# CS 1571 Introduction to AI
## Lecture 7

- IDA*
- Constraint satisfaction search

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## Search methods

- Uninformed search methods
  - Breadth-first search (BFS)
  - Depth-first search (DFS)
  - Iterative deepening (IDA)
  - Bi-directional search
  - Uniform cost search
- Informed (or heuristic) search methods:
  - Best first search with the heuristic function

## Best-first search

**Best-first search**

- Driven by the evaluation function $f(n)$ to guide the search.
- incorporates a **heuristic function** $h(n)$ in $f(n)$
- heuristic function measures a potential of a state (node) to reach a goal

**Special cases** (differ in the design of evaluation function):

- **Greedy search**

$$f(n) = h(n)$$

- **A* algorithm**

$$f(n) = g(n) + h(n)$$

+ **iterative deepening** version of A* : **IDA***



## A* search

- The problem with the **greedy search** is that it can keep expanding paths that are already very expensive.
- The problem with the **uniform-cost search** is that it uses only past exploration information (path cost), no additional information is utilized
- **A* search**

$$f(n) = g(n) + h(n)$$

$g(n)$ - cost of reaching the state

$h(n)$ - estimate of the cost from the current state to a goal

$f(n)$ - estimate of the path length

- **Additional A*condition**: admissible heuristic

$$h(n) \leq h^*(n) \quad \text{for all } n$$

## Optimality of A*

- In general, a heuristic function $h(n)$ :

  Can overestimate, be equal or underestimate the true distance of a node to the goal $h^*(n)$

- **Admissible heuristic condition**
  - **Never overestimate the distance to the goal !!!**

$$h(n) \le h^*(n) \quad \text{for all } n$$

**Example:** the straight-line distance in the travel problem never overestimates the actual distance



## Iterative deepening algorithm (IDA)

- Based on the idea of the limited-depth search, but
- It resolves the difficulty of knowing the depth limit ahead of time.

**Idea: try all depth limits in an increasing order.**

**That is,** search first with the depth limit *l=0*, then *l=1*, *l=2*, and so on until the solution is reached

**Iterative deepening** combines advantages of the depth-first and breadth-first search with only moderate computational overhead

## Properties of IDA

- **Completeness:** **Yes.** The solution is reached if it exists. (the same as BFS)
- **Optimality:** **Yes**, for the shortest path. (the same as BFS)
- **Time complexity:**

$$O(1) + O(b^1) + O(b^2) + \ldots + O(b^d) = O(b^d)$$

  **exponential in the depth of the solution *d***

  **worse than BFS, but asymptotically the same**
- **Memory (space) complexity:**

$$O(db)$$

  **much better than BFS**



## IDA*

- **Solves minimum cost-path problems with heuristics**
- **Iterative deepening version of A***

**Idea:**

- Performs **limited-cost depth-first search** for the current evaluation function limit
  - Keeps expanding nodes in the depth-first manner up to the evaluation function limit
- Progressively increases the **evaluation function limit** (instead of the depth limit)

**Problem:** the amount by which the evaluation limit should be progressively increased

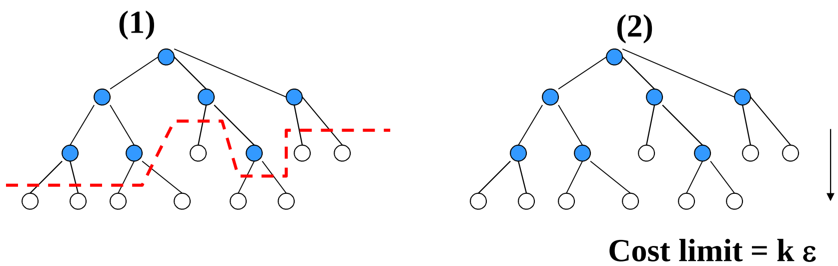
# IDA*

**Problem:** the amount by which the evaluation limit should be progressively increased

**Solutions:**

**(1) peak over the previous step boundary** to guarantee that in the next cycle some number of nodes are expanded

**(2) Increase the limit by a fixed cost increment – say ε**

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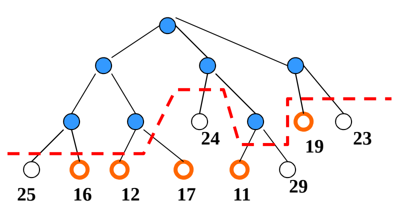
# IDA*

**Solution 1: peak over the previous step boundary to guarantee that in the next cycle more nodes are expanded**




**Properties:**

- the choice of the new cost limit influences how many nodes are expanded in each iteration
- Assume I choose a limit such that at least 5 new nodes are examined in the next DFS run
- What is the problem here?

