Adversarial search

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Game search problem

- **Game problem formulation:**
  - **Initial state:** initial board position + info whose move it is
  - **Operators:** legal moves a player can make
  - **Goal (terminal test):** determines when the game is over
  - **Utility (payoff) function:** measures the outcome of the game and its desirability

- **Search objective:**
  - find the sequence of player’s decisions (moves) maximizing its utility (payoff)
  - Consider the opponent’s moves and their utility
Game problem formulation (Tic-tac-toe)

Objectives:
• Player 1: maximize outcome
• Player 2: minimize outcome

Minimax algorithm. Example
Minimax algorithm. Example

Minimax algorithm

What assumption does the minimax algorithm make about the opponent?
Minimax algorithm

What assumption does the minimax algorithm make about the opponent?

- the opponent is rational; we assume the best opponent’s response

Complexity of the minimax algorithm

- We need to explore the complete game tree before making the decision

- Impossible for large games
  - Chess: 35 operators, game can have 50 or more moves
Solution to the complexity problem

Two solutions:
1. **Dynamic pruning of redundant branches** of the search tree
   - identify a provably suboptimal branch of the search tree before it is fully explored
   - Eliminate the suboptimal branch
   **Procedure:** Alpha-Beta pruning

2. **Early cutoff of the search tree**
   - uses imperfect minimax value estimate of non-terminal states (positions)

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Alpha beta pruning. Example

MAX

MIN

MAX

= 4

≥ 6

= 2

≤ 2

= 5

= 4

≥ 6

= 2

≤ 2

= 5

≥ 7

4 3 6 2 2 1 9 5 3 1 5 4 7 5

(nodes that were never explored !!!)
Using minimax value estimates

- **Idea:**
  - Cutoff the search tree before the terminal state is reached
  - Use imperfect estimate of the minimax value at the leaves

- **(Heuristic) evaluation function**

![Diagram](image)

Design of evaluation functions

- **Heuristic estimate** of the value for a sub-tree
- **Examples of a heuristic functions:**

  - **Material advantage in chess, checkers**
    - Gives a value to every piece on the board, its position and combines them
  
  - More general **feature-based evaluation function**
    - Typically a linear evaluation function:
      \[
      f(s) = f_1(s)w_1 + f_2(s)w_2 + \cdots + f_k(s)w_k
      \]
    
    \[f_i(s)\] - a feature of a state \(s\)
    \[w_i\] - feature weight
Further extensions to real games

- Play a restricted game: a restricted set of moves is considered under the **cutoff level** to reduce branching and improve the evaluation function
  - E.g., consider only the capture moves in chess