CS 1571 Introduction to AI
Lecture 5

Uninformed search methods II.

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Uninformed methods

- Uninformed search methods use only information available in the problem definition
  - **Breadth-first search (BFS)**
  - **Depth-first search (DFS)**
  - **Iterative deepening (IDA)**
  - **Bi-directional search**
- **For the minimum cost path problem:**
  - **Uniform cost search**

Elimination of state repeats

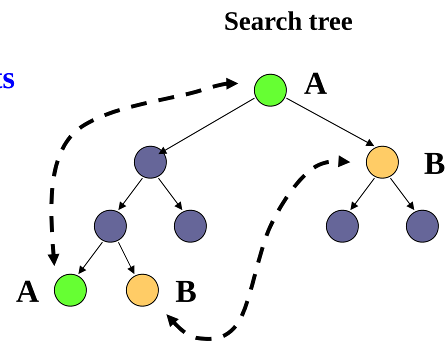
While searching the state space for the solution we can encounter the same state many times.

Question: Is it necessary to keep and expand all copies of states in the search tree?

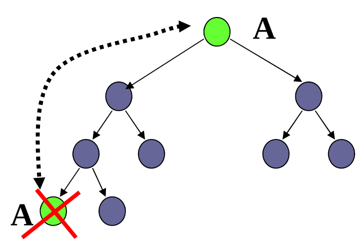
Two possible cases:

(A) Cyclic state repeats

(B) Non-cyclic state repeats



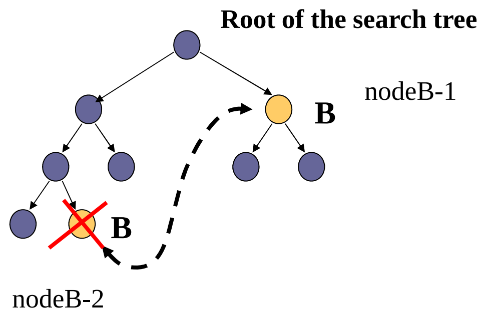
Elimination of cycles



Case A: Corresponds to the path with a cycle

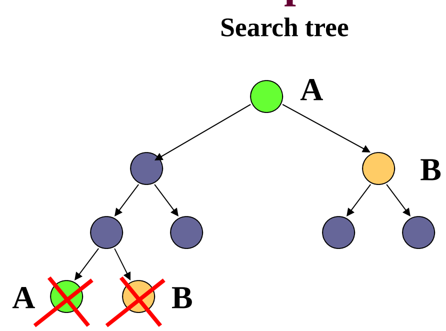
A branch of the tree representing a path with a cycle cannot be the part of the shortest solution and can be safely eliminated.

Elimination of non-cyclic state repeats



A state B is reached by a longer than optimal path than it cannot be the part of the shortest solution and can be safely eliminated.

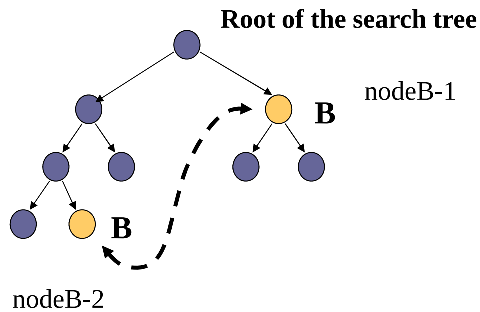
Elimination of state repeats with BFS



Breadth FS is well behaved with regard to all state repeats:

- we can safely eliminate the node that is associated with the state that has been expanded before

Elimination of state repeats with DFS



Caveat: The order of node expansion does not imply correct elimination strategy

Solution: we need to remember the length of the path in order to safely eliminate any of the nodes

Properties of breadth-first search

- **Completeness: Yes.** The solution is reached if it exists.
- **Optimality: Yes**, for the shortest path.
- **Time complexity:**

$$O(b^d)$$

exponential in the depth of the solution *d*

- **Memory (space) complexity:**

$$O(b^d)$$

nodes are kept in the memory

Properties of depth-first search

- **Completeness:** **No.** If infinite loops can occur.
- **Yes.** If we prevent them.
- **Optimality:** **No.** Solution found first may not be the shortest possible.