  We may find a sub-optimal solution
  - **Fix:** complete the search up to the limit to find the best

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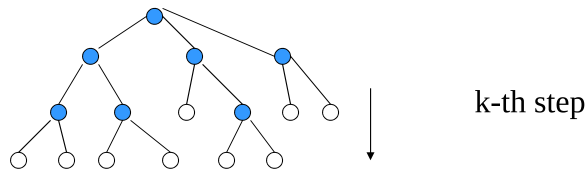
## IDA*

**Solution 2:** **Increase the limit by a fixed cost increment (ε)**




**Cost limit = k ε**

**Properties:**

What is bad? Too many or too few nodes expanded – no control of the number of nodes

How good is the first solution found ? Is it optimal?

– The solution found first may differ by < ε from the optimal solution



## next

# Constraint satisfaction search

## Search problem

**A search problem:**

- **Search space (or state space):** a set of objects among which we conduct the search;
- **Initial state:** an object we start to search from;
- **Operators (actions):** transform one state in the search space to the other;
- **Goal condition:** describes the object we search for

- **Possible metric on the search space:**
  - measures the quality of the object with respect to the goal



## Constraint satisfaction problem (CSP)

**Two types of search:**

- **path search** (a path from the initial state to a state satisfying the goal condition)
- **configuration search (**a configuration satisfying goal conditions)

**Constraint satisfaction problem (CSP)**

= **a configuration search problem** where**:**

- A **state** is defined by **a set of variables and their values**
- **Goal condition** is represented by **a set constraints on possible variable values**

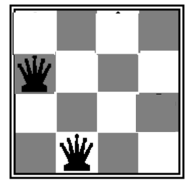
Special properties of the CSP lead to special search procedures we can design to solve them

## Example of a CSP: N-queens

**Goal:** n queens placed in non-attacking positions on the board

**Variables:**

- Represent queens, one for each column:
  - $Q_1, Q_2, Q_3, Q_4$
- Values:
  - Row placement of each queen on the board
    $\{1, 2, 3, 4\}$



$Q_1 = 2, Q_2 = 4$

**Constraints:** $Q_i \neq Q_j$ Two queens not in the same row

$|Q_i - Q_j| \neq |i - j|$ Two queens not on the same diagonal



## Satisfiability (SAT) problem

Determine whether a sentence in the conjunctive normal form (CNF) is satisfiable (can evaluate to true)

- Used in the propositional logic (covered later)

$$(P \vee Q \vee \neg R) \wedge (\neg P \vee \neg R \vee S) \wedge (\neg P \vee Q \vee \neg T) \ldots$$

**Variables:**

- Propositional symbols (P, R, T, S)
- Values: *True*, *False*

**Constraints:**

- Every conjunct must evaluate to true, at least one of the literals must evaluate to true

$$(P \vee Q \vee \neg R) \equiv True, (\neg P \vee \neg R \vee S) \equiv True, \ldots$$

## Other real world CSP problems

**Scheduling problems:**
- E.g. telescope scheduling
- High-school class schedule

**Design problems:**
- Hardware configurations
- VLSI design

**More complex problems may involve:**
- **real-valued variables**
- **additional preferences on variable assignments** – the optimal configuration is sought



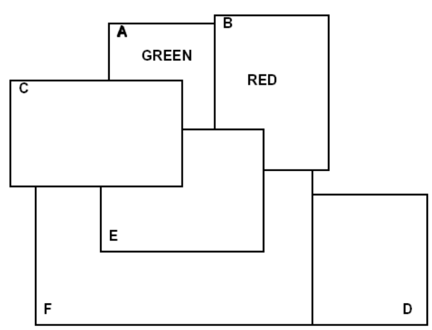
## Exercise: Map coloring problem

**Color a map using k different colors** such that no adjacent countries have the same color

**Variables: ?**

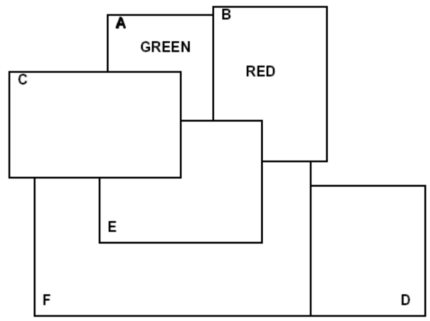
- Variable values: ?

