Knowledge-based agent

Knowledge base
Inference engine

- Knowledge base (KB):
  - Knowledge that describe facts about the world in some formal (representational) language
    - Domain specific
- Inference engine:
  - A set of procedures that use the representational language to infer new facts from known ones or answer a variety of KB queries. Inferences typically require search.
    - Domain independent
Example: MYCIN

• MYCIN: an expert system for diagnosis of bacterial infections
• Knowledge base represents
  – Facts about a specific patient case
  – Rules describing relations between entities in the bacterial infection domain

<table>
<thead>
<tr>
<th>If</th>
<th>Then</th>
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<tbody>
<tr>
<td>1. The stain of the organism is gram-positive, and</td>
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<td>2. The morphology of the organism is coccus, and</td>
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<td>3. The growth conformation of the organism is chains</td>
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<tr>
<td>Then the identity of the organism is streptococcus</td>
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</tbody>
</table>

• Inference engine:
  – manipulates the facts and known relations to answer diagnostic queries (consistent with findings and rules)

Knowledge representation

• Objective: express the knowledge about the world in a computer-tractable form
• Knowledge representation languages (KRLs)
  Key aspects:
  – Syntax: describes how sentences in KRL are formed in the language
  – Semantics: describes the meaning of sentences, what is it the sentence refers to in the real world
  – Computational aspect: describes how sentences and objects in KRL are manipulated in concordance with semantic conventions

Many KB systems rely on and implement some variant of logic
Logic

A formal language for expressing knowledge and for making logical inferences

Defined by:

- **A set of sentences**: A sentence is constructed from a set of primitives according to **syntactic rules**
- **A set of interpretations**: An interpretation \( I \) gives a semantic to primitives. It associates primitives with objects or values
  - \( I: \) primitives \( \rightarrow \) objects/values
- **The valuation (meaning) function** \( V: \)
  - Assigns a value (typically the truth value) to a given sentence under some interpretation
  
  \[ V : \text{sentence} \times \text{interpretation} \rightarrow \{ \text{True}, \text{False} \} \]

---

Propositional logic

- **The simplest logic**

- **Definition:**
  - A proposition is a statement that is either true or false.

- **Examples:**
  - Pitt is located in the Oakland section of Pittsburgh.
    - \( (T) \)
Propositional logic

• The simplest logic

• **Definition:**
  – A proposition is a statement that is either true or false.

• Examples:
  – Pitt is located in the Oakland section of Pittsburgh.
    • (T)
  – $5 + 2 = 8$.
    • (F)
  – It is raining today.
    • ?
Propositional logic

• The simplest logic

• **Definition:**
  – A proposition is a statement that is either true or false.

• Examples:
  – Pitt is located in the Oakland section of Pittsburgh.
    • (T)
  – $5 + 2 = 8$.
    • (F)
  – It is raining today.
    • (either T or F)

• Examples (cont.):
  – How are you?
    • ?
Propositional logic

- Examples (cont.):
  - How are you?
    - a question is not a proposition
  - $x + 5 = 3$
    - ?
  - $2$ is a prime number.
    - ?
Propositional logic

- Examples (cont.):
  - How are you?
    - a question is not a proposition
  - $x + 5 = 3$
    - since $x$ is not specified, neither true nor false
  - 2 is a prime number.
    - (T)

Propositional logic. Syntax

- Formally propositional logic $P$:
  - Is defined by Syntax+interpretation+semantics of $P$

Syntax:
- Symbols (alphabet) in $P$:
  - Constants: True, False
  - Propositional symbols
    - Examples:
      - $P$
      - Pitt is located in the Oakland section of Pittsburgh, etc.
    - A set of connectives:
      - $\neg, \land, \lor, \rightarrow, \leftrightarrow$
Propositional logic. Syntax

Sentences in the propositional logic:

- **Atomic sentences:**
  - Constructed from constants and propositional symbols
  - True, False are (atomic) sentences
  - $P, Q$ or Light in the room is on, It rains outside are (atomic) sentences

- **Composite sentences:**
  - Constructed from valid sentences via logical connectives
  - If $A, B$ are sentences then
    - $\neg A$ ( $A \land B$ ) ( $A \lor B$ ) ( $A \Rightarrow B$ ) ( $A \Leftrightarrow B$ )
    - or ( $A \lor B$ ) $\land$ ( $A \lor \neg B$ )
    - are sentences

Propositional logic. Semantics.