- **Time complexity:**

  $O(b^m)$

  exponential in the maximum depth of the search tree *m*

- **Memory (space) complexity:**

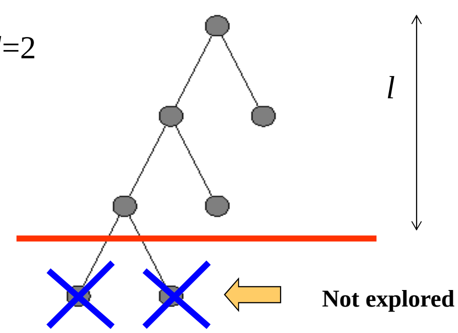
  $O(bm)$

  linear in the maximum depth of the search tree *m*

Limited-depth depth first search

- How to eliminate infinite depth-first exploration?
- Put the limit *(l)* on the depth of the depth-first exploration

Limit l=2



- **Time complexity:** $O(b^l)$
- **Memory complexity**: $O(bl)$

l - is the given limit

Iterative deepening algorithm (IDA)

- Based on the idea of the limited-depth search, but
- It resolves the difficulty of knowing the depth limit ahead of time.

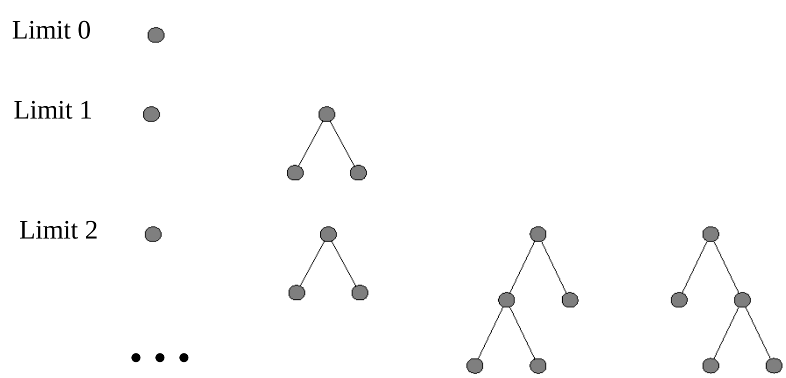
Idea: try all depth limits in an increasing order.

That is, search first with the depth limit $l=0$, then $l=1$, $l=2$, and so on until the solution is reached

Iterative deepening combines advantages of the depth-first and breadth-first search with only moderate computational overhead

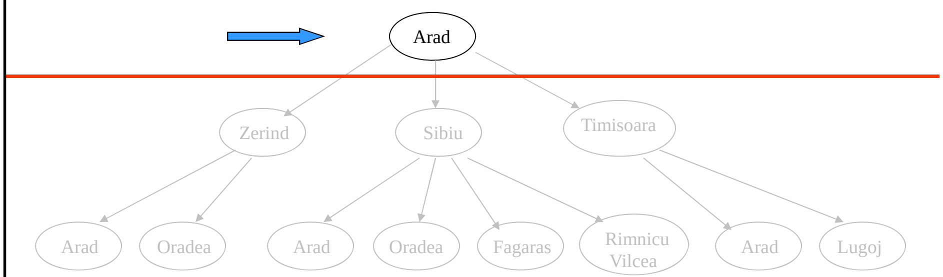
Iterative deepening algorithm (IDA)