**Constraints: ?**



## Map coloring

Color a map using k different colors such that no adjacent countries have the same color




**Variables:**

- Represent countries
  - $A, B, C, D, E$
- Values:
  - K -different colors

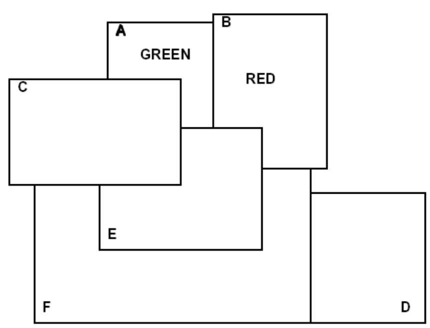
  {Red, Blue, Green,..}

**Constraints: ?**



## Map coloring

Color a map using k different colors such that no adjacent countries have the same color




**Variables:**

- Represent countries
  - $A, B, C, D, E$
- Values:
  - K -different colors

  {Red, Blue, Green,..}

**Constraints:** $A \neq B, A \neq C, C \neq E$, etc

This is an example of a problem with **binary constraints**

# Constraint satisfaction as a search problem

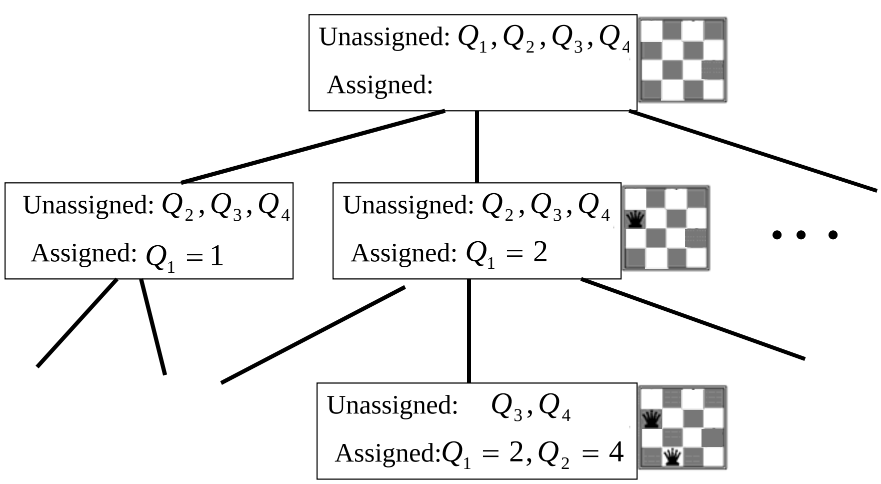
**A formulation of the search problem:**

- **States.** Assignment (partial or complete) of values to variables.
- **Initial state.** No variable is assigned a value.
- **Operators.** Assign a value to one of the unassigned variables.
- **Goal condition.** All variables are assigned, no constraints are violated.

- **Constraints** can be **represented**:
  - **Explicitly** by a set of allowable values
  - **Implicitly** by a function that tests for the satisfaction of constraints

