CS 1510 Midterm 2
Fall 2013

1. (20 points) We consider the problem of computing the longest increasing subsequence of a sequence $x_1, \ldots, x_n$ of integers.

   (a) (5 points) Show that a naive recursive algorithm can not work for this problem. To ask the same question in another way, explain why you need to strengthen the inductive hypothesis.

   (b) (5 points) Define the strengthened recursive subproblem used in the recursive algorithm developed in class. Recall that we computed terms of the form $LCS(i, j)$. I'm asking for the definition of $LCS(i, j)$.

   (c) (5 points) Give a recursive formula/expression for $LCS(i, j)$.

   (d) (5 points) Give iterative array-based pseudo-code to compute all the $LCS(i, j)$ terms.

2. (20 points) The input to this problem is two sequences $T = t_1, \ldots, t_n$ and $P = p_1, \ldots, p_k$ such that $k \leq n$, and a positive integer cost c_i associated with each t_i. The problem is to find a subsequence of T that matches P with minimum aggregate cost. That is, find the subsequence $i_1 < \ldots < i_k$ such that for all j, $1 \leq j \leq k$, we have $t_{i_j} = p_j$ and $\sum_{j=1}^{k} c_{i_j}$ is minimized.

   Give a polynomial time dynamic programming algorithm for this problem (note that the algorithm needs to return the subsequence, and not just its cost.).

   In order to get positive credit, you need to formally define what the table entries your algorithm is trying to compute, and you must give some English explanation justifying the correctness of your algorithm. To satisfy the second requirement, it is sufficient to explain the development of the algorithm using either recursion or the pruning method.

3. (20 points) Consider the following problem:

   INPUT: A tree T with integer profits on the edges.

   OUTPUT: The most profit that one can obtain from a simple path P in T.

   The profits may be positive or negative or zero. A negative profit can be thought of as a cost. A path is simple if it doesn't repeat any edge. The profit of a path is the sum of the profits of the edges. The profit of a path consisting of one vertex, and no edges, is 0.

   Give a linear time algorithm to solve the above problem.

   In order to get positive credit, you need to formally define what the table entries your algorithm is trying to compute, and you must give some English explanation justifying the correctness of your algorithm. To satisfy the second requirement, it is sufficient to explain the development of the algorithm using either recursion or the pruning method.