CS 1675 Machine Learning
Lecture 15

Multilayer neural networks: applications

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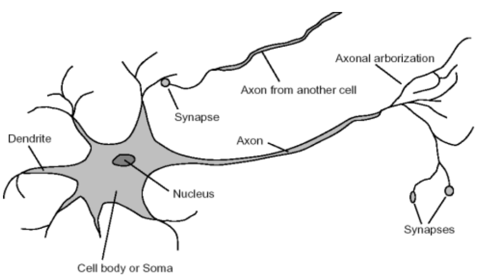
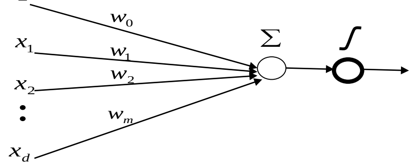
Multi-layered neural networks

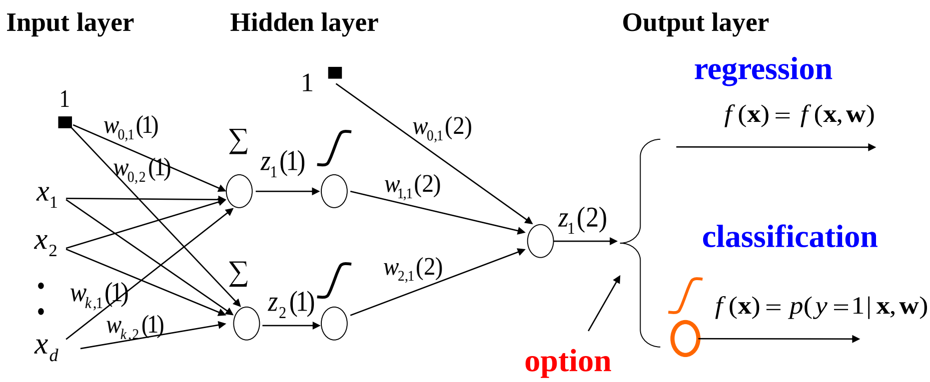
- An alternative way to model **nonlinearities for regression /classification problems**
- **Idea:** Cascade several simple nonlinear models (e.g. logistic units) **to approximate nonlinear functions** for regression /classification. Learn/adapt these simple models.
- **Motivation:** neuron connections





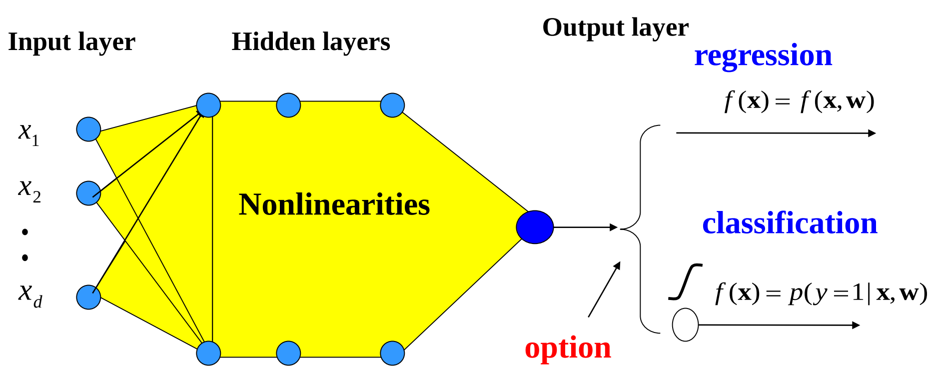
Multilayer neural network

- Models **non-linearity through nonlinear switching units**
- Can be applied to both **regression and binary classification problems**



Multilayer neural network

- **Non-linearities are modeled using multiple hidden nonlinear units (organized in layers)**
- The output layer determines whether it is a **regression or a binary classification problem**

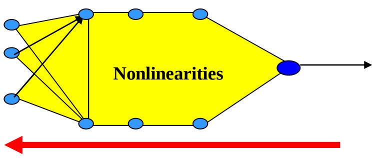


Learning with MLP

- How to learn the parameters of the neural network?
- **Gradient descent algorithm**
  - Weight updates based on the error: $J(D, \mathbf{w})$

$$\mathbf{w} \leftarrow \mathbf{w} - \alpha \nabla_{\mathbf{w}} J(D, \mathbf{w})$$

- We need to **compute gradients for weights in all units**
- **Can be computed in one backward sweep through the net !!!**