- Progressively increases the limit of the limited-depth depth-first search



Iterative deepening

Cutoff depth = 0



Iterative deepening

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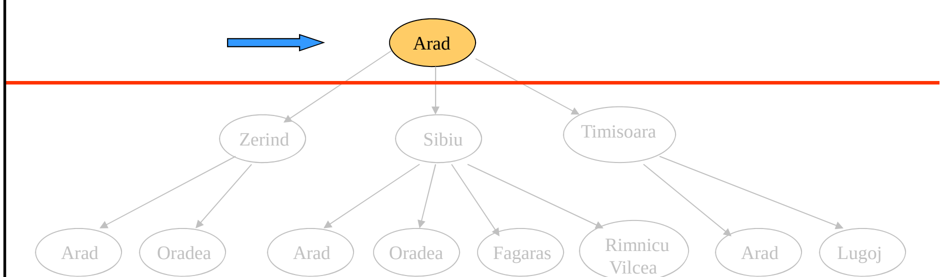
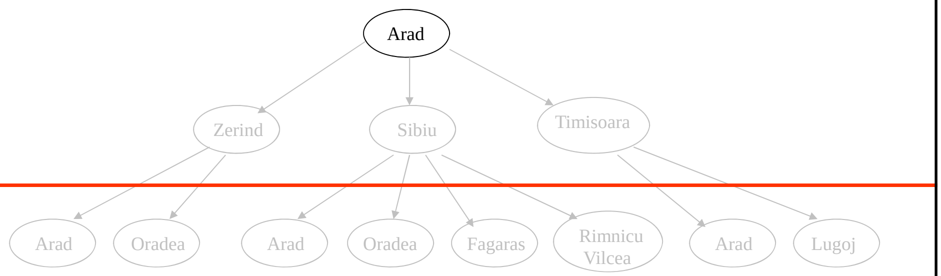