The semantic gives the meaning to sentences.

the semantics in the propositional logic is defined by:

1. **Interpretation of propositional symbols and constants**
   - Semantics of atomic sentences
2. **Through the meaning of logical connectives**
   - Meaning (semantics) of composite sentences
Semantic: propositional symbols

A propositional symbol

- a statement about the world that is either true or false

Examples:
- Pitt is located in the Oakland section of Pittsburgh
- It rains outside
- Light in the room is on

- An interpretation maps symbols to one of the two values: True (T), or False (F), depending on whether the symbol is satisfied in the world

\[ I: \text{Light in the room is on} \rightarrow \text{True}, \quad \text{It rains outside} \rightarrow \text{False} \]
\[ I': \text{Light in the room is on} \rightarrow \text{False}, \quad \text{It rains outside} \rightarrow \text{False} \]

The meaning (value) of the propositional symbol for a specific interpretation is given by its interpretation

\[ V(\text{Light in the room is on}, I) = \text{True} \]
\[ V(\text{It rains outside}, I) = \text{False} \]

\[ V(\text{Light in the room is on}, I') = \text{False} \]

<table>
<thead>
<tr>
<th>Interpretations</th>
<th>Meanings (values)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light in the room is on</td>
<td>It rains outside</td>
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<tr>
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</table>
Semantics: constants

- **The meaning (truth) of constants:**
  - True and False constants are always (under any interpretation) assigned the corresponding *True, False* value

\[
\begin{align*}
V(\text{True}, I) &= \text{True} \\
V(\text{False}, I) &= \text{False}
\end{align*}
\]

For any interpretation \( I \)

Semantics: composite sentences.

- **The meaning (truth value) of complex propositional sentences.**
  - Determined using the standard rules of logic:

<table>
<thead>
<tr>
<th></th>
<th>( P )</th>
<th>( Q )</th>
<th>( \neg P )</th>
<th>( P \land Q )</th>
<th>( P \lor Q )</th>
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</table>
Translation

Translation of English sentences to propositional logic:
(1) identify atomic sentences that are propositions
(2) Use logical connectives to translate more complex composite sentences that consist of many atomic sentences

Assume the following sentence:
• It is not sunny this afternoon and it is colder than yesterday.

Atomic sentences:
• p = It is sunny this afternoon
• q = it is colder than yesterday

Translation: \( \neg p \land q \)

Translation

Assume the following sentences:
• It is not sunny this afternoon and it is colder than yesterday.
• We will go swimming only if it is sunny.
• If we do not go swimming then we will take a canoe trip.
• If we take a canoe trip, then we will be home by sunset.

Denote:
• p = It is sunny this afternoon
• q = it is colder than yesterday
• r = We will go swimming
• s = we will take a canoe trip
• t = We will be home by sunset
Translation

Assume the following sentences:
- It is not sunny this afternoon and it is colder than yesterday. \( \neg p \land q \)
- We will go swimming only if it is sunny. \( r \rightarrow p \)
- If we do not go swimming then we will take a canoe trip. \( \neg r \rightarrow s \)
- If we take a canoe trip, then we will be home by sunset. \( s \rightarrow t \)

Denote:
- \( p \) = It is sunny this afternoon
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Translation

Assume the following sentences:
- It is not sunny this afternoon and it is colder than yesterday. \( \neg p \land q \)
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Denote:
- \( p \) = It is sunny this afternoon
- \( q \) = it is colder than yesterday
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- \( t \) = We will be home by sunset
Contradiction and Tautology

Some composite sentences may always (under any interpretation) evaluate to a single truth value:

- **Contradiction** (always *False*)
  \[ P \land \neg P \]

- **Tautology** (always *True*)
  \[ P \lor \neg P \]

\[
\begin{align*}
\neg (P \lor Q) & \iff (\neg P \land \neg Q) \\
\neg (P \land Q) & \iff (\neg P \lor \neg Q)
\end{align*}
\]

DeMorgan’s Laws

Model, validity and satisfiability

- An interpretation is a model for a set of sentences if it assigns true to each sentence in the set.

- **Example 1:**
  - **Primitives:** \(P, Q\)  
  - **Sentence:** \(P \lor Q\)  
  - **Interpretations:**  
    - \(P \rightarrow True, Q \rightarrow True\)  
    - **Model?**  

<table>
<thead>
<tr>
<th>(P)</th>
<th>(Q)</th>
<th>(P \lor Q)</th>
<th>((P \lor Q) \land \neg Q)</th>
<th>((P \lor Q) \land \neg Q) (\Rightarrow P)</th>
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<tr>
<td>True</td>
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Model, validity and satisfiability

• An interpretation is a model for a set of sentences if it assigns true to each sentence in the set.

• Example 1:
  • Primitives: P,Q
  • Sentence: \( P \vee Q \)
  - Interpretations: Model?
    • \( P \to True, Q \to True \) Yes
    • \( P \to True, Q \to False \)
    • \( P \to False, Q \to False \)

\[
\begin{array}{|c|c|c|c|c|}
\hline
P & Q & P \vee Q & (P \vee Q) \land \neg Q & ((P \vee Q) \land \neg Q) \Rightarrow P \\
\hline
True & True & True & False & True \\
True & False & True & True & True \\
False & True & True & False & True \\
False & False & False & False & False \\
\hline
\end{array}
\]
Model, validity and satisfiability

- An interpretation is a model for a set of sentences if it assigns true to each sentence in the set.

- **Example 1:**
  - Primitives: P, Q
  - Sentence: \( P \lor Q \)
  - Interpretations:
    - \( P \Rightarrow True, Q \Rightarrow True \) Yes
    - \( P \Rightarrow True, Q \Rightarrow False \) Yes
    - \( P \Rightarrow False, Q \Rightarrow False \) No

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Model, validity and satisfiability

- An interpretation is a model for a set of sentences if it assigns true to each sentence in the set.

