Problem from Section 4.1

18. a) Plugging in \( n = 2 \), we see that \( P(2) \) is the statement \( 2! < 2^2 \).

b) Since \( 2! = 2 \), this is the true statement \( 2 < 4 \).

c) The inductive hypothesis is the statement that \( k! < k^k \).

d) For the inductive step, we want to show for each \( k \geq 2 \) that \( P(k) \) implies \( P(k+1) \). In other words, we want to show that assuming the inductive hypothesis (see part (c)) we can prove that \( (k+1)! < (k+1)^{k+1} \).

e) \( (k+1)! = (k+1)k! < (k+1)k^k < (k+1)(k+1)^k = (k+1)^{k+1} \).

f) We have completed both the basis step and the inductive step, so by the principle of mathematical induction, the statement is true for every positive integer \( n \) greater than 1.

32. The statement is true for the base case, \( n = 1 \), since \( 3 \mid 3 \). Suppose that \( 3 \mid (k^3 + 2k) \). We must show that \( 3 \mid ((k + 1)^3 + 2(k + 1)) \). If we expand the expression in question, we obtain \( k^3 + 3k^2 + 3k + 1 + 2k + 2 = (k^3 + 2k) + 3(k^2 + k + 1) \). By the inductive hypothesis, 3 divides \( k^3 + 2k \), and certainly 3 divides \( 3(k^2 + k + 1) \), so 3 divides their sum, and we are done.