CS 1571 Introduction to AI
Lecture 3

Problem solving by searching

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A search problem

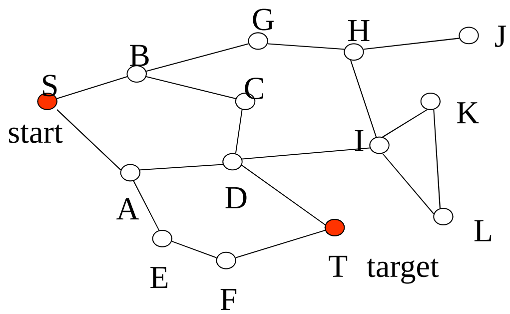
Many interesting problems in science and engineering are solved using search

A search problem is defined by:

- **A search space:**
  - The set of objects among which we search for the solution
    Examples: routes between cities, or n-queens configuration
- **A goal condition**
  - Characteristics of the object we want to find in the search space?
  - **Examples:**
    - Path between cities A and B
    - Non-attacking n-queen configuration

Graph representation of a search problem

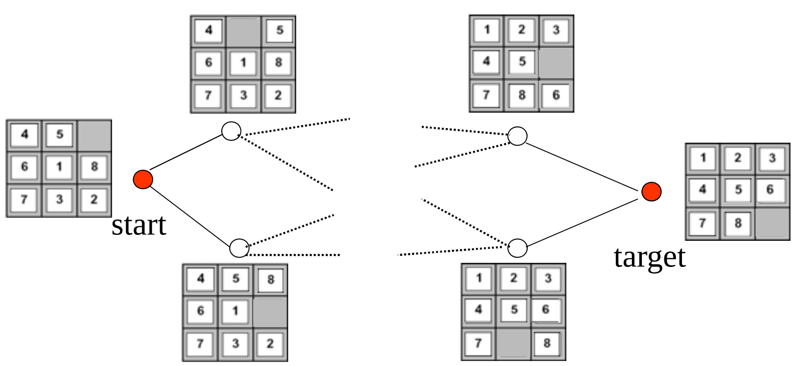
- **Search problems can be often represented using graphs**
- **Typical example: Route finding**
  - Map corresponds to the graph, nodes to cities, links valid moves via available connections
  - **Goal:** find a route (sequence of moves) in the graph from S to T



Graph search

- **Less obvious conversion:**

  Puzzle 8. Find a sequence of moves from the initial configuration to the goal configuration.
  - nodes corresponds to states of the game,
  - links to valid moves made by the player



Graph Search Problems

Search problems can be often represented as graph search problems:

- **Initial state**
  - State (configuration) we start to search from (e.g. start city, initial game position)
- **Operators**:
  - Transform one state to another (e.g. valid connections between cities, valid moves in Puzzle 8)
- **Goal condition:**
  - Defines the target state (destination, winning position)

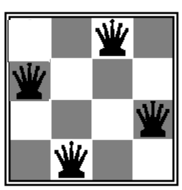
Search space is now defined indirectly through**:**

The initial state + Operators

N-queens

Some problems are easy to convert to the graph search problems

- **But some problems are harder and less intuitive**
  - Take e.g. N-queens problem**.**



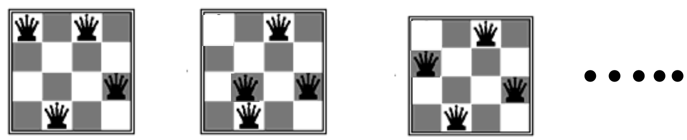
Goal configuration

- **Problem:**
  - **We look for a configuration, not a sequence of moves**
  - **No distinguished initial state, no operators (moves)**

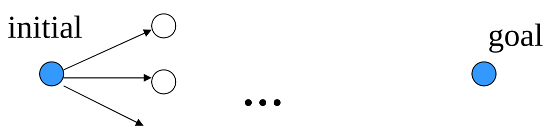
N-queens

How to choose the search space for N-queens?

