**Uncertainty: Wumpus World**

Chapter 13

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**Wumpus World**

\[
P_{ij} = true \text{ iff } [i, j] \text{ contains a pit}
\]

\[
B_{ij} = true \text{ iff } [i, j] \text{ is breezy}
\]

Include only \( B_{1,1}, B_{1,2}, B_{2,1} \) in the probability model
Specifying the probability model

The full joint distribution is $P(P_{1,1}, \ldots, P_{4,1}, B_{1,1}, B_{1,2}, B_{2,1})$

Apply product rule: $P(B_{1,1}, B_{1,2}, B_{2,1} | P_{1,1}, \ldots, P_{4,1})P(P_{1,1}, \ldots, P_{4,1})$

(Do it this way to get $P(Effect|Cause)$.)

First term: 1 if pits are adjacent to breezes, 0 otherwise

Second term: pits are placed randomly, probability 0.2 per square:

$P(P_{1,1}, \ldots, P_{4,4}) = \prod_{i,j=1,1}^{4,4} P(P_{i,j}) = 0.2^n \times 0.8^{16-n}$

for $n$ pits.

Observations and query

We know the following facts:

$b = \neg b_{1,1} \land b_{1,2} \land b_{2,1}$

$known = \neg p_{1,1} \land \neg p_{1,2} \land \neg p_{2,1}$

Query is $P(P_{1,3}|known, b)$

Define $Unknown = P_{ij}$s other than $P_{1,3}$ and $Known$

For inference by enumeration, we have

$P(P_{1,3}|known, b) = \alpha \sum_{unknown} P(P_{1,3}, unknown, known, b)$

Grows exponentially with number of squares!
Using conditional independence

Basic insight: observations are conditionally independent of other hidden squares given neighbouring hidden squares.

Define $Unknown = Fringe \cup Other$

$P(b|P_{1,3}, Known, Unknown) = P(b|P_{1,3}, Known, Fringe)$

Manipulate query into a form where we can use this!

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Using conditional independence contd.

$P(P_{1,3}|known, b) = \alpha \sum_{unknown} P(P_{1,3}, unknown, known, b)$

$= \alpha \sum_{unknown} P(b|P_{1,3}, known, unknown)P(P_{1,3}, known, unknown)$

$= \alpha \sum_{fringe} \sum_{other} P(b|known, P_{1,3}, fringe, other)P(P_{1,3}, known, fringe, other)$

$= \alpha \sum_{fringe} P(b|known, P_{1,3}, fringe) \sum_{other} P(P_{1,3}, known, fringe, other)$

$= \alpha P(known)P(P_{1,3}) \sum_{fringe} P(b|known, P_{1,3}, fringe) P(fringe) \sum_{other} P(other)$

$= \alpha' P(P_{1,3}) \sum_{fringe} P(b|known, P_{1,3}, fringe) P(fringe)$
Using conditional independence contd.

\[ P(P_{1,3} | \text{known}, b) = \alpha' \langle 0.2(0.04 + 0.16 + 0.16), 0.8(0.04 + 0.16) \rangle \approx \langle 0.31, 0.69 \rangle \]

\[ P(P_{2,2} | \text{known}, b) \approx \langle 0.86, 0.14 \rangle \]