planning problems
Solutions

• Complex state description and local action effects:
  – avoid the enumeration and inference of every state component, focus on changes only

• Many possible actions:
  – Apply actions that make progress towards the goal
  – Understand what the effect of actions is and reason with the consequences of these action

• Sequences of actions in the plan can be too long:
  – Many goals consists of independent or nearly independent sub-goals
  – Allow goal decomposition & divide and conquer strategies

STRIPS planner

Defines a restricted representation language when compared to the situation calculus

Advantage: leads to more efficient planning algorithms.
  – State-space search with structured representations of states, actions and goals
  – Action representation avoids the frame problem

STRIPS planning problem:
• much like a standard search problem
• State descriptions use first order logic
STRIPS planner

- **States:**
  - conjunction of ground literals,
    - e.g. $\text{On}(A,B)$, $\text{On}(B,\text{Table})$, $\text{Clear}(A)$
    - represent facts that are true at a specific point in time
- **Actions (operators):**
  - **Action:** $\text{Move} (x,y,z)$
  - **Preconditions:** conjunctions of literals with variables
    - $\text{On}(x,y)$, $\text{Clear}(x)$, $\text{Clear}(z)$
  - **Effects.** Two lists:
    - **Add list:** $\text{On}(x,z)$, $\text{Clear}(y)$
    - **Delete list:** $\text{On}(x,y)$, $\text{Clear}(z)$
    - Everything else remains untouched (is preserved)

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STRIPS planning

**Operator:** $\text{Move} (x,y,z)$

- **Preconditions:** $\text{On}(x,y)$, $\text{Clear}(x)$, $\text{Clear}(z)$
- **Add list:** $\text{On}(x,z)$, $\text{Clear}(y)$
- **Delete list:** $\text{On}(x,y)$, $\text{Clear}(z)$

![Diagram showing STRIPS planning](M. Hauskrecht)
STRIPS planning

Initial state:
• Conjunction of literals that are true

Goals in STRIPS:
• A goal is a partially specified state
• Is defined by a conjunction of ground literals
  – No variables allowed in the description of the goal

Example:
On(A,B) \land On(B,C)

Search in STRIPS

Objective:
Find a sequence of operators (a plan) from the initial state to the state satisfying the goal

Two approaches to build a plan:
• Forward state space search (goal progression)
  – Start from what is known in the initial state and apply operators in the order they are applied
• Backward state space search (goal regression)
  – Start from the description of the goal and identify actions that help us to reach the goal
Forward search (goal progression)

- **Idea**: Given a state \( s \)
  - Unify the preconditions of some operator \( a \) with \( s \)
  - Add and delete sentences from the add and delete list of an operator \( a \) from \( s \) to get a new state

Search tree:

Initial state

- \( \text{Move}(\text{A, Table}, B) \)
- \( \text{Move}(\text{B, Table}, C) \)
- \( \text{Move}(\text{A, Table}, C) \)
- \( \text{Move}(\text{A, Table}, B) \)

Goal
Forward search (goal progression)

- Use operators to generate new states to search
- Check new states whether they satisfy the goal

**Search tree:**

```
A  B  C
```

Initial state: $\text{Move}(A, \text{Table}, B)$

Goal: $A$ $B$ $C$

Heuristics?

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Backward search (goal regression)

**Idea:** Given a goal $G$

- Unify the add list of some operator $a$ with a subset of $G$
- If the delete list of $a$ does not remove elements of $G$, then the goal regresses to a new goal $G'$ that is obtained from $G$ by:
  - deleting add list of $a$
  - adding preconditions of $a$

**New goal ($G'$)**

- $\text{On}(A, \text{Table})$
- $\text{Clear}(B)$
- $\text{Clear}(A)$
- $\text{On}(B, C)$
- $\text{On}(C, \text{Table})$

**Mapped from $G$**

```
A  B  C
```

**Goal ($G$)**

- $\text{On}(A, B)$
- $\text{On}(B, C)$
- $\text{On}(C, \text{Table})$
Backward search (goal regression)

- Use operators to generate new goals
- Check whether the initial state satisfies the goal

**Search tree:**

```
Initial state

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
</table>

Move(B,Table,C)  Move(A,Table,B)  goal

Move(A,B,Table)
```

State-space search

- **Forward and backward state-space planning approaches:**
  - Work with strictly linear sequences of actions

```

```

- **Disadvantages:**
  - They cannot take advantage of the problem decompositions in which the goal we want to reach consists of a set of independent or nearly independent sub-goals
  - Action sequences cannot be built from the middle
  - No mechanism to represent least commitment in terms of the action ordering
Divide and conquer

- **Divide and conquer strategy:**
  - divide the problem to a set of smaller sub-problems,
  - solve each sub-problem independently
  - combine the results to form the solution

In planning we would like to satisfy a set of goals

- **Divide and conquer in planning:**
  - Divide the planning goals along individual goals
  - Solve (find a plan for) each of them independently
  - Combine the plan solutions in the resulting plan

- Is it always safe to use divide and conquer?
  - No. There can be interacting goals.