- Ideas? **Search space:**
  - all configurations of N queens on the board



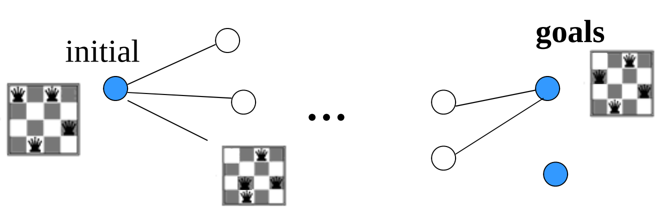
- **Can we convert it to a graph search problem?**
- We need states, operators, initial state and goal condition.



N-queens

Search space:
- all configurations of N queens on the board

- **To convert it to a graph search problem** we need states, operators, initial state and goal condition.

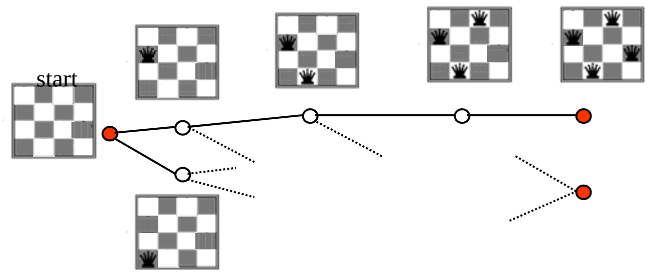


Initial state: an arbitrary N-queen configuration
Operators (moves): change a position of one queen

N-queens

An alternative way to formulate the N-queens problem as a search problem:

- **Search space:** configurations of 0,1,2, … N queens
- **Graph search:**
  - States configurations of 0,1,2,…N queens
  - Operators: additions of a queen to the board
  - Initial state: no queens on the board

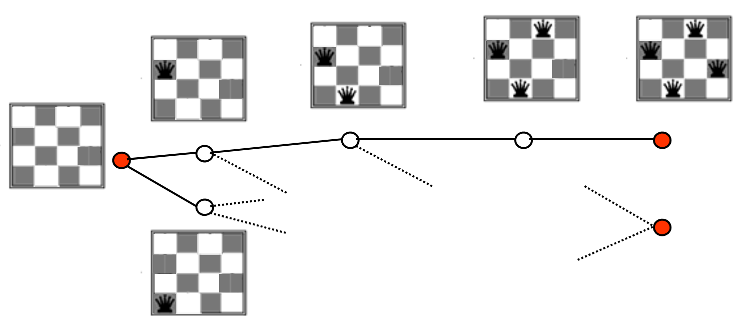


Graph search

N-queens problems

- This is a different graph search problem when compared to Puzzle 8 or Route planning:

  We want to find only the target configuration, not a path

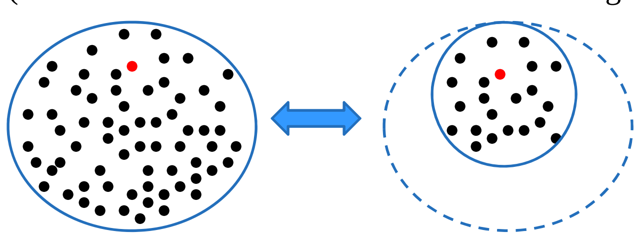


Two types of graph search problems

- **Path search**
  - Find a path between states S and T
  - **Example:** traveler problem, Puzzle 8
  - **Additional goal criterion:** minimum length (cost) path
- **Configuration search (constraint satisfaction search)**
  - Find a state (configuration) satisfying the goal condition
  - **Example:** n-queens problem
  - **Additional goal criterion:** “soft” preferences for configurations, e.g. minimum cost design

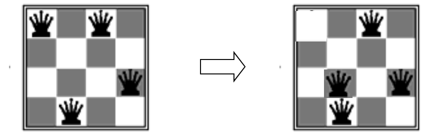
Search

- **Search (process)**
  - The process of exploration of the search space
- **The efficiency of the search depends on:**
  - **The search space and its size**
  - Method used to explore (traverse) the search space
  - Condition to test the satisfaction of the search objective (what it takes to determine I found the desired goal object



Comparison of two problem formulations

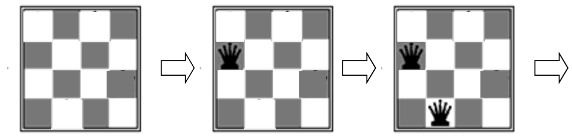
Solution 1:



Operators: switch one of the queens

$\binom{16}{4}$ - all configurations

Solution 2:

