Propositional logic

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Knowledge representation
Knowledge-based agent

- **Knowledge base (KB):**
  - A set of sentences that describe facts about the world in some formal (representational) language
  - **Domain specific**
- **Inference engine:**
  - A set of procedures that use the representational language to infer new facts from known ones or answer a variety of KB queries. Inferences typically require search.
  - **Domain independent**

**Example: MYCIN**

- MYCIN: an expert system for diagnosis of bacterial infections
- **Knowledge base** represents
  - Facts about a specific patient case
  - Rules describing relations between entities in the bacterial infection domain

<table>
<thead>
<tr>
<th>If</th>
<th>Then</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The stain of the organism is gram-positive, and 2. The morphology of the organism is coccus, and 3. The growth conformation of the organism is chains</td>
<td>the identity of the organism is streptococcus</td>
</tr>
</tbody>
</table>

- **Inference engine:**
  - manipulates the facts and known relations to answer diagnostic queries (consistent with findings and rules)
Knowledge representation

- The objective of knowledge representation is to express the knowledge about the world in a computer-tractable form.

- Key aspects of knowledge representation languages:
  - **Syntax:** describes how sentences are formed in the language.
  - **Semantics:** describes the meaning of sentences, what is it the sentence refers to in the real world.
  - **Computational aspect:** describes how sentences and objects are manipulated in concordance with semantical conventions.

  *Many KB systems rely on some variant of logic.*

Logic

- **Logic:**
  - defines a formal language for logical reasoning.

- A tool that helps us to understand how to construct a valid argument.

- **Logic Defines:**
  - the meaning of statements
  - the rules of logical inference
Logic
A formal language for expressing knowledge and ways of reasoning.

Logic is defined by:

• A set of sentences
  – A sentence is constructed from a set of primitives according to syntax rules.

• A set of interpretations
  – An interpretation gives a semantic to primitives. It associates primitives with values.

• The valuation (meaning) function $V$
  – Assigns a value (typically the truth value) to a given sentence under some interpretation

$V : \text{sentence} \times \text{interpretation} \rightarrow \{\text{True}, \text{False}\}$

Propositional logic

• The simplest logic

• Definition:
  – A proposition is a statement that is either true or false.

• Examples:
  – Pitt is located in the Oakland section of Pittsburgh.
    • (T)
Propositional logic

• The simplest logic

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  – A proposition is a statement that is either true or false.

• Examples:
  – Pitt is located in the Oakland section of Pittsburgh.
    • (T)
  – $5 + 2 = 8$.
    • (F)
  – It is raining today.
    • ?
Propositional logic

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Propositional logic

• Examples (cont.):
  – How are you?
    • ?
Propositional logic

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  - How are you?
    - a question is not a proposition
  - \( x + 5 = 3 \)
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- Propositional logic

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  - How are you?
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  - \( x + 5 = 3 \)
    - since \( x \) is not specified, neither true nor false
  - 2 is a prime number.
    - ?
Propositional logic

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  - How are you?
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  - She is very talented.
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- There are other life forms on other planets in the universe.
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Propositional logic

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  – How are you?
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  – \( x + 5 = 3 \)
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  – 2 is a prime number.
    • (T)
  – She is very talented.
    • since she is not specified, neither true nor false
  – There are other life forms on other planets in the universe.
    • either T or F

Propositional logic. Syntax

• Formally propositional logic \( P \):
  – Is defined by Syntax+interpretation+semantics of \( P \)

Syntax:
• Symbols (alphabet) in \( P \):
  – Constants: \textit{True}, \textit{False}
  – Propositional symbols
    Examples:
    • \( P \)
    • \textit{Pitt is located in the Oakland section of Pittsburgh.},
    • \textit{It rains outside}, etc.
  – A set of connectives:
    \( \neg, \wedge, \vee, \Rightarrow, \Leftrightarrow \)
Propositional logic. Syntax

Sentences in the propositional logic:

- **Atomic sentences:**
  - **Constructed from constants and propositional symbols**
  - True, False are (atomic) sentences
  - $P \cdot Q$ or Light in the room is on, It rains outside are (atomic) sentences

- **Composite sentences:**
  - **Constructed from valid sentences via connectives**
  - If $A$, $B$ are sentences then
    
    \[
    \neg A \quad (A \land B) \quad (A \lor B) \quad (A \Rightarrow B) \quad (A \Leftrightarrow B)
    \]
    
    or \quad $(A \lor B) \land (A \lor \neg B)$
    
    are sentences

Propositional logic. Semantics.

The semantic gives the meaning to sentences.

the semantics in the propositional logic is defined by:

1. **Interpretation of propositional symbols and constants**
   - Semantics of atomic sentences

2. **Through the meaning of connectives**
   - Meaning (semantics) of composite sentences
Semantic: propositional symbols

A propositional symbol

- a statement about the world that is either true or false

Examples:
- Pitt is located in the Oakland section of Pittsburgh
- It rains outside
- Light in the room is on

- An interpretation maps symbols to one of the two values: True (T), or False (F), depending on whether the symbol is satisfied in the world

\[ I: \text{Light in the room is on} \rightarrow \text{True}, \ \text{It rains outside} \rightarrow \text{False} \]

\[ I': \text{Light in the room is on} \rightarrow \text{False}, \ \text{It rains outside} \rightarrow \text{False} \]

Semantic: propositional symbols

The meaning (value) of the propositional symbol for a specific interpretation is given by its interpretation

\[ I: \text{Light in the room is on} \rightarrow \text{True}, \ \text{It rains outside} \rightarrow \text{False} \]

\[ V(\text{Light in the room is on}, I) = \text{True} \]

\[ V(\text{It rains outside}, I) = \text{False} \]

\[ I': \text{Light in the room is on} \rightarrow \text{False}, \ \text{It rains outside} \rightarrow \text{False} \]

\[ V(\text{Light in the room is on}, I') = \text{False} \]
Semantics: constants

- The meaning (truth) of constants:
  - True and False constants are always (under any interpretation) assigned the corresponding True, False value

\[ V(\text{True}, I) = \text{True} \quad \left\{ \begin{array}{l} V(\text{False}, I) = \text{False} \end{array} \right. \]

For any interpretation $I$

Semantics: composite sentences.

- The meaning (truth value) of complex propositional sentences.
  - Determined using the standard rules of logic:

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg P$</th>
<th>$P \land Q$</th>
<th>$P \lor Q$</th>
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Translation

Assume the following sentences:
• It is not sunny this afternoon and it is colder than yesterday.
• We will go swimming only if it is sunny.
• If we do not go swimming then we will take a canoe trip.
• If we take a canoe trip, then we will be home by sunset.

Denote:
• p = It is sunny this afternoon
• q = it is colder than yesterday
• r = We will go swimming
• s= we will take a canoe trip
• t= We will be home by sunset