CS 1675 Machine Learning
Lecture 15

Multiclass classification

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Multiclass classification

- **Binary classification:**
  - **Number of classes = 2**
  - **A special case of multiclass classification**

  Multiclass classification
  - **Number of classes is > 2**

Multiclass classification

- **Discriminative approach**
  - Parametric discriminant functions $g_i(\mathbf{x}, \mathbf{w})$
  - Learns discriminant functions for each class *i* **directly using a loss function**
    - A logistic regression model
- **Generative model approach**
  - Generative model of the distribution p(**x**,y) = p(**x**|y).p(y)
  - Learns the parameters Θ of the models p(**x**|y) and p(y) using the density estimation techniques
  - Discriminant functions are based on the model

$$g_i(\mathbf{x}, \Theta) = p(y = i \mid x, \Theta)$$

$$y^* = \arg\max_i \; g_i(\mathbf{x})$$

Generative model approach: multiple classes

Indirect:

1. **Represent and learn the distribution** $p(\mathbf{x}, y)$
2. **Define and use probabilistic discriminant functions**

$$g_i(\mathbf{x}) = \log\, p(y = i \mid \mathbf{x})$$

Model $p(\mathbf{x}, y) = p(\mathbf{x} \mid y)\, p(y)$

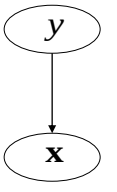
- $p(\mathbf{x} \mid y)$ = **Class-conditional distributions (densities)**

  k class-conditional distributions

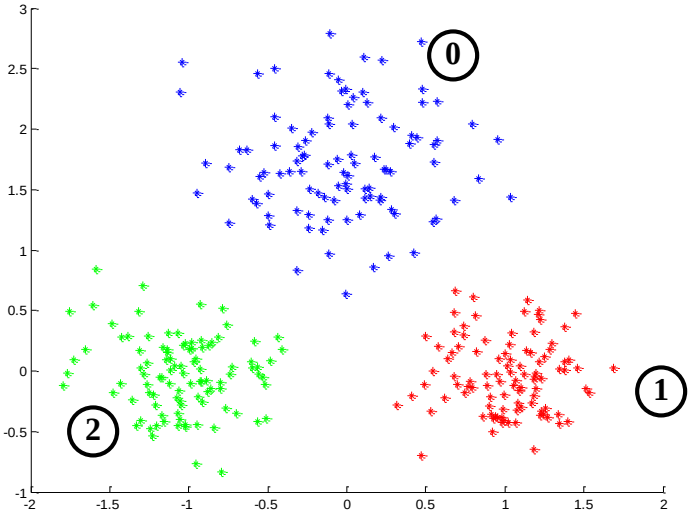
$$p(\mathbf{x} \mid y = i) \qquad \forall i \quad 0 \le i \le K - 1$$

- $p(y)$ = **Priors on classes**
- - probability of class *y*

$$\sum_{i=1}^{K-1} p(y = i) = 1$$



Multi-way classification. Example



Making class decision

Discriminant functions:

- **Posterior of a class** – choose the class with the highest posterior probability

Choice: $$i = \arg\max_{i=0,\ldots k-1} p(y = i \mid \mathbf{x}, \boldsymbol{\theta}_i)$$

$$p(y = i \mid \mathbf{x}) = \frac{p(\mathbf{x} \mid \Theta_i)\, p(y = i)}{\sum_{j=0}^{k-1} p(\mathbf{x} \mid \Theta_j)\, p(y = j)}$$

Discriminative approach

- **Parametric models** of discriminant functions:
  - $g_0(x), g_1(x), .. g_{K-1}(x)$
- Learns the discriminant functions directly

Key issues:

- How to design the discriminant functions?
- How to train them?

Another question:

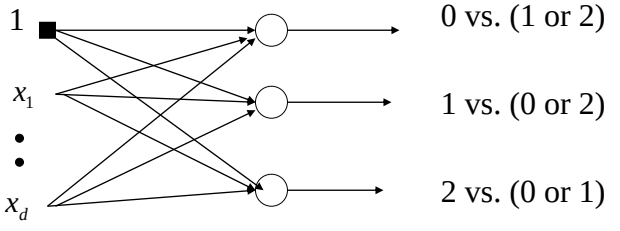
- Can we use binary classifiers to build the multi-class models?