- The process is called **back-propagation**

Learning with MLP

- **Online gradient descent algorithm**
  - Weight update:

$$w_{i,j}(k) \leftarrow w_{i,j}(k) - \alpha \frac{\partial}{\partial w_{i,j}(k)} J_{\text{online}}(D_u, \mathbf{w})$$

$$\frac{\partial}{\partial w_{i,j}(k)} J_{online}(D_u, \mathbf{w}) = \frac{\partial J_{online}(D_u, \mathbf{w})}{\partial z_i(k)} \frac{\partial z_i(k)}{\partial w_{i,j}(k)} = \delta_i(k) x_j(k-1)$$

$$\boxed{w_{i,j}(k) \leftarrow w_{i,j}(k) - \alpha \delta_i(k) x_j(k-1)}$$

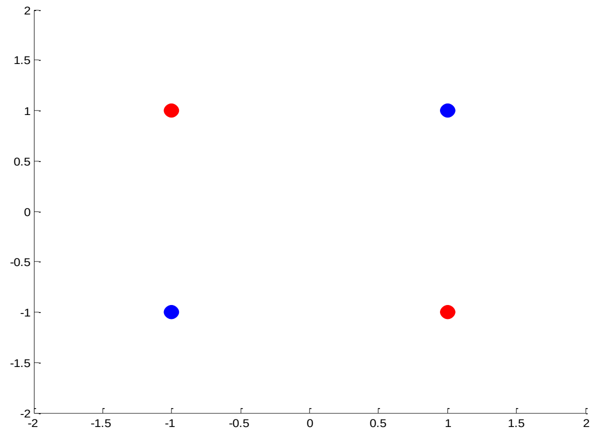
$x_j(k-1)$ - j-th output of the (k-1) layer

$\delta_i(k)$ - A derivative computed via backpropagation

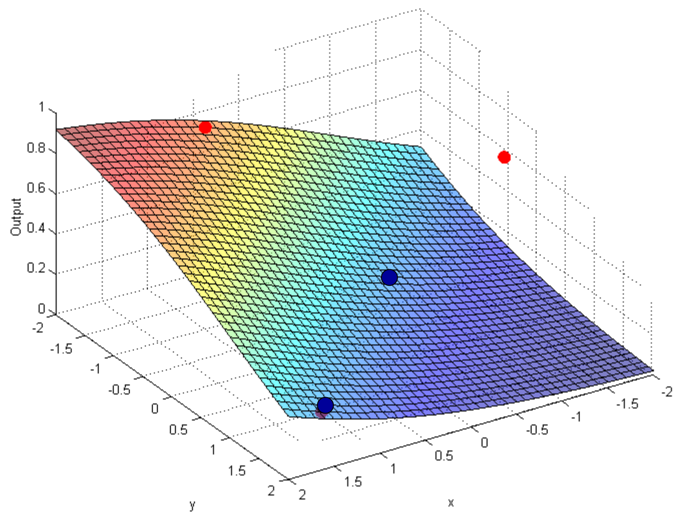
α - a learning rate

Xor Example.

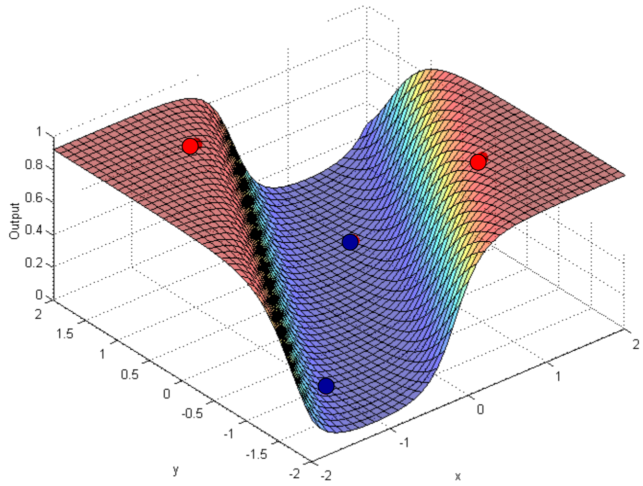
- linear decision boundary does not exist



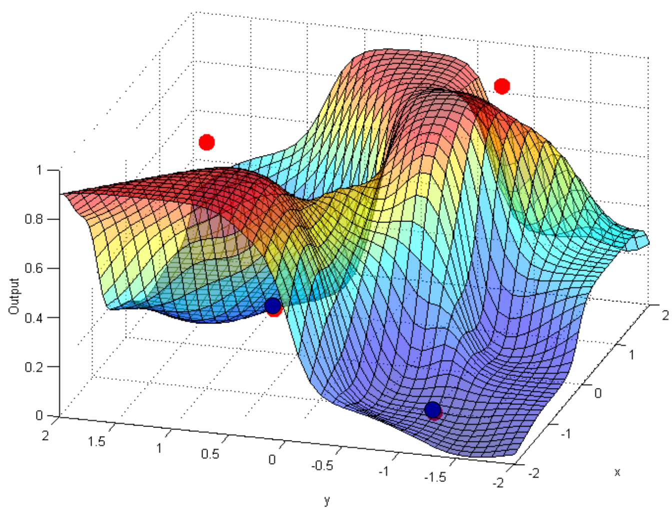
Xor example. Linear unit



Xor example.
Neural network with 2 hidden units



Xor example.
Neural network with 10 hidden units



Neural networks

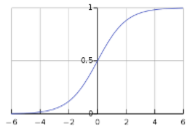
Activation (transfer) functions

- **Determine how inputs are transformed to output**

Possible choices of nonlinear transfer functions:

