1. Problem 3-3 part a from the CLRS text. You need not provide justifications for your order.

Due Friday January 12.

2. Problem 3-4. For conjectures that are untrue, explain which reasonable class of functions the statement is true for. For example, you might say ”The statement is untrue in general, and here is an example. But the statement is true if the functions are strictly increasing, and here is why.

Due Friday January 12.

3. Consider the following definitions for $f(n, m) = O(g(n, m))$. State which definitions are logically equivalent. Fully justify your answers. Argue about which definition you think is the best one in the context where $f$ and $g$ are run times of algorithms and the input size is monotonically growing in $n$ and $m$ (for example $n$ might be the number of vertices in a graph and $m$ might be the number of edges in a graph).

(a) There exists positive constants $c > 0, n_0, m_0$ such that for all $(n, m)$ where $n \geq n_0$ and $m \geq m_0$ it is the case that $f(n, m) \leq c g(n, m)$

(b) There exists positive constants $c > 0, n_0, m_0$ such that for all $(n, m)$ where $n \geq n_0$ or $m \geq m_0$ it is the case that $f(n, m) \leq c g(n, m)$

(c) There exists a constant $c > 0$ such that

$$\limsup_{n \to \infty} \limsup_{m \to \infty} \frac{f(n, m)}{g(n, m)} < c$$

Note that this means that you first take the limit superior with respect to $m$. The result will be a function of just $n$. You then take the limit superior of this function with respect to $n$. If you don’t know what limit superior means, you can just assume that the limit exists, in which case the limit and limit superior are the same.

(d) There exists a constant $c > 0$ such that

$$\limsup_{m \to \infty} \limsup_{n \to \infty} \frac{f(n, m)}{g(n, m)} < c$$

Note that this means that you first take the limit superior with respect to $n$. The result will be a function of just $m$. You then take the limit superior of this function with respect to $m$. If you don’t know what limit superior means, you can just assume that the limit exists, in which case the limit and limit superior are the same. Note that on the surface that this definition is different than the last on in that the order that you take the limits is switched.

(e) There exists a constant $c > 0$ such that for all but finitely many pairs $(m, n)$ it is the case that $f(n, m) \leq c g(n, m)$.

Due Friday January 12.
4. Skim/Read section 16.4 in the text. Consider a hereditary set system $M = (S, \mathcal{I})$. Assume that for every possible collection of positive weights on the elements of $S$, it is the case that the natural greedy algorithm (given on page 440 of the CLRS text) returns the maximum weight independent set. Show that $M$ is a matroid.

Hint: Assume to reach a contradiction that $M$ is not a matroid. Thus $M$ must not have the exchange property. Let $A$ and $B$ be elements of $\mathcal{I}$, where $|A| < |B|$, that do not satisfy the exchange property. Give the elements weights so that the greedy algorithm produces $A$. It's sufficient to have only 4 different weights. A weight $w_1$ for the elements of $A \cap B$, a weight $w_2$ for the elements of $A - B$, a weight $w_3$ for the elements of $B - A$, and a weight $w_4$ for the remaining elements. Ask yourself what property you want of these weights.

Due Wednesday January 17

5. Problem 16-5 part c. You must use an exchange argument.

Due Wednesday January 17

6. Problem 4-3 from the CLRS text except parts d and f. Apply the Master Theorem (Theorem 4.1) whenever applicable. Otherwise, draw the recursive call tree and sum up the costs level by level.

Due Friday January 19

7. Problem 4-3 part f. Use induction; You will need 2 inductive proofs, one for the upper bound and one for the lower bound.

Due Friday January 19

8. (a) Problem 30.2-6 from the CLRS text.

(b) What would be the running time of the running time of the FFT algorithm if one used the field of integers modulo $m$ as suggested in problem 30.2-6? You can assume arithmetic operations are implemented in the most naive/natural way that you learned in elementary school.

(c) Problem 30.2-7 from the CLRS text.

(d) Problem 30.2-8 from the CLRS text.

Due Monday January 22