One versus the rest (OVR)

Methods based on binary classification methods

- **Assume:** we have 3 classes labeled 0,1,2
- **Approach 1:**

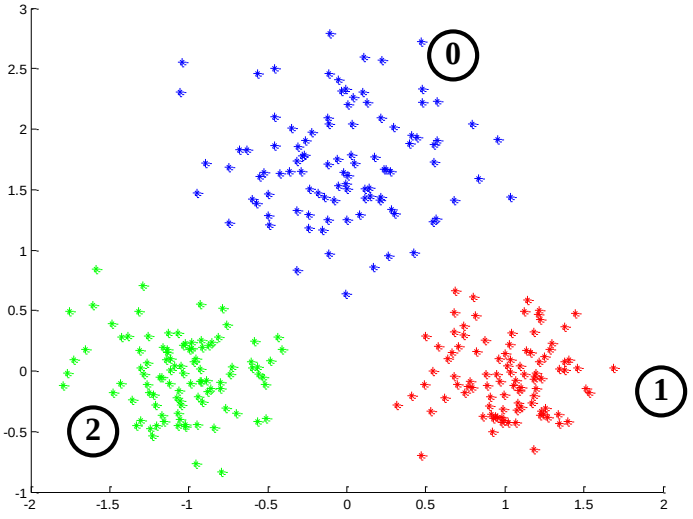
  A binary logistic regression on every class versus the rest (OvR)



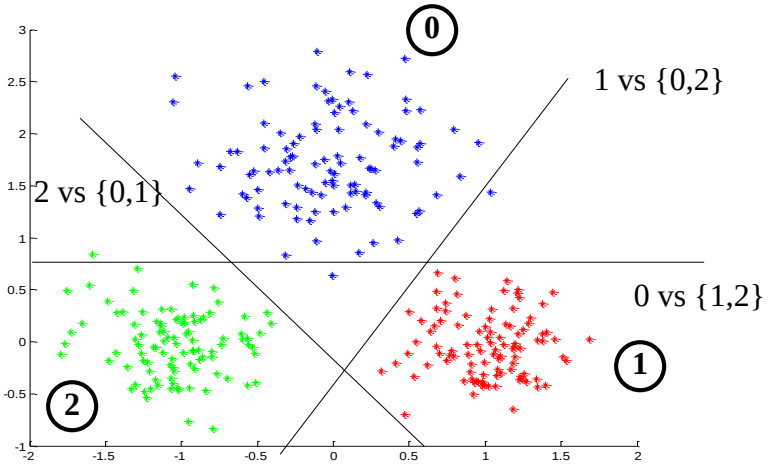
Class decision: class label for a 'singleton' class

- Does not work all the time

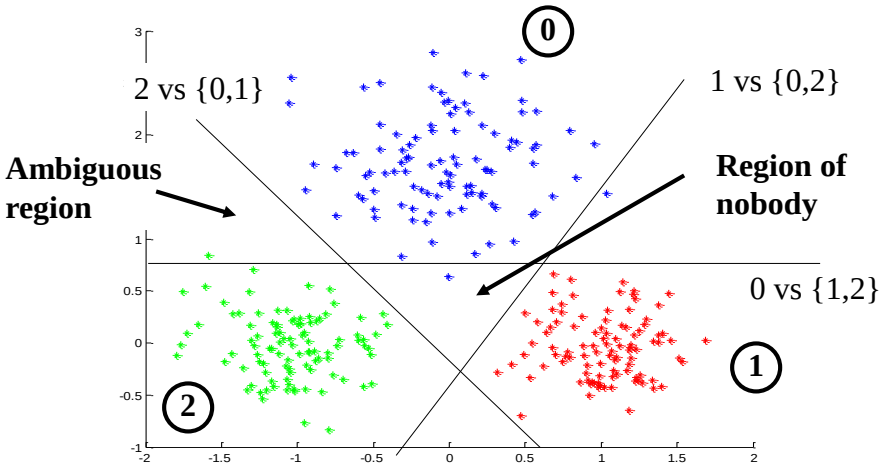
Multiclass classification. Example



Multiclass classification. Approach 1.



Multiclass classification. Approach 1.