Sussman’s anomaly

- An example from the blocks world in which the divide and conquer strategy for goals fails due to interacting goals

Initial state

\[
\begin{array}{c}
C \\
A \\
B
\end{array}
\]

Goal

\[
\begin{array}{c}
A \\
B \\
C
\end{array}, \quad On(A, B), \quad On(B, C)
\]
Sussman’s anomaly

1. Assume we want to satisfy $On(A, B)$ first

```
Initial state
C
A
B

```

But now we cannot satisfy $On(B, C)$ without undoing $On(A, B)$

2. Assume we want to satisfy $On(B, C)$ first.

```
Initial state
C
A
B

```

But now we cannot satisfy $On(A, B)$ without undoing $On(B, C)$
State space vs. plan space search

- An alternative to planning algorithms that search states (configurations of world)
- **Plan:** Defines a sequence of operators to be performed
- **Partial plan:**
  - plan that is not complete
    - Some plan steps are missing
  - some orderings of operators are not finalized
    - Only relative order is given
- **Benefits of working with partial plans:**
  - We do not have to build the sequence from the initial state or the goal
  - We do not have to commit to a specific action sequence
  - We can work on sub-goals individually (divide and conquer)

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State-space vs. plan-space search

**State-space search**

- **STRIPS operator**
  - State (set of formulas)

**Plan-space search**

- **Start**
  - Incomplete (partial) plan
  - Finish
  - Plan transformation operators

M. Hauskrecht
Plan transformation operators

Examples of:
- Add an operator to a plan so that it satisfies some open condition
- Add link (+ instantiate)
- Order (reorder) operators

Partial-order planners (POP)

- also called Non-linear planners
- Use STRIPS operators

Graphical representation of an operator $\text{Move}(x,y,z)$

Delete list is not shown !!!
Illustration of a POP on the Sussman’s anomaly case
Partial order planning. Start and finish.

Open conditions: conditions yet to be satisfied
Partial order planning. Add operator.

We want to satisfy **an open condition**

Always select an operator that helps to satisfy one of the open conditions.
Partial order planning. Add link.

Add link

Satisfies an open condition

Satisfies an open condition

\[ \text{instantiates } y/\text{Fl} \]
Partial order planning. Add operator.

Partial order planning. Add links.
Partial order planning. Interactions.

Start

On(C,A) Clear(Fl) Clear(C) On(A,Fl) Clear(B) Clear(B) On(B,Fl) On(A,B) Clear(Fl) On(B,Fl) Clear(C) Move(B,Fl,C)

Finish

On(A,B) On(B,C)

Delete Clear(B) A is stacked on B

Move(A,Fl,Fl)

Start

On(A,B) Clear(Fl)

Move(B,Fl,C)

Goal

A B C

Partial order planning. Order operators.

Start

On(C,A) Clear(Fl) Clear(C) On(A,Fl) Clear(B) Clear(B) On(B,Fl) On(A,B) Clear(Fl) On(B,Fl) Clear(C)

Finish

On(A,B) On(B,C)

Move(B,Fl,C) comes before Move(A,Fl,B)

Move(A,Fl,B)

Goal

A B C

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Partial order planning. Add operator

Partial order planning. Add links.
Partial order planning. Threats.

- **Start**: On(C,A)
- **Finish**: On(B,C)
- **Goal**: Move(B,A,Fl)

**Deletes** Clear(C)

B moved on top of C

Partial order planning. Order operators.

- **Start**: On(C,A)
- **Finish**: On(B,C)
- **Goal**: Move(B,A,Fl)

**Move(C,A,Fl)** comes before Move(B,A,Fl)
POP planning. Directions.

Consistent POP plan.
Partial order planning. Result plan.

Plan: a topological sort of a graph

Partial order planning.

- **Remember** we search the space of partial plans

- **POP:** is sound and complete