Decision making in the presence of uncertainty

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Decision-making in the presence of uncertainty

• Computing the probability of some event may not be our ultimate goal
• Instead we are often interested in making decisions about our future actions so that we satisfy some goals
• Example: medicine
  – Diagnosis is typically only the first step
  – The ultimate goal is to manage the patient in the best possible way. Typically many options available:
    • Surgery, medication, collect the new info (lab test)
    • There is an uncertainty in the outcomes of these procedures: patient can be improve, get worse or even die as a result of different management choices.
Decision-making in the presence of uncertainty

Main issues:
• How to model the decision process with uncertain outcomes in the computer?
• How to make decisions about actions in the presence of uncertainty?

The field of **decision-making** studies ways of making decisions in the presence of uncertainty.

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Decision making example.

Assume we want to invest $100 for 6 months
• **We have 4 choices:**
  1. Invest in Stock 1
  2. Invest in Stock 2
  3. Put money in bank
  4. Keep money at home

Stock 1 value can go **up** or **down**:
- **Up:** with probability 0.6
- **Down:** with probability 0.4
Decision making example.

Assume we want to invest $100 for 6 months

- We have 4 choices:
  1. Invest in Stock 1
  2. Invest in Stock 2
  3. Put money in bank
  4. Keep money at home

### Monetary Outcomes for up and down states

- Stock 1 value can go **up** or **down**:
  - **Up**: with probability 0.6
  - **Down**: with probability 0.4

### Monetary outcomes for different states

- **Up**:
  - Stock 1: 110
  - Stock 2: 140
  - Bank: 80
  - Home: 101

- **Down**:
  - Stock 1: 90
  - Stock 2: 80
  - Bank: 80
  - Home: 100
Decision making example.

We need to make a choice whether to invest in Stock 1 or 2, put money into bank or keep them at home. But how?

<table>
<thead>
<tr>
<th></th>
<th>Stock 1</th>
<th>Stock 2</th>
<th>Bank</th>
<th>Home</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monetary outcomes for different scenarios</td>
<td>(up)</td>
<td>(down)</td>
<td>(up)</td>
<td>(down)</td>
</tr>
<tr>
<td>Stock 1</td>
<td>0.6</td>
<td>0.4</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>Stock 2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Bank</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Home</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Decision making example.

Assume a simplified problem with the Bank and Home choices only.

The result is guaranteed – the outcome is deterministic

What is the rational choice assuming our goal is to make money?
Decision making. Deterministic outcome.

Assume a simplified problem with the Bank and Home choices only.
These choices are deterministic.

Our goal is to make money. What is the rational choice?
Answer: Put money into the bank. The choice is always strictly better in terms of the outcome.

But what to do if we have uncertain outcomes?

Decision making. Stochastic outcome

- How to quantify the goodness of the stochastic outcome?
We want to compare it to deterministic and other stochastic outcomes.

?
Decision making. Stochastic outcome

- How to quantify the goodness of the stochastic outcome?
  We want to compare it to deterministic and other stochastic outcomes.

Idea: Use the expected value of the outcome

Expected value

- Let $X$ be a random variable representing the monetary outcome with a discrete set of values $\Omega_X$.
- **Expected value** of $X$ is:
  $$E(X) = \sum_{x \in \Omega_X} xP(X = x)$$

**Intuition:** Expected value summarizes all stochastic outcomes into a single quantity.

- **Example:**

  What is the expected value of the outcome of Stock 1 option?
Expected value

- Let $X$ be a random variable representing the monetary outcome with a discrete set of values $\Omega_X$.
- **Expected value** of $X$ is:
  \[
  E(X) = \sum_{x \in \Omega_X} x P(X = x)
  \]
- **Expected value** summarizes all stochastic outcomes into a single quantity

- Example:
  
  ![Diagram of expected value calculation for Stock 1]

  Expected value for the outcome of the Stock 1 option is:
  \[
  0.6 \times 110 + 0.4 \times 90 = 66 + 36 = 102
  \]

Expected values

**Investing $100 for 6 months**

- Stock 1
  - **Up**: 0.6, 110
  - **Down**: 0.4, 90

- Stock 2
  - **Up**: 0.4, 140
  - **Down**: 0.6, 80

- Bank
  - **Up**: 1.0, 101
  - **Down**: 1.0, 100

  \[
  0.6 \times 110 + 0.4 \times 90 = 66 + 36 = 102
  \]
Expected values

Investing $100 for 6 months

Stock 1

Stock 2

Bank

Home

$102 = 0.6 \times 110 + 0.4 \times 90 = 66 + 36 = 102$

$104 = 0.4 \times 140 + 0.6 \times 80 = 56 + 48 = 104$

$101 = 1.0 \times 101$

$100 = 1.0 \times 100$

Expected values

Investing $100 for 6 months

Stock 1

Stock 2

Bank

Home

$102 = 0.6 \times 110 + 0.4 \times 90 = 66 + 36 = 102$

$104 = 0.4 \times 140 + 0.6 \times 80 = 56 + 48 = 104$

$101 = 1.0 \times 101$

$100 = 1.0 \times 100$
Expected values

Investing $100 for 6 months

- Stock 1: 102
  - 0.6 (up) 110
  - 0.4 (down) 90
  - $0.6 \times 110 + 0.4 \times 90 = 66 + 36 = 102$