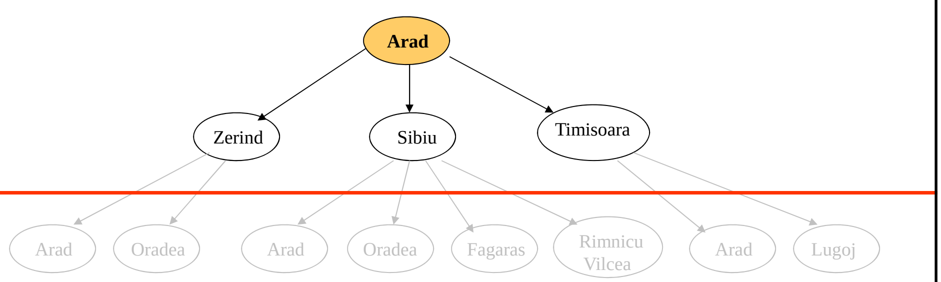
Iterative deepening

Cutoff depth = 1



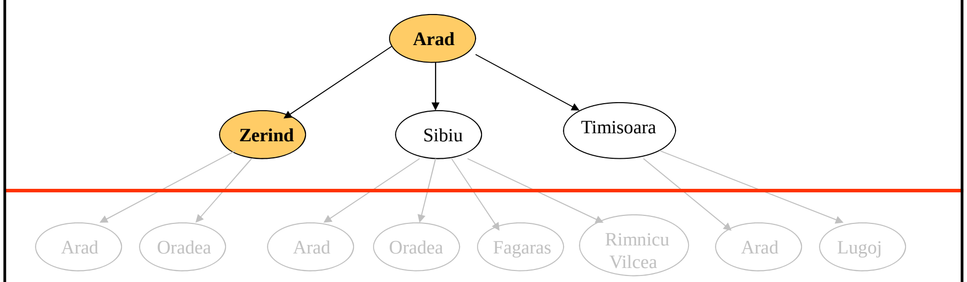
Iterative deepening

Cutoff depth = 1



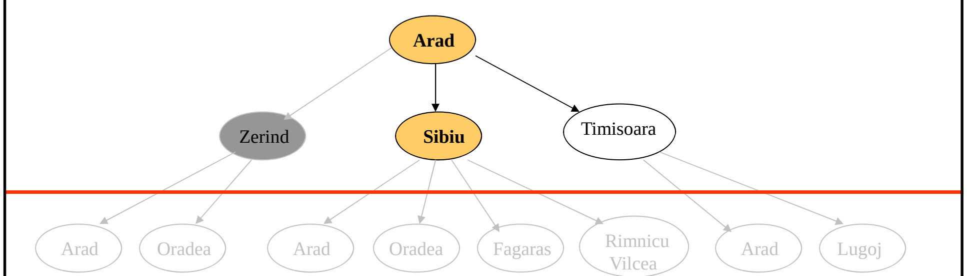
Iterative deepening

Cutoff depth = 1



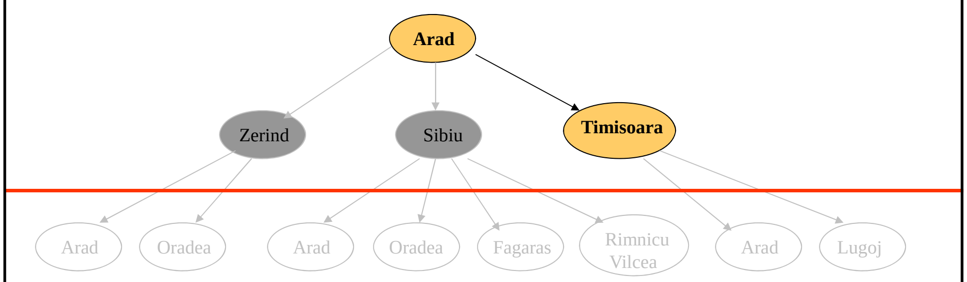
Iterative deepening

Cutoff depth = 1



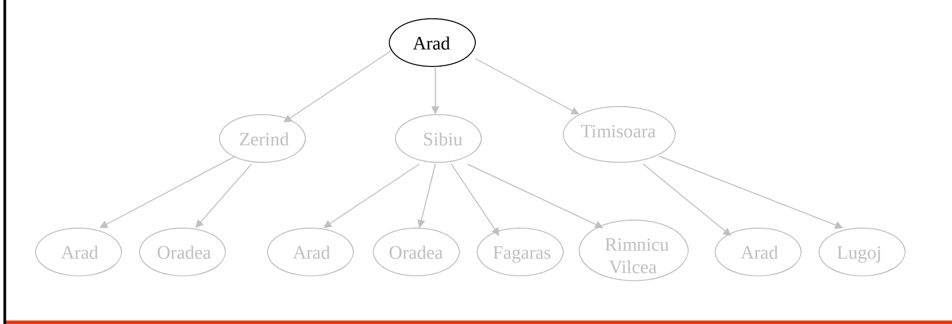
Iterative deepening

Cutoff depth = 1



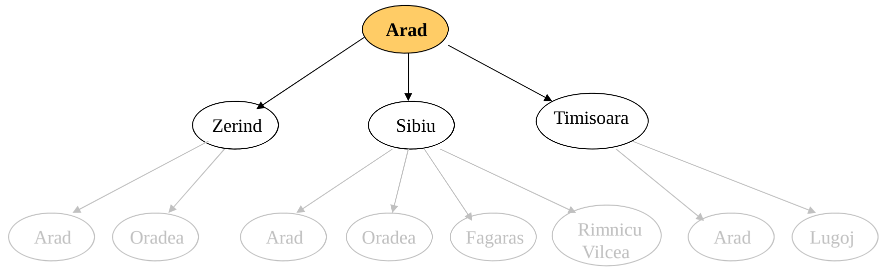
Iterative deepening

Cutoff depth = 2



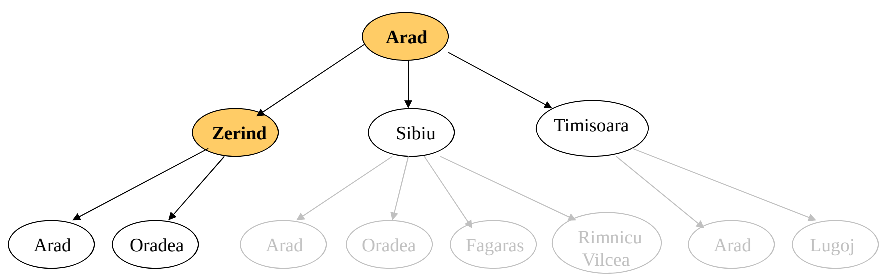
Iterative deepening

Cutoff depth = 2



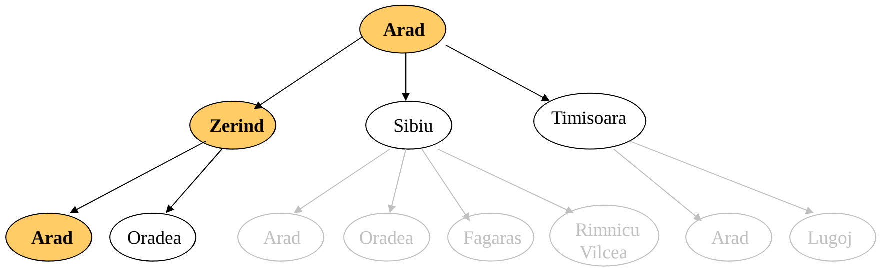
Iterative deepening

Cutoff depth = 2



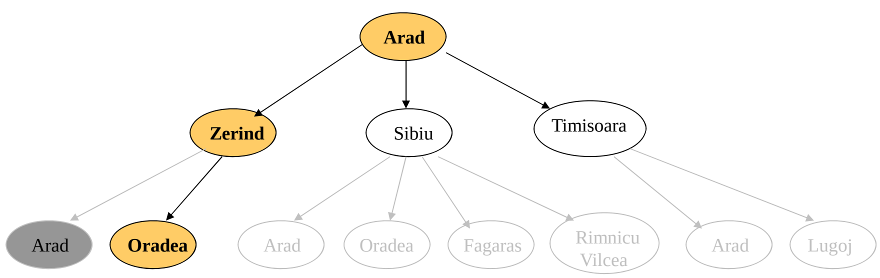
Iterative deepening

Cutoff depth = 2