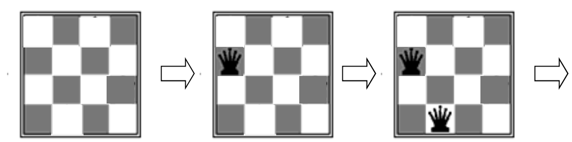


Operators: add a queen to the leftmost unoccupied column

$1 + 4 + 4^2 + 4^3 + 4^4 < 4^5$ - configurations altogether

Even better solution to the N-queens

Solution 2:



Operators: add a queen to the leftmost unoccupied column

$< 4^5$ - configurations altogether

Improved solution with a smaller search space

Operators: add a queen to the leftmost unoccupied column such that it does not attack already placed queens

$$\leq 1 + 4 + 4*3 + 4*3*2 + 4*3*2*1 = 65$$

- configurations altogether

Search

- **Search (process)**
  - The process of exploration of the search space
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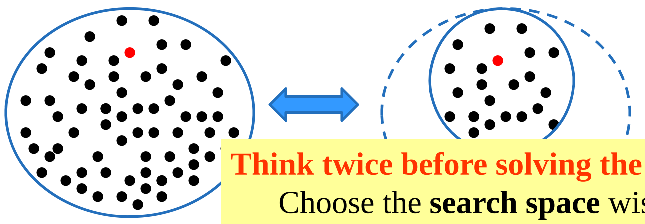

Think twice before solving the problem:
Choose the **search space** wisely

Search

- **Search (process)**
  - The process of exploration of the search space
- **The efficiency of the search depends on:**
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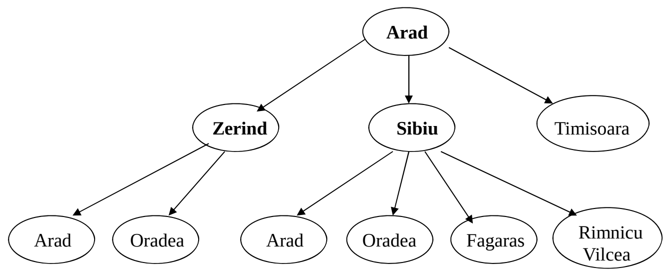
  - **Method used to explore (traverse) the search space**
  - Condition to test the satisfaction of the search objective (what it takes to determine I found the desired goal object

Search process

Exploration of the state space through successive application of operators from the initial state

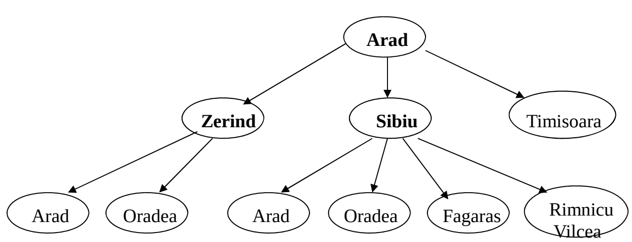
- **Search tree = structure representing the exploration trace**
  - Is built on-line during the search process
  - Branches correspond to explored paths, and leaf nodes to the exploration fringe



Search tree

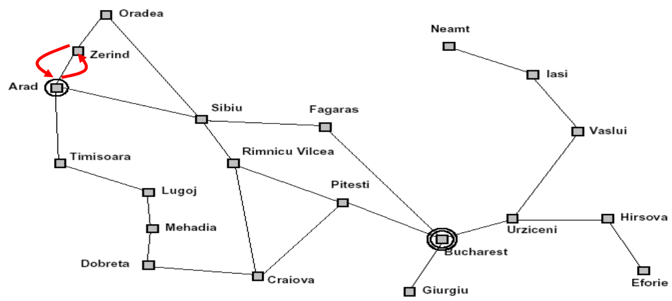
- A **search tree = (search) exploration trace**
  - **different from the graph representation of the problem**
  - states can repeat in the search tree