- **Example 2:**
  - Primitives: P, Q
  - Sentences:
    - \( P \lor Q \)
    - \(( P \lor Q ) \land \lnot Q \)
    - \(( ( P \lor Q ) \land \lnot Q ) \Rightarrow P \)

- Is there a model?

<table>
<thead>
<tr>
<th>P</th>
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Model, validity and satisfiability

- An interpretation **is a model for a set of sentences** if it assigns true to each sentence in the set.

- **Example 2:**
  - Primitives: $P, Q$
  - Sentences: $P \lor Q$
    
    $(P \lor Q) \land \neg Q$
    
    $((P \lor Q) \land \neg Q) \Rightarrow P$

  - Is there a model? **Yes** $P \Rightarrow True$ $Q \Rightarrow False$

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Model, validity and satisfiability

- An interpretation **is a model for a set of sentences** if it assigns true to each sentence in the set.

- A sentence is **satisfiable** if it has a model;
  - There is at least one interpretation under which the sentence can evaluate to True

- **Example:**
  - Sentence: $(P \lor Q) \land \neg Q$

  - Satisfiable?

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Model, validity and satisfiability

• An interpretation is a model for a set of sentences if it assigns true to each sentence in the set.
• A sentence is satisfiable if it has a model;
  – There is at least one interpretation under which the sentence can evaluate to True
• Example:
  – Sentence: \((P \lor Q) \land \lnot Q\)
  – Satisfiable? Yes, True for \(P \rightarrow True, Q \rightarrow False\)

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Model, validity and satisfiability

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• A sentence is satisfiable if it has a model;
  – There is at least one interpretation under which the sentence can evaluate to True.
• A sentence is valid if it is True in all interpretations
  – i.e., if its negation is not satisfiable (leads to contradiction)

valid sentence?

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Entailment

- Entailment reflects the relation of one fact in the world following from the others

- Entailment $KB \models \alpha$
- Knowledge base KB entails sentence $\alpha$ if and only if $\alpha$ is true in all worlds where KB is true
Sound and complete inference

**Inference** is a process by which new sentences are derived from existing sentences in the KB
- the inference process is implemented on a computer

Assume an **inference procedure** $i$ that
- derives a sentence $\alpha$ from the KB: $KB \vdash_i \alpha$

**Properties of the inference procedure in terms of entailment**
- **Soundness**: An inference procedure is **sound**
  
  If $KB \vdash_i \alpha$ then it is true that $KB \models \alpha$

- **Completeness**: An inference procedure is **complete**
  
  If $KB \models \alpha$ then it is true that $KB \vdash_i \alpha$

Logical inference problem

**Logical inference problem:**
- **Given:**
  - a knowledge base KB (a set of sentences) and
  - a sentence $\alpha$ (called a **theorem**),
- **Does a KB semantically entail $\alpha$?** $KB \models \alpha$ ?

  In other words: In all interpretations in which sentences in the KB are true, is also $\alpha$ true?

  **Question:** Is there a procedure (program) that can decide this problem in a finite number of steps?

  **Answer:** Yes. Logical inference problem for the propositional logic is **decidable**.
Solving logical inference problem

In the following:

**How to design the procedure that answers:**

\[ KB \models \alpha \] ?

**Three approaches:**

- Truth-table approach
- Inference rules
- Conversion to the inverse SAT problem
  - Resolution-refutation

---

**Truth-table approach**

**Problem:** \[ KB \models \alpha \] ?

- We need to check all possible interpretations for which the KB is true (models of KB) whether \( \alpha \) is true for each of them

**Truth table:**

- enumerates truth values of sentences for all possible interpretations (assignments of True/False values to propositional symbols)

**Example:**

<table>
<thead>
<tr>
<th>( P )</th>
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<th>( P \iff Q )</th>
<th>( (P \lor \neg Q) \land \neg Q )</th>
</tr>
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$\alpha = |KB \models Q \iff \neg P \lor \neg Q)$
Truth-table approach

A two steps procedure:

1. Generate table for all possible interpretations
2. Check whether the sentence $\alpha$ evaluates to true whenever $KB$ evaluates to true

Example: $KB = (A \lor C) \land (B \lor \neg C)$ $\alpha = (A \lor B)$

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>$A \lor C$</th>
<th>$(B \lor \neg C)$</th>
<th>KB</th>
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Truth-table approach

A two steps procedure:
1. Generate table for all possible interpretations
2. Check whether the sentence $\alpha$ evaluates to true whenever $KB$ evaluates to true

Example: $KB = (A \lor C) \land (B \lor \neg C)$, $\alpha = (A \lor B)$

<table>
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<tr>
<th>$A$</th>
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<th>$A \lor C$</th>
<th>$(B \lor \neg C)$</th>
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• The truth-table approach is sound and complete for the propositional logic!!
Limitations of the truth table approach.

$KB \models \alpha$?

- What is the computational complexity of the truth table approach?

$2^n$ Rows in the table has to be filled

- the truth table is exponential in the number of propositional symbols (we checked all assignments)