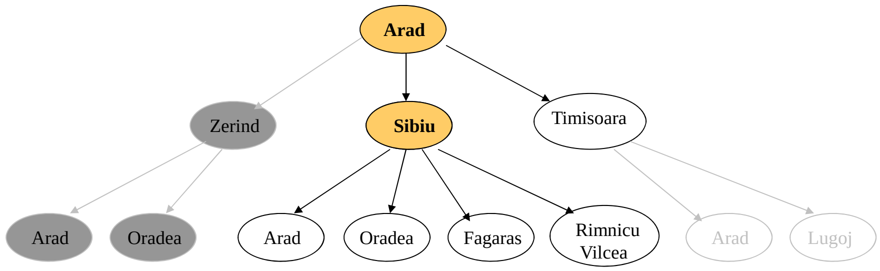
Iterative deepening

Cutoff depth = 2



Iterative deepening

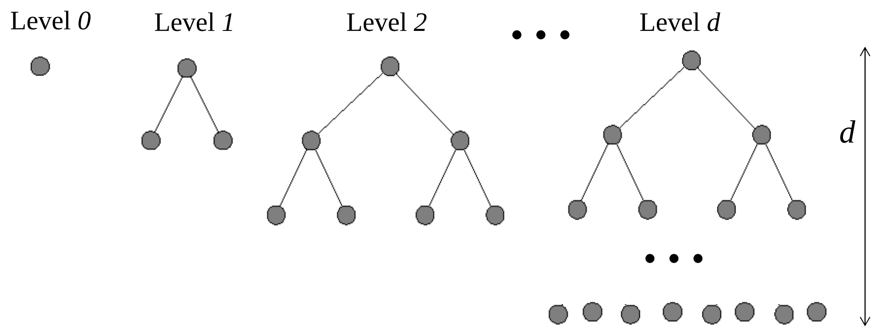
Cutoff depth = 2



Properties of IDA

- **Completeness:** **Yes.** The solution is reached if it exists.
  (the same as BFS when limit is always increased by 1)
- **Optimality:** **Yes**, for the shortest path.
  (the same as BFS)
- **Time complexity:**
  ?
- **Memory (space) complexity:**
  ?

IDA – time complexity



$O(1)$ $O(b)$ $O(b^2)$ $O(b^d)$

$$\underbrace{\qquad\qquad\qquad\qquad}_{O(b^d)}$$

Properties of IDA

- **Completeness:** **Yes.** The solution is reached if it exists. (the same as BFS)
- **Optimality:** **Yes**, for the shortest path. (the same as BFS)
- **Time complexity:**

