Exam II
(closed book)
CS 441
Spring 2005, Dr. Litman

1. Check the pages, there should be 5 (multi-part) questions.

2. Please remember to put your name below.

3. Put your initials on the bottom of each page.

4. Pace yourself!

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1. **Functions**

(a) Let \( f(n) = 2n + 1 \). Answer the following questions AND explain your reasons behind the answers.

- Is \( f \) a one-to-one function from the set of integers to the set of integers?

- Is \( f \) an onto function from the set of integers to the set of integers?

- Is \( f \) a bijection?

- What are the domain, codomain, and range of \( f \)?
(b) Suppose \( g : A \to B \) and \( f : B \to C \) where \( A = B = C = \{1,2,3,4\} \), and 
\[ g = \{(1,4),(2,1),(3,1),(4,2)\}, \text{ and } f = \{(1,3),(2,2),(3,4),(4,2)\} \]

- Find \( f \circ g \)

- Find \( g \circ f \)

- Find \( g \circ g \)

- Find \( g \circ (g \circ g) \)
2. Sequences and Summations

(a) Find the formulas that generate each of the following sequences $a_1, a_2, a_3, \ldots$

- 5, 9, 13, 17, 21, \ldots

- 1, 1/3, 1/5, 1/7, 1/9, \ldots

(b) Find the sum that generates each of:

- $\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots$

- $2 + 4 + 8 + 16 + 32 + \ldots + 2^{28}$
(c) Find the values of:

\[ \sum_{j=2}^{8} 3 \]

and

\[ \sum_{j=0}^{4} (2j + 1) \]

(d) What are the values of the terms \(a_1, a_3\) and \(a_5\) of the sequence \(a_n\), where

- \(a_n = n^2 + n\)

- \(a_n = 2\)
3. **Mathematical Induction**

(a) Suppose you wish to prove that the following is true for all positive integers \( n \) by using the Principle of Mathematical Induction:

\[
1 + 3 + 5 + \ldots + (2n - 1) = n^2.
\]

- Write \( P(1) \)
- Write \( P(72) \)
- Write \( P(73) \)
- Use \( P(72) \) to prove \( P(73) \)
- Write \( P(k) \)
- Write \( P(k+1) \)
- Use Induction to prove that \( P(n) \) is true for all positive integers \( n \).
4. Recursion

(a) Find \( f(2) \) and \( f(3) \) if \( f(n) = \frac{f(n-1)}{f(n-2)} \), \( f(0) = 2 \), \( f(1) = 5 \)

(b) Suppose that \( \{a_n\} \) is defined recursively by \( a_n = a_{n-1}^2 - 1 \) and that \( a_0 = 2 \). Find \( a_2 \) and \( a_3 \).

(c) Write a recursive definition for the function \( f(n) = an \) (using addition), where \( n \) is a positive integer and \( a \) is a real number.

(d) Give a recursive definition (with initial condition(s)) of \( \{a_n\} \) (where \( n = 1, 2, 3, \ldots \) for \( \{a_n\} = 2^n \).
5. Miscellaneous

(a) What is wrong with the following proof that every positive integer equals the next larger positive integer?

“Proof.” Let \( P(n) \) be the proposition that \( n=n+1 \). Assume that \( P(k) \) is true, so that \( k=k+1 \). Add 1 to both sides of this equation to obtain \( k+1=k+2 \). Since this is the statement \( P(k+1) \), it follows that \( P(n) \) is true for all positive integers \( n \).

(b) Does the following rule for \( g \) describe a function: \( g: \mathbb{N} \to \mathbb{N} \) where \( g(n) = \text{any integer} > n \). State yes or no, and explain.

(c) Suppose \( f: \mathbb{R} \to \mathbb{R} \) and \( g: \mathbb{R} \to \mathbb{R} \) where \( g(x)=2x+1 \) and \( g \circ f(x) = 2x+11 \). Find the rule for \( f \).
(d) Find the value of:

\[ \sum_{k=1}^{2} \sum_{j=0}^{1} (2j + 2k) \]

(e) Suppose that \( f \) is the function from the set \( \{a,b,c,d\} \) to itself with \( f(a)=d, f(b)=a, f(c)=b, f(d)=c \). Find the inverse of \( f \).