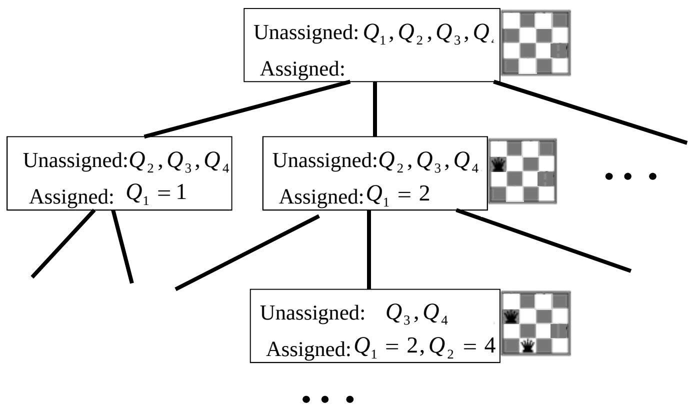


# Search strategies for solving CSP



# Search strategies for solving CSP

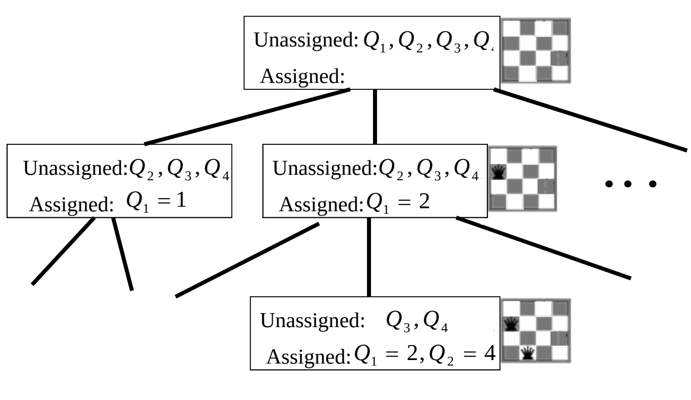
- **Maximum depth of the tree (m): ?**
- **Depth of the solution (d) : ?**
- **Branching factor (b) : ?**






# Search strategies for solving CSP

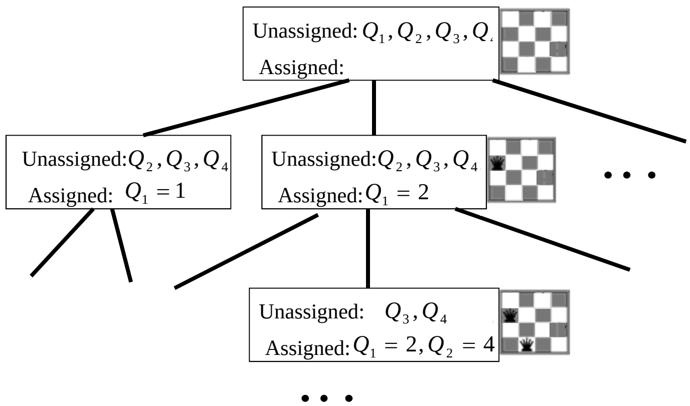
- **Maximum depth of the tree:** Number of variables in the CSP
- **Depth of the solution:** Number of variables in the CSP
- **Branching factor:** if we fix the order of variable assignments the branch factor depends on the number of their values



# Search strategies for solving CSP

- **What search algorithm to use: ?**

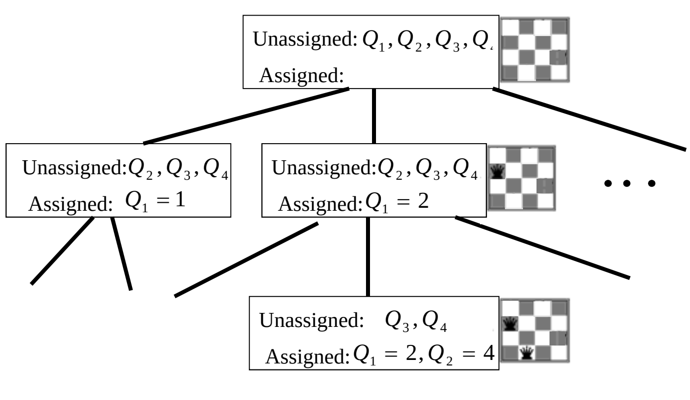
  Depth of the tree = Depth of the solution=number of vars






# Search strategies for solving CSP

- **What search algorithm to use: Depth first search !!!**
  - Since we know the depth of the solution
  - We do not have to keep large number of nodes in queues