- Stock 2: 104
  - 0.4 (up) 140
  - 0.6 (down) 80
  - $0.4 \times 140 + 0.6 \times 80 = 56 + 48 = 104$

- Bank: 101
  - 1.0
  - $1.0 \times 101 = 101$

- Home: 100
  - 1.0

Expected values

Investing $100 for 6 months

- Stock 1: 102
  - 0.6 (up) 110
  - 0.4 (down) 90
  - $0.6 \times 110 + 0.4 \times 90 = 66 + 36 = 102$

- Stock 2: 104
  - 0.4 (up) 140
  - 0.6 (down) 80
  - $0.4 \times 140 + 0.6 \times 80 = 56 + 48 = 104$

- Bank: 101
  - 1.0
  - $1.0 \times 101 = 101$

- Home: 100
  - 1.0

?
Expected values

Investing $100 for 6 months

<table>
<thead>
<tr>
<th>Action</th>
<th>Stock 1</th>
<th>Stock 2</th>
<th>Bank</th>
<th>Home</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected</td>
<td>110</td>
<td>140</td>
<td>101</td>
<td>100</td>
</tr>
<tr>
<td>Probability</td>
<td>0.6</td>
<td>0.4</td>
<td>0.6</td>
<td>1.0</td>
</tr>
</tbody>
</table>

\[0.6 \times 110 + 0.4 \times 90 = 66 + 36 = 102\]
\[0.4 \times 140 + 0.6 \times 80 = 56 + 48 = 104\]
\[1.0 \times 101 = 101\]
\[1.0 \times 100 = 100\]

Selection based on expected values

The optimal action is the option that maximizes the expected outcome:

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\[0.6 \times 110 + 0.4 \times 90 = 66 + 36 = 102\]
\[0.4 \times 140 + 0.6 \times 80 = 56 + 48 = 104\]
\[1.0 \times 101 = 101\]
\[1.0 \times 100 = 100\]
Relation to the game search

- Game search: minimax algorithm
  - considers the rational opponent and its best move
- Decision making: maximizes the expectation
  - play against the nature – a stochastic non-malicious “opponent”

\[
\begin{array}{c}
\text{Stock 1} \\
0.6 \\
102 \\
0.4 \\
104 \\
\text{Stock 2} \\
0.4 \\
101 \\
0.6 \\
100 \\
\text{Bank} \\
1.0 \\
101 \\
1.0 \\
100 \\
\text{Home} \\
\end{array}
\]

\[
\begin{array}{c}
\uparrow \\
\downarrow \\
\uparrow \\
\downarrow \\
\end{array}
\]

(Stochastic) Decision tree

- Decision tree:

\[
\begin{array}{c}
\text{Stock 1} \\
0.6 \\
102 \\
0.4 \\
104 \\
\text{Stock 2} \\
0.4 \\
101 \\
0.6 \\
100 \\
\text{Bank} \\
1.0 \\
101 \\
1.0 \\
100 \\
\text{Home} \\
\end{array}
\]

- decision node
- chance node
- outcome (value) node
Sequential (multi-step) problems

The decision tree can be build to capture multi-step decision problems:
• Choose an action
• Observe the stochastic outcome
• And repeat

How to make decisions for multi-step problems?
• Start from the leaves of the decision tree (outcome nodes)
• Compute expectations at chance nodes
• Maximize at the decision nodes
Algorithm is sometimes called **expectimax**

Multi-step problem example

Assume:
• Two investment periods
• Two actions: stock and bank

```
Stock   Bank
  0.5   0.5
  0.4   0.5
  0.6   0.5

Stock   Bank
  0.5   0.5
  0.5   0.5
  1.0   1.0
```

```
200 130 60 90 140 100 125 130 60 90 140 80 105
```
Multi-step problem example

Assume:

• Two investment periods
• Two actions: stock and bank

### Multi-step problem example

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Multi-step problem example

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- Two investment periods
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![Diagram](attachment:image.png)
Multi-step problem example

Assume:
- Two investment periods
- Two actions: stock and bank

```
<table>
<thead>
<tr>
<th></th>
<th>Stock</th>
<th>Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>117</td>
<td>110</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>(up)</td>
<td>(down)</td>
</tr>
<tr>
<td>2nd</td>
<td>150</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>(down)</td>
<td>(up)</td>
</tr>
</tbody>
</table>
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