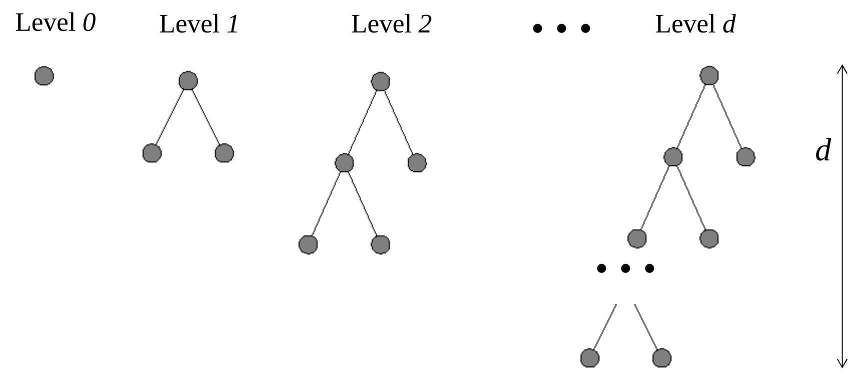
  $$O(1) + O(b^1) + O(b^2) + \ldots + O(b^d) = O(b^d)$$

  exponential in the depth of the solution *d*

  worse than BFS, but asymptotically the same
- **Memory (space) complexity:**

  ?

IDA – memory complexity



$$\underbrace{O(1) \qquad O(b) \qquad O(2b) \qquad\qquad O(db)}_{O(db)}$$

Properties of IDA

- **Completeness:** **Yes.** The solution is reached if it exists.
  (the same as BFS)
- **Optimality:** **Yes**, for the shortest path.
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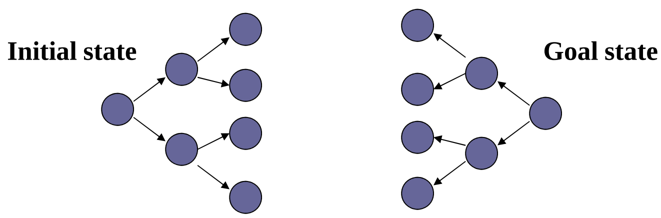
- **Time complexity:**
  $$O(1) + O(b^1) + O(b^2) + \ldots + O(b^d) = O(b^d)$$
  exponential in the depth of the solution *d*
  worse than BFS, but asymptotically the same
- **Memory (space) complexity:**
  $$O(db)$$
  much better than BFS

Bi-directional search

- In some search problems we want to find the path from the initial state to the **unique goal state** (e.g. traveler problem)
- **Bi-directional search idea:**

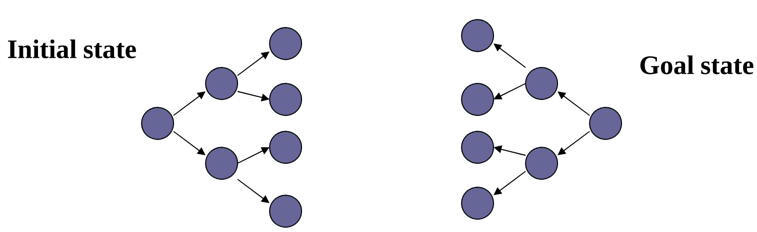


  - Search both from the initial state and the goal state;
  - Use **inverse operators** for the goal-initiated search.

Bi-directional search

Why bidirectional search? What is the benefit? Assume BFS.

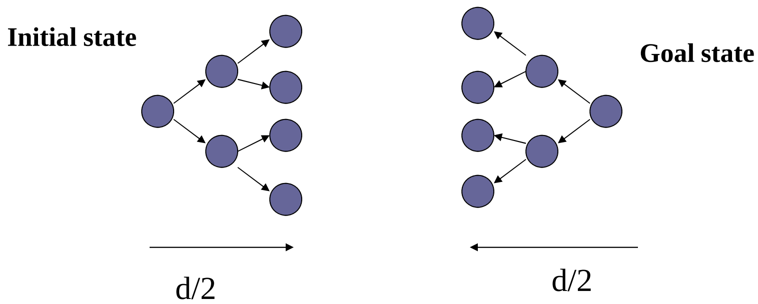
- ?



Bi-directional search

Why bidirectional search? What is the benefit? Assume BFS.