# Search strategies for solving CSP

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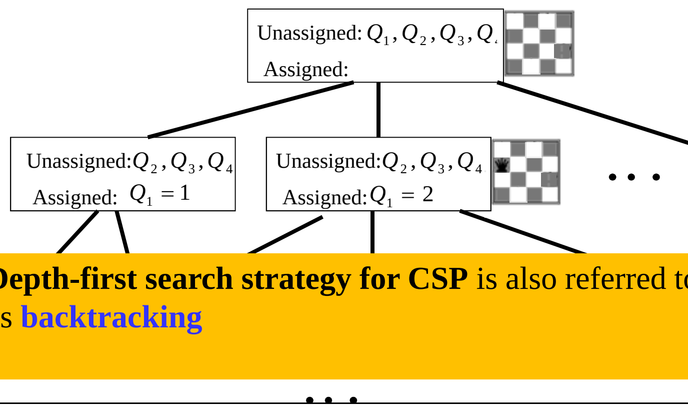



**Depth-first search strategy for CSP** is also referred to as **backtracking**



# Constraint consistency

**Question:**

- **When to check the constraints defining the goal condition?**
- The violation of constraints can be checked:
  - at the end (for the leaf nodes)
  - for each node of the search tree during its generation or before its expansion

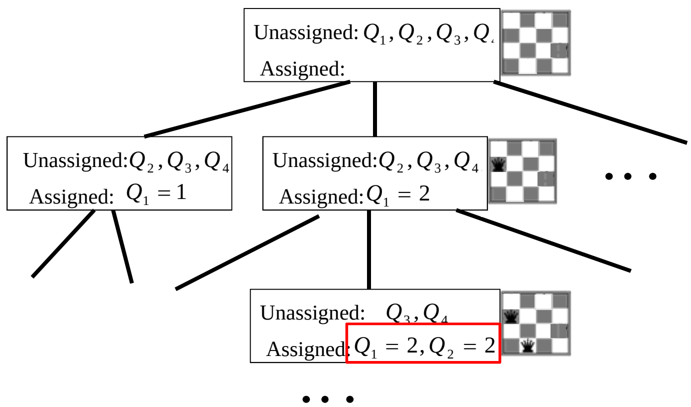
**Checking the constraints for intermediate nodes:**

- More efficient: cuts branches of the search tree early

# Constraint consistency

**Checking the constraints for intermediate nodes:**

- More efficient: cuts branches of the search tree early

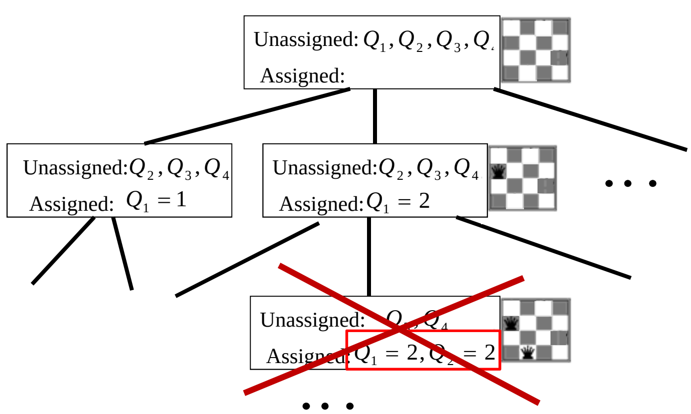





# Constraint consistency

**Checking the constraints for intermediate nodes:**

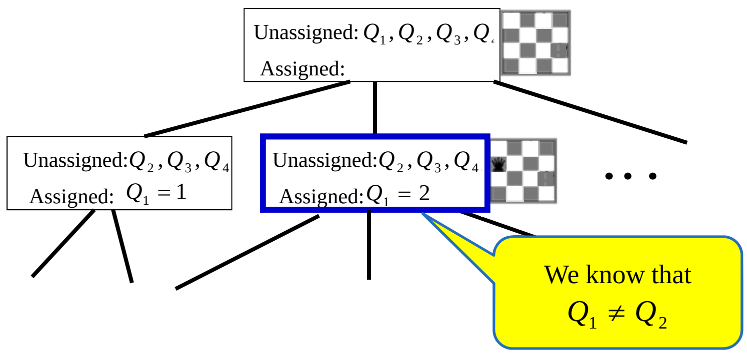
- More efficient: cuts branches of the search tree early



# Constraint consistency

**Another way to cut the search space and tree exploration**

- Current **variable assignments** together **with constraints restrict remaining legal values of unassigned variables**
- The remaining **legal and illegal values of variables may be inferred** (effect of constraints propagates)