Graph

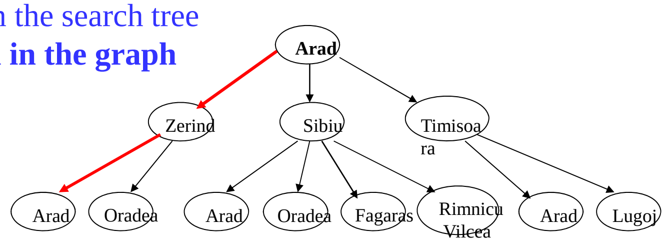


Search tree
built by exploring paths starting from city Arad

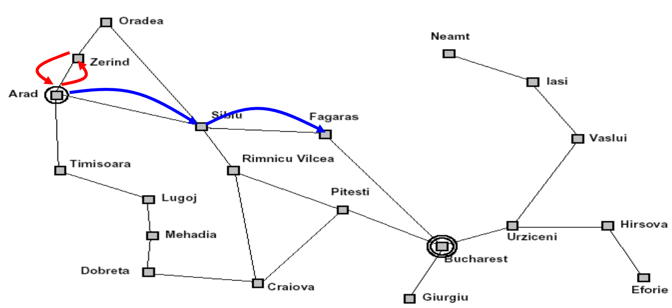
Search tree



A branch in the search tree
= **a path in the graph**



Search tree



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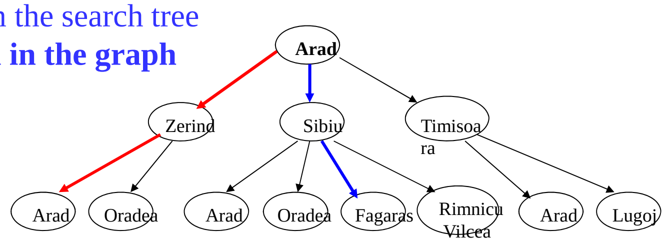

General search algorithm

General-search (*problem, strategy*)
initialize the search tree with the initial state of *problem*
loop
  if there are no candidate states to explore **return** failure
  choose a leaf node of the tree to expand next according to *strategy*
  if the node satisfies the goal condition **return** the solution
  expand the node and add all of its successors to the tree
end loop

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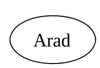

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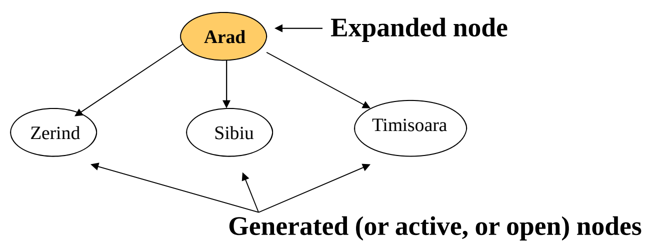

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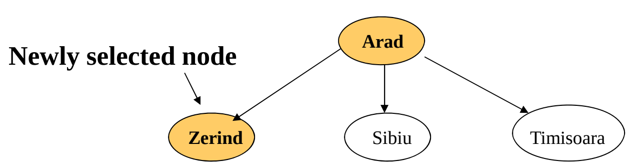

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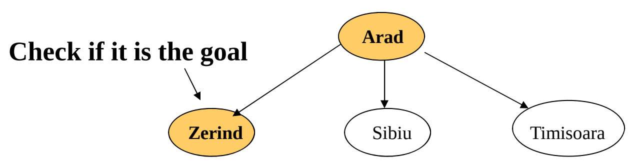

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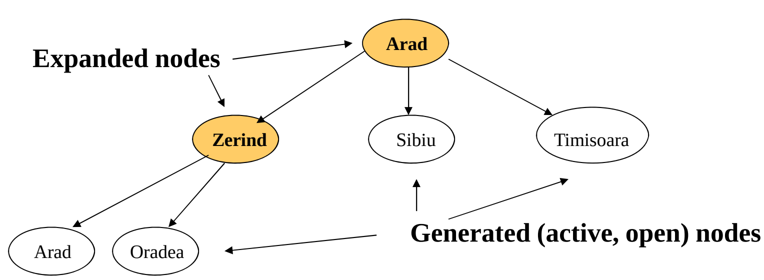