One versus the rest (OVR)

Unclear how to decide on class in some regions

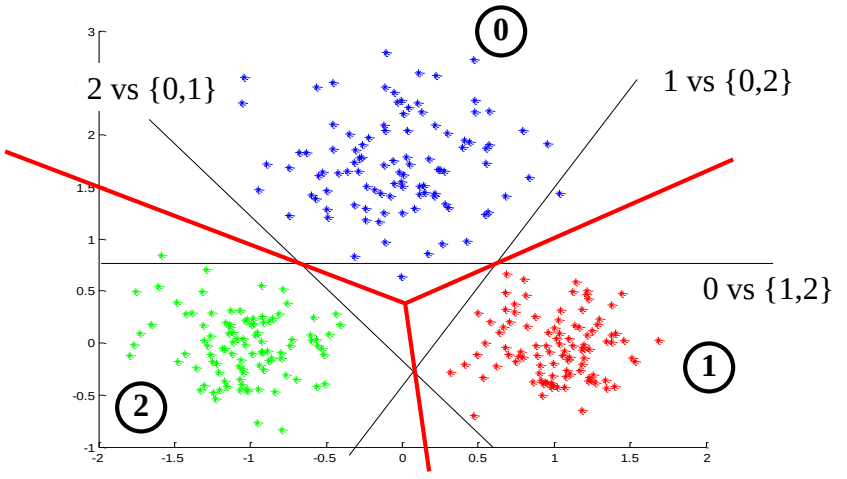
- **Ambigous region:**
  - 0 vs. (1 or 2) classifier says 0
  - 1 vs. (0 or 2) classifier says 1
- **Region of nobody:**
  - 0 vs. (1 or 2) classifier says (1 or 2)
  - 1 vs. (0 or 2) classifier says (0 or 2)
  - 2 vs (1 or 2) classifier says (1 or 2)

- **One solution:** compare discriminant functions defined on binary classifiers for single option:

  $g_i(\mathbf{x}) = g_{\text{i vs rest}}(\mathbf{w_i}^T\mathbf{x})$

  - discriminant function for i trained on i vs. rest

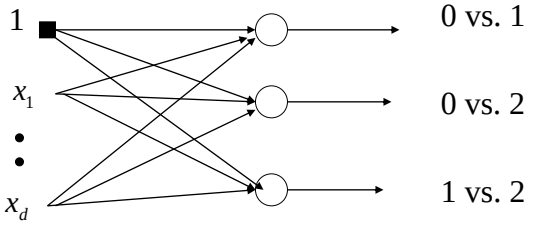
Multiclass classification. Approach 1.



One vs One (OVO)

Methods based on binary classification methods

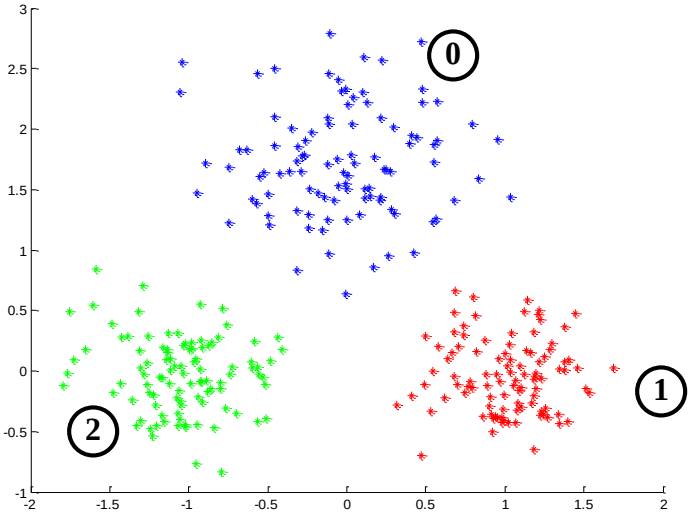
- **Assume:** we have 3 classes labeled 0,1,2
- **Approach 2:**
  - A binary logistic regression on all pairs