- **Cuts the depth of the search space by half**



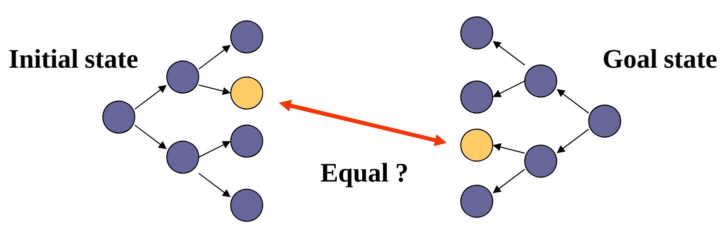
$O(b^{d/2})$ Time and memory complexity

Bi-directional search

Why bidirectional search? Assume BFS.

- **It cuts the depth of the search tree by half.**

Caveat: Merge the solutions.



- How?

Bi-directional search

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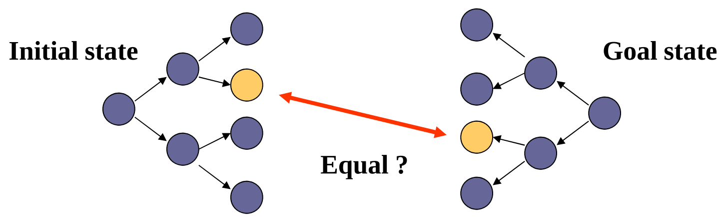
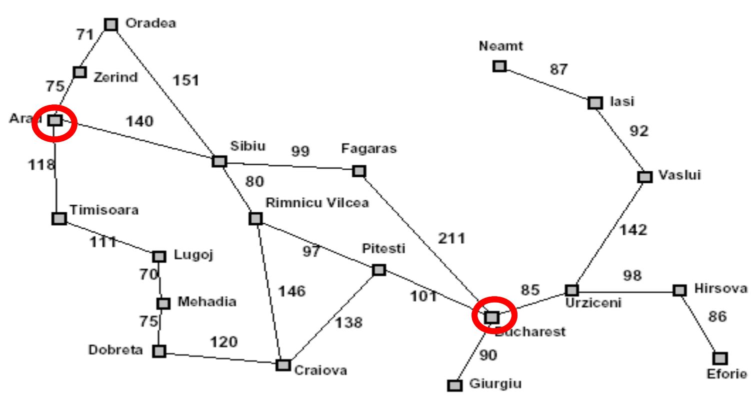


- How? The hash structure remembers the side of the tree the state was expanded first time. If the same state is reached from other side we have a solution.

Minimum cost path search

Traveler example with distances [km]



Optimal path: the shortest distance path between the initial and destination city

Searching for the minimum cost path

- **General minimum cost path-search problem:**
  - **adds weights or costs** to operators (links)
- **Search strategy:**
  - "Intelligent" expansion of the search tree should be driven by the cost of the current (partially) built path
- **Implementation:**
  - **Path cost function for node *n* :** $g(n)$
    - length of the path represented by the search tree branch starting at the root of the tree (initial state) to n
  - **Search strategy:**
    - Expand the leaf node with the minimum $g(n)$ first
    - Can be implemented by the priority queue

Searching for the minimum cost path

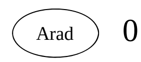
- The basic algorithm for finding the minimum cost path:
  - **Dijkstra's shortest path**
- In AI, the strategy goes under the name
  - **Uniform cost search**
- **Note:**
  - When operator costs are all equal to 1 the uniform cost search is equivalent to the breadth first search BFS

Uniform cost search

- Expand the node with the minimum path cost first
- **Implementation:** **a priority queue**

queue ⇨

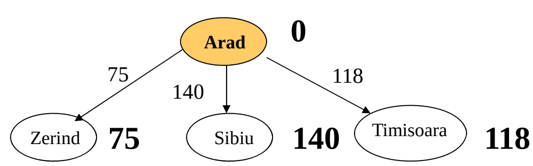
	g(n)
Arad	**0**



Uniform cost search

queue ⇨

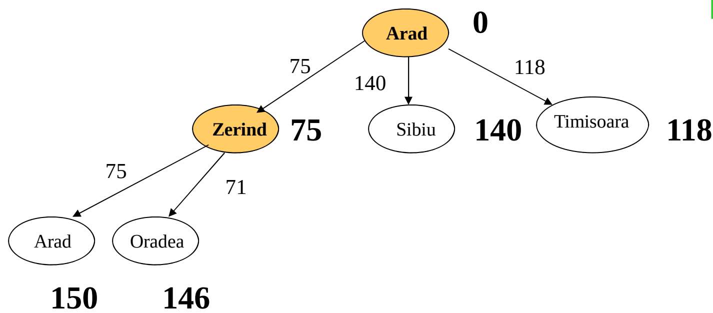
	g(n)
Zerind	**75**
Timisoara	**118**
Sibiu	**140**



