1 Basic Definitions

A *optimization problem* has the following form: output a best solution $S$ satisfying some property $P$. Best usually means least cost, a minimization problem, or most benefit, a maximization problem. A best solution is called an *optimal solution*. Note that for many problems there may be many different optimal solutions. A *feasible solution* is a solution that satisfies the property $P$. Most of the problems that we consider can be viewed as optimization problems.

An algorithm $A$ for a minimization problem $P$ has *performance ratio* $c$ if for every input $I$, the cost of the output of $A$ on input $I$ has cost at most $c$ time the cost of the least cost feasible solution to $I$.

An algorithm $A$ for a maximization problem $P$ has *performance ratio* $c$ if for every input $I$, the benefit of most beneficial feasible solution to $I$ is at most $c$ times the benefit of the output of $A$ on input $I$.

2 MST doubling algorithm for TSP with triangle inequality

See section 9.5.1

3 Christofides algorithm for TSP with triangle inequality

See section 9.5.1
4 Without triangle inequality, there is no algorithm with constant performance ratio

Theorem For all $c$, if there is a polynomial time algorithm for TSP with a constant performance ratio $c$ then every NP-complete problem has a polynomial time algorithm.

Proof: We use a many-to-one reduction from the Hamiltonian cycle problem (i.e. the problem of deciding whether a graph contains a simple spanning cycle). Let $H$ a weighted complete graph constructed from an unweighted graph $G$ with $n$ vertices in the following manner:

- each edge in $G$ is given weight 1, and
- a edge $(x, y)$ with weight $cn$ is added between each pair of nonadjacent vertices in $G$.

Note that if $G$ has a Hamiltonian cycle then $H$ has a tour of weight $n$, and if $G$ does not have a Hamiltonian cycle then every tour in $H$ has weight greater than $cn$. Hence, if the approximation algorithm for TSP finds a tour with length at most $cn$ then we may be sure the $G$ has a Hamiltonian cycle, and if the approximation algorithm for TSP finds a tour with length more than $cn$ then we may be sure the $G$ does not have a Hamiltonian cycle.

5 Analysis of First Fit for Bin Packing

6 Analysis of First Fit Decreasing for Bin Packing

See section 9.5.2