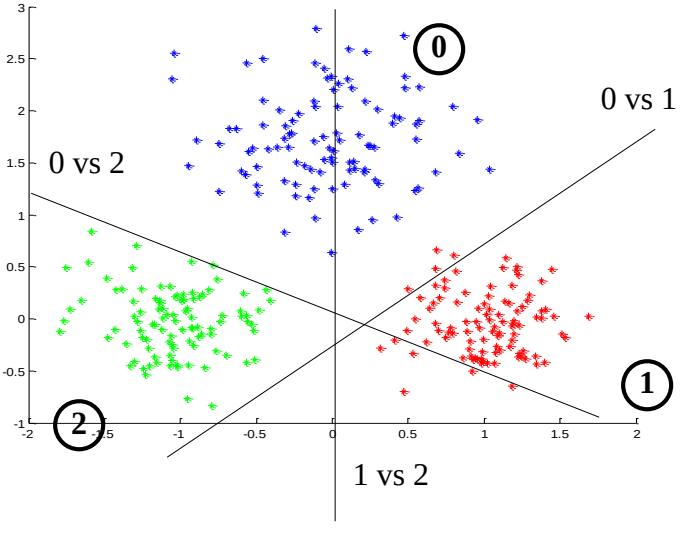
Class decision: class label based on who gets the majority

- Does not work all the time

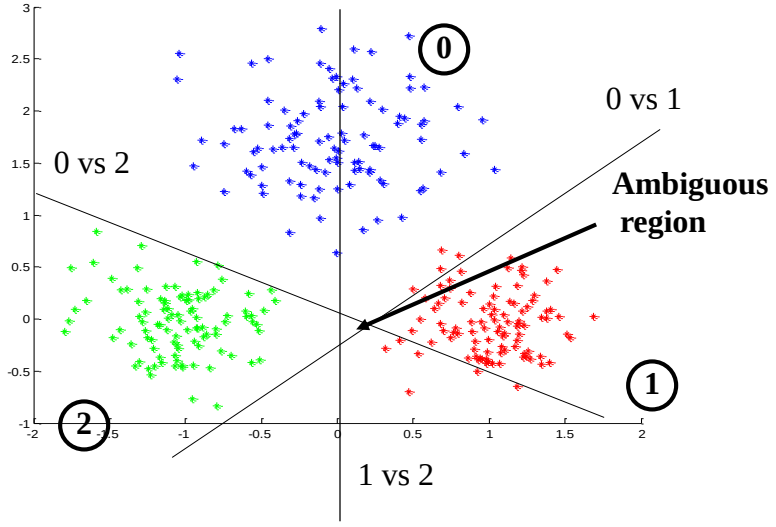
Multiclass classification. Example



Multiclass classification (OVO)



Multiclass classification OVO



One vs one (OVO) model

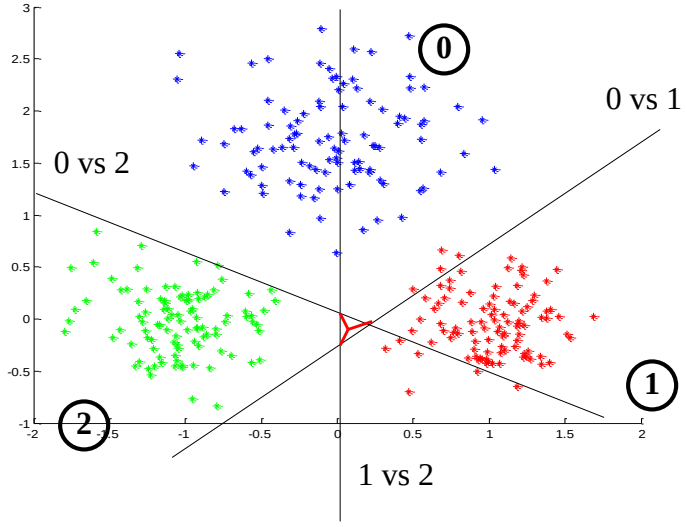
Unclear how to decide on class in some regions

- **Ambiguous region:**
  - 0 vs. 1 classifier says 0
  - 1 vs. 2 classifier says 1
  - 2 vs. 0 classifier says 2

- **One solution:** define a new discriminant function by adding the discriminant functions for pairwise classifiers

$g_i(\mathbf{x}) = \text{sum\_j}\ (\ g_{i \text{ vs } j}(\mathbf{w}^T\mathbf{x}))$

Multiclass classification.



Multiclass classification

OVR and OVO:

- define multiclass classifier using output classes of binary classifiers

Problems: ambiguous regions, regions of nobody
Solution: define discriminant functions for the multiclass case using the discriminant functions from binary classification problems

A Concern:

- Calibration of the discriminant functions
  - Discriminant functions from independently trained binary classification models may not be directly comparable

Solution:

- joint learning of discriminant functions