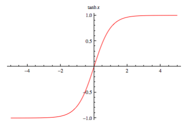
- **Logistic function**

$$f(z) = \frac{1}{1+e^{-z}} \qquad f(z)' = f(z)(1-f(z))$$



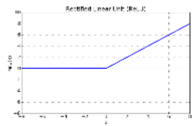
- **Hyperbolic tangent**

$$f(z) = \tanh(z) = \frac{2}{1+e^{-2z}} - 1 \qquad f(z)' = 1 - f(z)^2$$



- **Rectified linear units (Relu)**

$$f(z) = \begin{cases} 0 & z < 0 \\ z & z \geq 0 \end{cases}$$



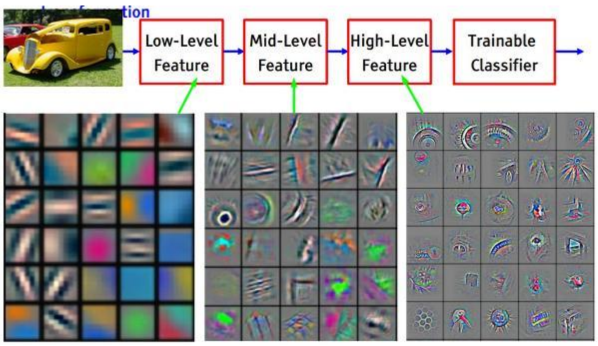
Limitation of standard NNs

Standard NN:

- **do not scale well to high dimensional data (e.g. images)**
  - 100x100 image + 100 hidden units = 1 million parameters.
  - Overfitting;
  - Tremendous requirements of computation and storage.
- **Sensitive to small translation of inputs**
  - Images: objects can have size, slant or position variations
  - Speech: varying speed, pitch or intonation.
- **Ignores the topology of the input**
  - i.e. the input variables can be presented in any order without affecting the outcome of training.
  - However, images or speech have a strong local structure
    - E.g. pixels nearby are highly correlated.

Deep learning

- **Deep learning.** Machine learning algorithms based on learning multiple levels of representation / abstraction. More than one layer of non-linear feature transformation.

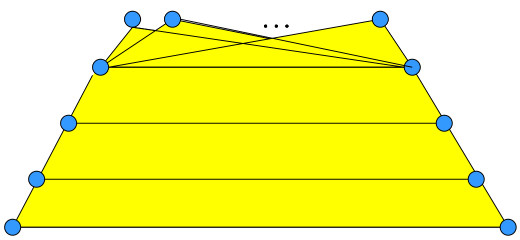


Deep neural networks

Early efforts

- **Optical character recognition** – digits 20x20
  - **MNIST** dataset
  - Automatic sorting of mails
  - 5 layer network with multiple output functions

10 outputs (0,1,…9)



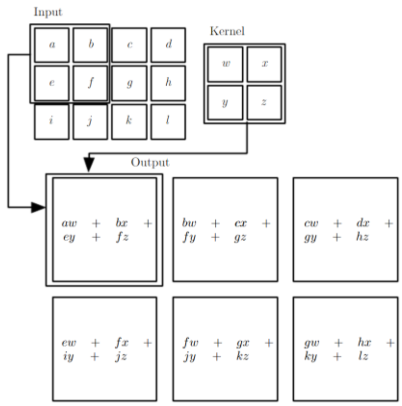
20x20 = 400 inputs

layer	Neurons	Weights
5	10	3000
4	300	1200
3	1200	50000
2	784	3136
1	3136	78400

Convolutional NN

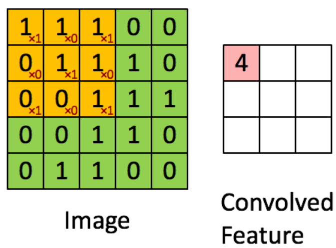
Take advantage of the local structure of the data (image, speech)

Convolution in Machine Learning
- the **input** array
  - e.g. image pixels.
- a **filter or kernel**
  - a smaller (local) matrix of parameters
- Output: a **feature map**
  - Filter applied to the image



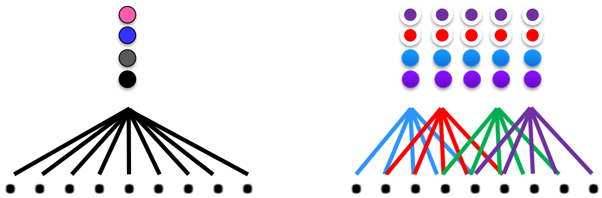
Feature Extraction using Convolution

- The statistics of one part of the image are the same as any other part.
- Meaning that different parts of an image can share the same feature parameters (**kernel**).
- Use this kernel to **convolve** a set of features.
- This is called one feature mapping.