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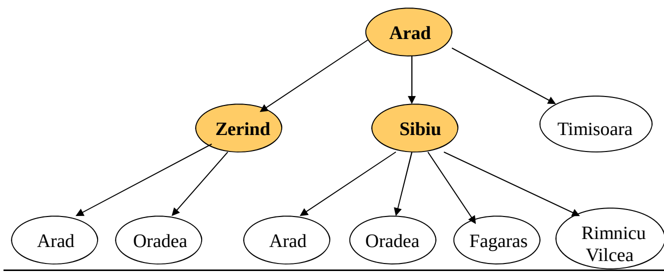

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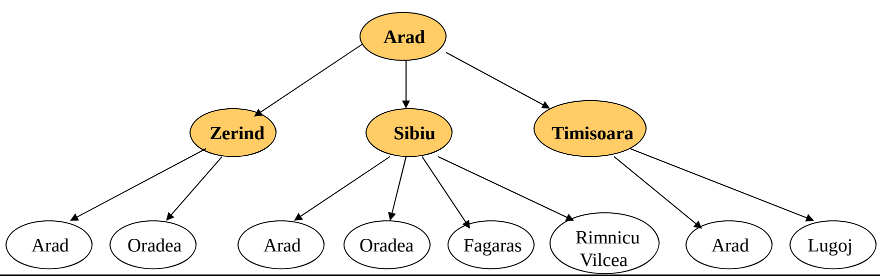

General search algorithm

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  end loop
```

- **Search methods differ in how they explore the space, that is how they choose the node to expand next !!!!!**

Implementation of search

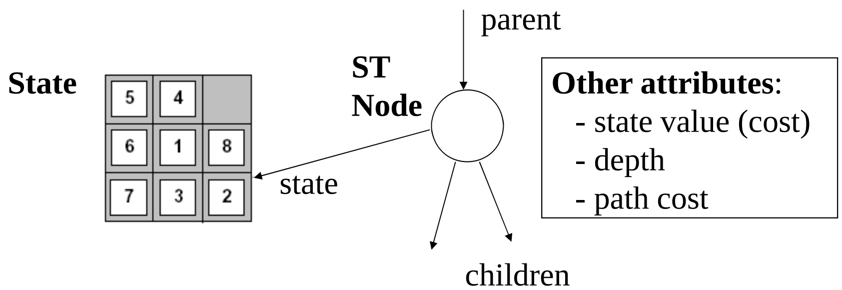
- Search methods can be implemented using the **queue** structure