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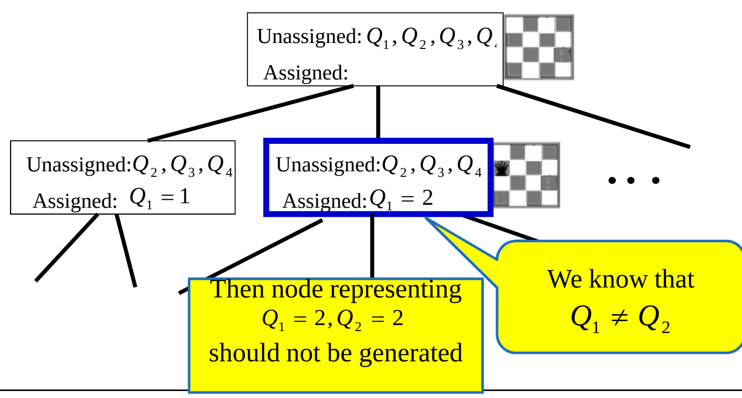


## Constraint consistency

**Another way to cut the search space and tree exploration**

- Current **variable assignments** together **with constraints restrict remaining legal values of unassigned variables**
- The remaining **legal and illegal values of variables may be inferred** (effect of constraints propagates)

- To prevent "blind" exploration we can keep track of the remaining legal values, so we know when the constraints will be violated and when to terminate the search



## Constraint propagation

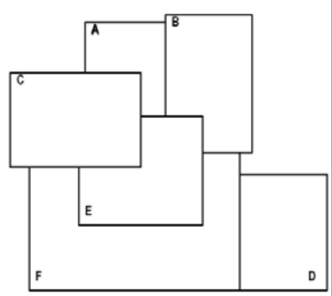
A **state** (more broadly) is defined:

- by a set of assigned variables, their values and
- a list of legal and illegal assignments for unassigned variables

Legal and illegal assignments can be represented:

- **equations** (value assignments) and
- **disequations (list of invalid assignments)**

$A$ = Red, Blue $\quad\quad C \neq$ Red



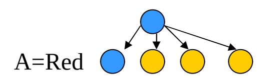

**Constraints + assignments**
can entail new equations and disequations

$A$ = Red $\rightarrow$ $B \neq$ Red

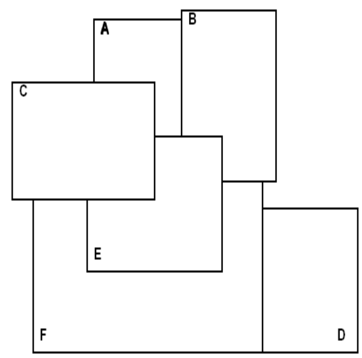
**Constraint propagation:** the process of inferring of new equations and disequations from existing equations and disequations

## Constraint propagation

- Assign A=Red




| | Red | Blue | Green |
|---|---|---|---|
| A | ✓ | | |
| B | | | |
| C | | | |
| D | | | |
| E | | | |
| F | | | |

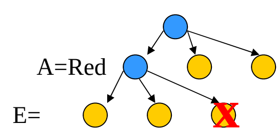



✓ - equations ✗ - disequations

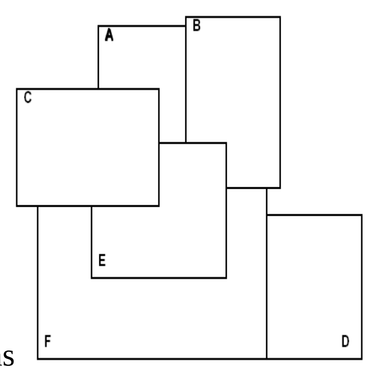


## Constraint propagation

- Assign A=Red



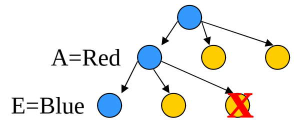

| | Red | Blue | Green |
|---|---|---|---|
| A | ✓ | | |
| B | ✗ | | |
| C | ✗ | | |
| D | | | |
| E | ✗ | | |
| F | | | |