Uniform cost search

queue

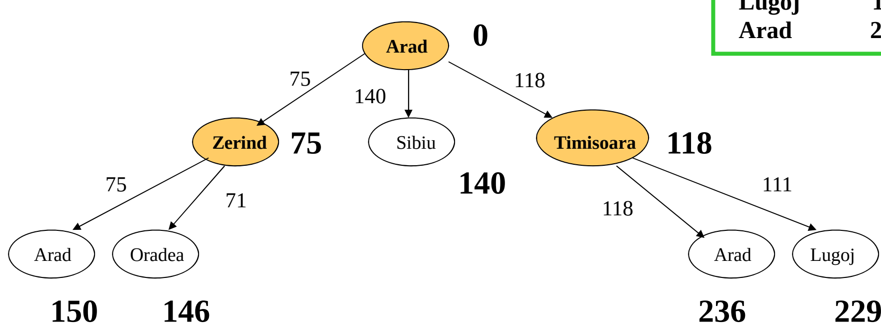
	g(n)
Timisoara	118
Sibiu	140
Oradea	**146**
Arad	**150**



Uniform cost search

queue

	g(n)
Sibiu	140
Oradea	146
Arad	150
Lugoj	**129**
Arad	**236**



Properties of the uniform cost search

- **Completeness: Yes**, assuming that operator costs are non-negative (the cost of path never decreases)

$$g(n) \leq g(\text{successor } (n))$$

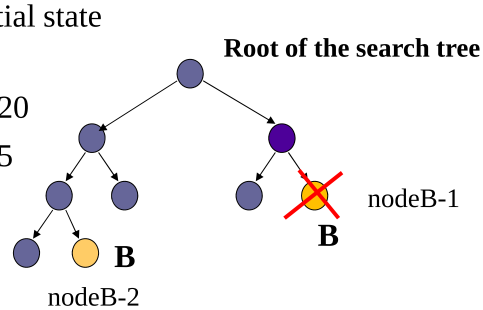
- **Optimality: Yes.** Returns the least-cost path.
- **Time complexity:**
  number of nodes with the cost *g(n)* smaller than the optimal cost
- **Memory (space) complexity:**
  number of nodes with the cost *g(n)* smaller than the optimal cost

Elimination of state repeats

Idea: A node is redundant and can be eliminated if there is another node with exactly the same state and a shorter path from the initial state

g(nodeB-1) =120

g(nodeB-2) =95



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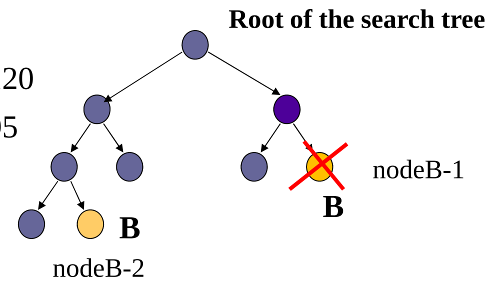

Assuming positive costs:

- If the state has already been expanded, is there a shorter path to that node ?

Elimination of state repeats

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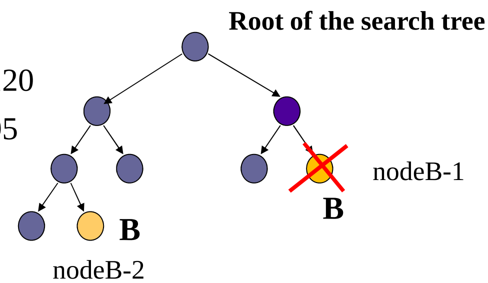

Assuming positive costs:

- If the state has already been expanded, is there a shorter path to that node ? **No !**

Implementation: Marking with the hash table