```
General search (problem, Queuing-fn)
   nodes ← Make-queue(Make-node(Initial-state(problem)))
   loop
      if nodes is empty then return failure
      node ← Remove-node(nodes)
      if Goal-test(problem) applied to State(node) is satisfied then return node
      nodes ← Queuing-fn(nodes, Expand(node, Operators(node)))
   end loop
```

- Candidates are added to the queue structure (named *nodes*)
- Queuing function determines what node will be selected next

Implementation of search

- A **search tree node** is a data-structure that is a part of the search tree



- **Expand function –** applies Operators to the state represented by the search tree *node*. Together with Queuing-fn it fills the attributes.

Search

- **Search (process)**
  - The process of exploration of the search space
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Uninformed search methods

- Search techniques that rely only on the information available in the problem definition
  - **Breadth first search**
  - **Depth first search**
  - **Iterative deepening**
  - **Bi-directional search**

For the minimum cost path problem:

  - **Uniform cost search**

Search methods

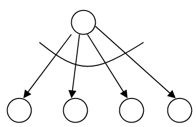
Properties of search methods :

- **Completeness.**
  - Does the method find the solution if it exists?
- **Optimality.**
  - Is the solution returned by the algorithm optimal? Does it give a minimum length path?
- **Space and time complexity.**
  - How much time it takes to find the solution?
  - How much memory is needed to do this?

Parameters to measure complexities.

- **Space and time complexity.**