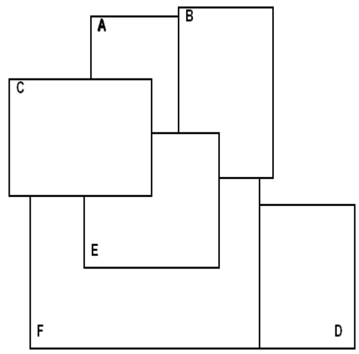

✓ - equations ✗ - disequations

# Constraint propagation

- Assign E=Blue



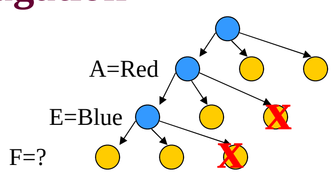

| | Red | Blue | Green |
|---|---|---|---|
| A | ✓ | | |
| B | ✗ | | |
| C | ✗ | | |
| D | | | |
| E | ✗ | ✓ | |
| F | | | |



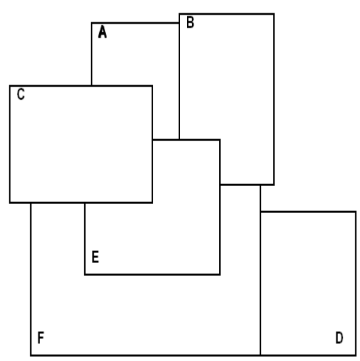



# Constraint propagation

- Assign E=Blue

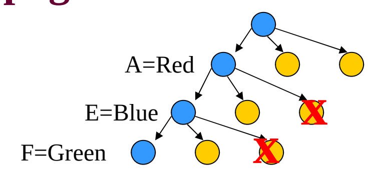



| | Red | Blue | Green |
|---|---|---|---|
| A | ✓ | ✗ | |
| B | ✗ | ✗ | |
| C | ✗ | ✗ | |
| D | | | |
| E | ✗ | ✓ | |
| F | | ✗ | |

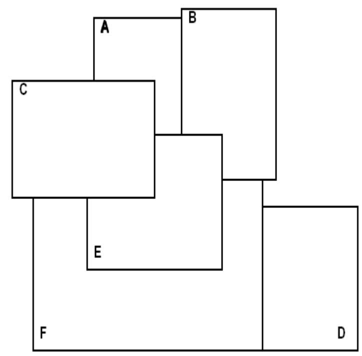


# Constraint propagation

- Assign F=Green



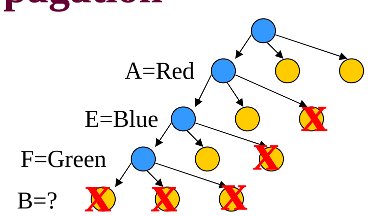

| | Red | Blue | Green |
|---|---|---|---|
| A | ✓ | ✗ | |
| B | ✗ | ✗ | |
| C | ✗ | ✗ | |
| D | | | |
| E | ✗ | ✓ | |
| F | | ✗ | ✓ |



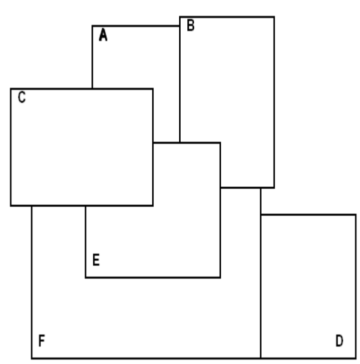



# Constraint propagation

- Assign F=Green

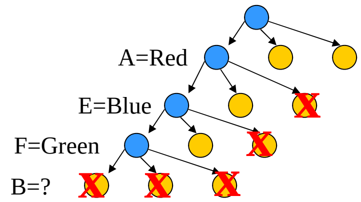



| | Red | Blue | Green |
|---|---|---|---|
| A | ✓ | ✗ | |
| B | ✗ | ✗ | ✗ |
| C | ✗ | ✗ | ✗ |
| D | | | ✗ |
| E | ✗ | ✓ | ✗ |
| F | | ✗ | ✓ |

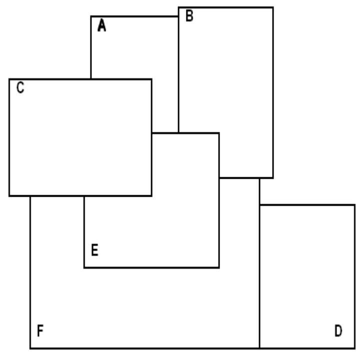


# Constraint propagation

- Assign F=Green




| | Red | Blue | Green |
|---|---|---|---|
| A | ✓ | ✗ | |
| B | ✗ | ✗ | ✗ |
| C | ✗ | ✗ | ✗ |
| D | | | ✗ |
| E | ✗ | ✓ | ✗ |
| F | | ✗ | ✓ |

