Solutions to problem set 6

Problem 1. Inference with propositional rules.

Assume a simplified animal identification problem due to P. Winston. The knowledge needed for the problem consists of the following set of rules:

1. If the animal has hair then it is a mammal
2. If the animal gives milk then it is a mammal
3. If the animal has feathers then it is a bird
4. If the animal flies and it lays eggs then it is a bird
5. If the animal is a mammal and it eats meat then it is a carnivore
6. If the animal is a mammal and it has pointed teeth and it has claws and its eyes point forward
   then it is a carnivore
7. If the animal is a mammal it has hoofs then it is an ungulate
8. If the animal is a mammal and it chews cud then it is an ungulate
9. If the animal is a mammal and it chews cud then it is even-toed
10. If the animal is a carnivore and it has a tawny color and it has dark spots then it is a cheetah
11. If the animal is a carnivore and it has a tawny color and it has black strips then it is a tiger
12. If the animal is an ungulate and it has long legs and it has a long neck and it has a tawny
    color and it has dark spots then it is a giraffe
13. If the animal is an ungulate and it has a white color and it has black stripes then it is a zebra
14. If the animal is a bird and it does not fly and it has long legs and it has a long neck and it is
    black and white then it is an ostrich,
15. If the animal is a bird and it does not fly and it swims and it is black and white then it is a
    penguin
16. If the animal is a bird and it is a good flyer then it is an albatross.

The above set of rules can be represented in the propositional logic using implications of
the form $A_1 \land A_2 \land \cdots \land A_k \rightarrow B$, that is, all the statements are in the Horn form. Recall
that inferences with modus ponens for KB in the Horn normal form are both sound and
complete.
Assume a set of initial facts: the animal gives milk, it chews cud, it has long legs, long neck, tawny cloor and dark spots are all TRUE for the animal we want to identify. Assume the following set of theorems:

- Theorem1: the animal is a giraffe;
- Theorem2: the animal is a penguin;
- Theorem3: the animal is a mammal.

Decide using the repeated application of the modus ponens inference rule whether Theorems 1-3 hold. For every theorem proved give a sequence of rules (their numbers) used to derive the conclusion.

**Answer.**

- Theorem1: the animal is a giraffe is TRUE. The sequence of rules to prove this: R2, R8, R12.
- Theorem2: the animal is a penguin cannot be proved. None of the conditions of rule R15 can be proved.
- Theorem3: the animal is a mammal is TRUE. The sequence of rules to prove this is: R2.

**Problem 2.**

Do exercise 8.9. from page 316 of the RN textbook.

**Answer.**

a. (i) 2 (ii) 1 (iii) 3
b. (i) 1 (ii) 3 (iii) 2 (iv) 2
c. (i) 1 (ii) 1 (iii) 3 (iv) 3
d. (i) 1 (ii) 2 (iii) 3 (iv) 1
e. (i) 3 (ii) 1 (iii) 3 (iv) 2
Problem 3.

Do exercise 8.10. from page 317 of the RN textbook.

Answer.

a. \(\text{Occupation}(Emily, \text{Surgeon}) \lor \text{Occupation}(Emily, \text{Lawyer})\)

b. \(\text{Occupation}(Joe, \text{Actor}) \land \exists x \text{Occupation}(Joe, x)\)

c. \(\forall x \text{Occupation}(x, \text{Surgeon}) \implies \text{Occupation}(x, \text{Doctor})\)

d. \(\forall x \text{Occupation}(x, \text{Lawyer}) \implies \neg \text{Customer}(Joe, x)\)

e. \(\exists x \text{Boss}(x, Emily) \land \text{Occupation}(x, \text{Lawyer})\)

f. \(\exists x \text{Occupation}(x, \text{Lawyer}) \land (\forall y \text{Customer}(y, x) \implies \text{Occupation}(y, \text{Doctor}))\)

g. \(\forall x \text{Occupation}(x, \text{Surgeon}) \implies (\exists y \text{Occupation}(y, \text{Lawyer}) \land \text{Customer}(x, y))\)