  - **Complexity** is measured in terms of the following tree parameters:
    - b – **maximum branching factor**
    - d – **depth of the optimal solution**
    - m – **maximum depth of the state space**

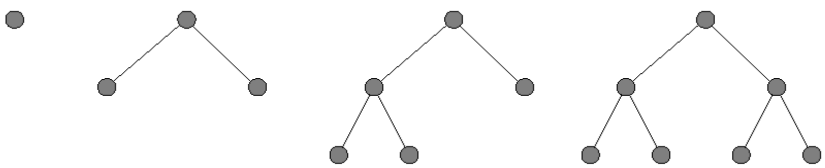
Branching factor

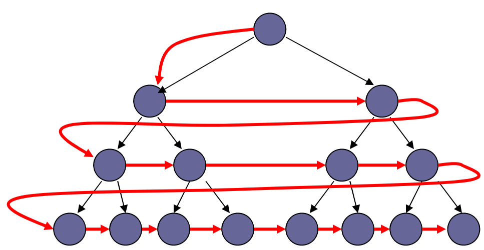


The number of applicable operators

Breadth first search (BFS)

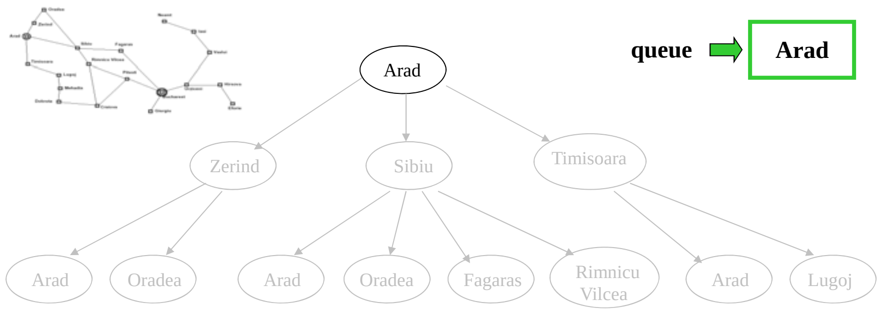
- **The shallowest node is expanded first**





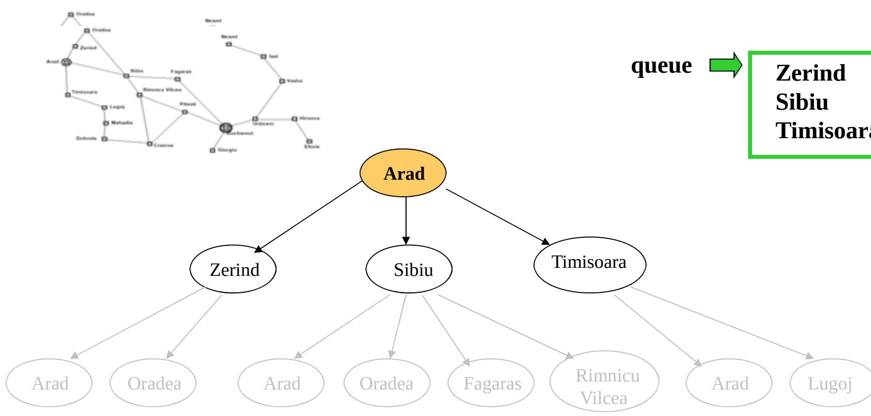
Breadth-first search

- **Expand the shallowest node first**
- Implementation: put successors to the end of the queue (FIFO)



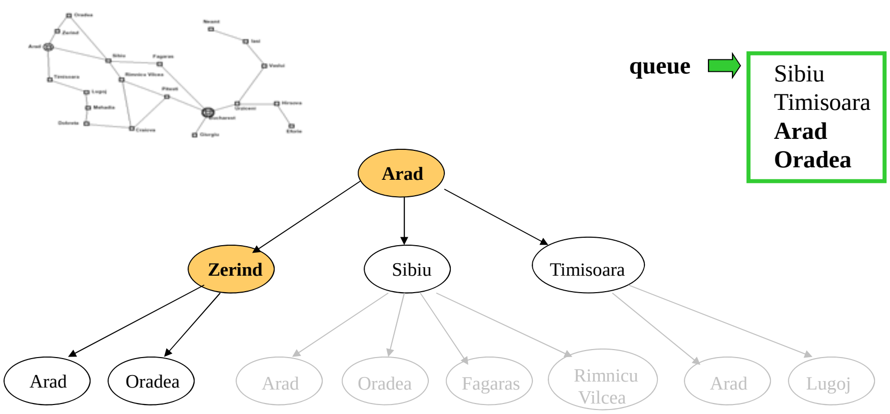
queue ➡ **Arad**

Breadth-first search



queue ➡ **Zerind Sibiu Timisoara**

Breadth-first search



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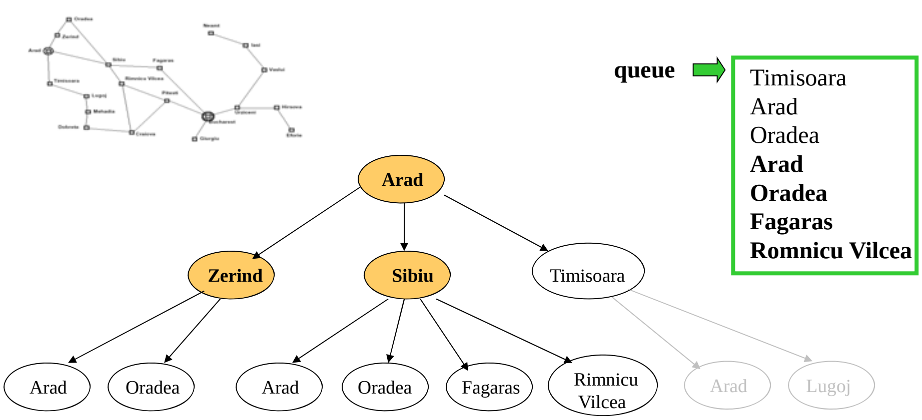

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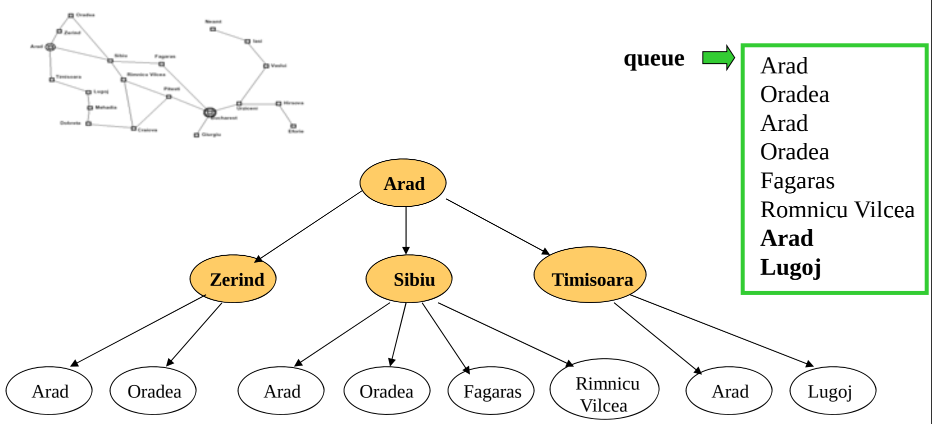
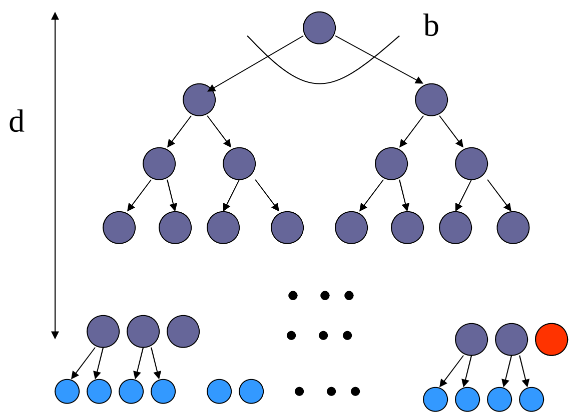


Properties of breadth-first search

- **Completeness:** **Yes.** The solution is reached if it exists.
- **Optimality:** **Yes**, for the shortest path.
- **Time complexity: ?**
- **Memory (space) complexity: ?**

BFS – time complexity



depth	number of nodes
0	1
1	$2^1=2$
2	$2^2=4$
3	$2^3=8$
d	2^d (b^d)
d+1	2^{d+1} (b^{d+1})

Expanded nodes: $O(b^d)$

Total nodes: $O(b^{d+1})$

Properties of breadth-first search

- **Completeness:** **Yes.** The solution is reached if it exists.
- **Optimality:** **Yes**, for the shortest path.
- **Time complexity:**

$$1+b+b^2+\ldots+b^d = O(b^d)$$

exponential in the depth of the solution *d*

- **Memory (space) complexity: ?**