Feature Extraction using Convolution

4 features on full data (image) | 4 features on the local data



Fully connected layer | Locally connected layer

9 weights per hidden unit
9 x 4 = 36 weights

5 weights per hidden unit
5 x 4 = 20 weights

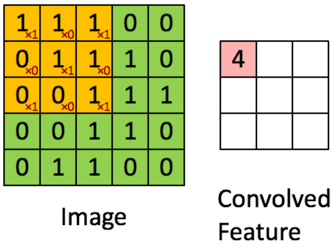
Increased #input, #hidden unit, but fewer weights

Pooling (Subsampling, Down-sampling)

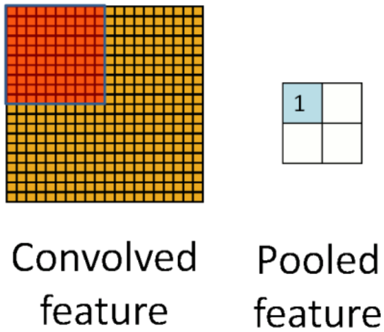
- **Assumption:** Features useful in one region are likely to be useful for other regions.
- To describe a large image, statistics can be **aggregated.**
- For example, one can calculate mean or max of a particular feature over a region.
  - Called **mean pooling**, **max pooling** respectively.
- These summary statistics are much lower in dimension.
- Also can improve results (less-overfitting).

Convolution and Pooling

Convolution

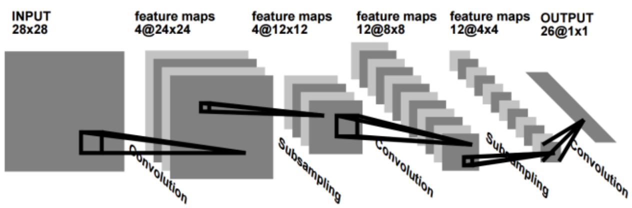


Pooling



Convolutional NN

- CNN = (≥ 1) convolution layer(s) + standard NN
- **One convolution layer is:**
  - Convolution operation + activation function + pooling
- You can view the convolution layer(s) as a feature extractor.
  - **Input:** raw image pixels, raw time series
  - **Output:** summarized features.



CNN vs. NN

- NN is sensitive to local distortions of unstructured data.
  - NN can theoretically be trained to be invariant to these distortions, probably resulting in multiple units with identical weights.
  - But such a training task requires a large number of training instances.
- CNN with pooling can be invariant to small translations:
  - Shifts (automatically)
  - Rotation (with extra mechanism)

Object Recognition Task

- **ImageNet Data** (2009 - 2016)

ImageNet 2012

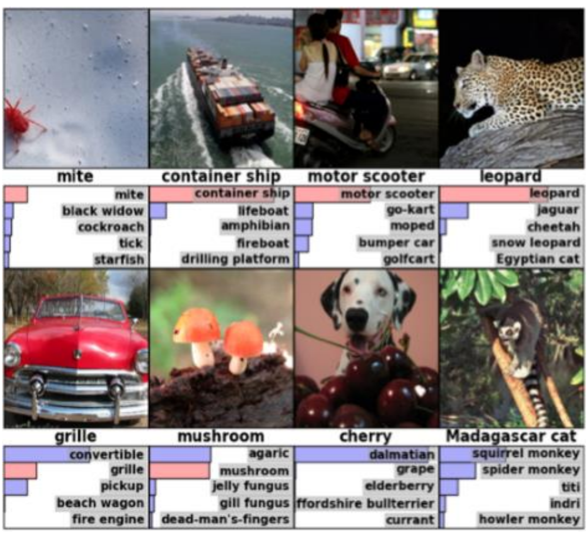
Data

- Size:
  - Number of images
    - 1.2 million training images
    - 50K validation images
    - 150K testing images
  - Variable image size
- Supervised task
  - Labeled using Amazon's Mechanical Turk
- Categories:
  - 1000 categories (objects)
    - Approximately 1000 in each categor
- RGB pictures

Goal

Provide a probability for different categories that an image can belong to

Object Recognition



ImageNet

- Achieves state-of-the-art on many object recognition tasks.