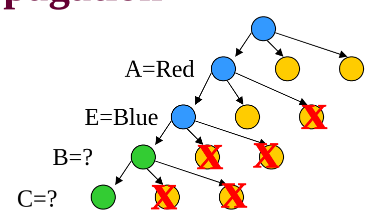



**Conflict !!! No legal assignments available for B and C**

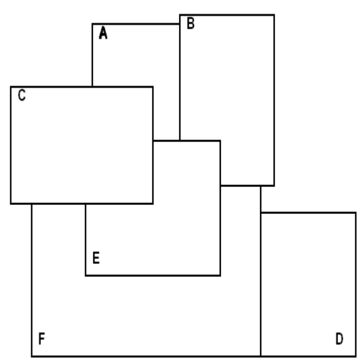


# Constraint propagation

- We can derive remaining legal values through propagation




| | Red | Blue | Green |
|---|---|---|---|
| A | ✓ | ✗ | |
| B | ✗ | ✗ | ✓ |
| C | ✗ | ✗ | ✓ |
| D | | | |
| E | ✗ | ✓ | |
| F | | ✗ | |



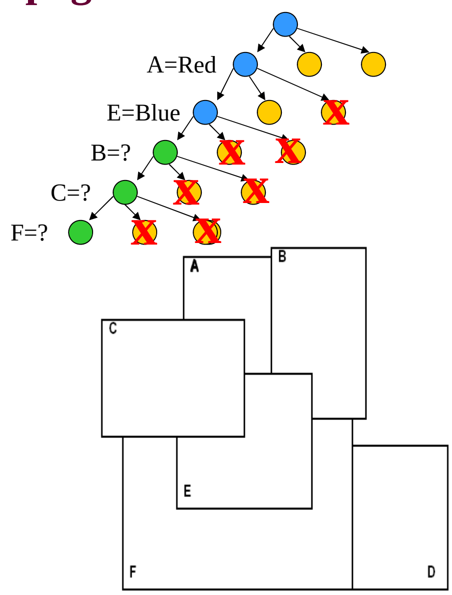

**B=Green**

**C=Green**

## Constraint propagation

- We can derive remaining legal values through propagation

| | Red | Blue | Green |
|---|---|---|---|
| A | ✓ | ✗ | ✗ |
| B | ✗ | ✗ | ✓ |
| C | ✗ | ✗ | ✓ |
| D | ✗ | | |
| E | ✗ | ✓ | ✗ |
| F | ✓ | ✗ | ✗ |



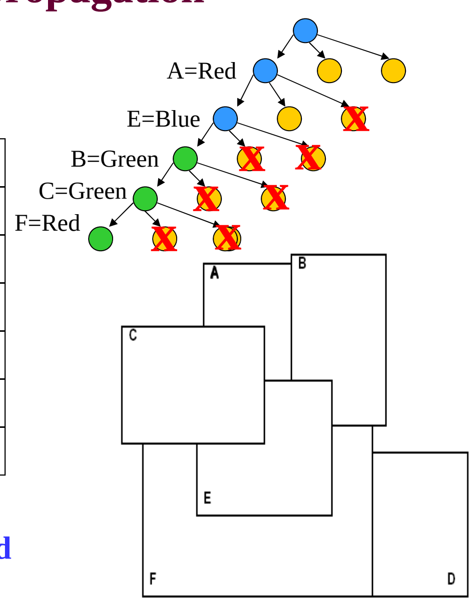

**B=Green**
**C=Green** ⇒ **F=Red**



## Constraint propagation

- We can derive remaining legal values through propagation

| | Red | Blue | Green |
|---|---|---|---|
| A | ✓ | ✗ | ✗ |
| B | ✗ | ✗ | ✓ |
| C | ✗ | ✗ | ✓ |
| D | ✗ | | |
| E | ✗ | ✓ | ✗ |
| F | ✓ | ✗ | ✗ |




**B=Green**
**C=Green** ⇒ **F=Red**