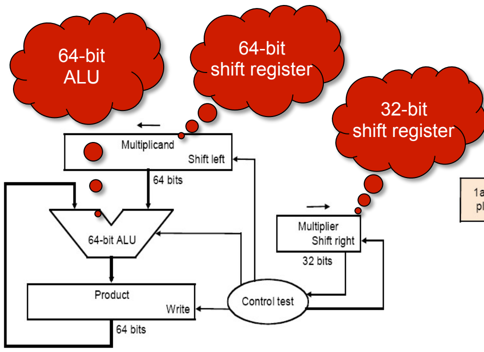
Multiplication

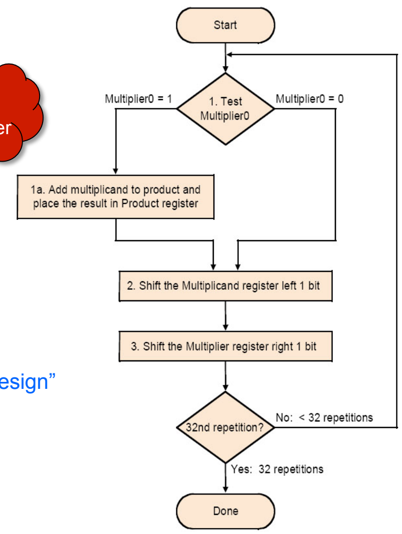
- More complicated than addition
  - A straightforward implementation will involve *addition* and *shifting*
- A "more complex operation" implies
  - More area (on silicon) and/or
  - More time (more clock cycles or longer clock cycle time)
- Let's begin from a simple, straightforward method
  - For now, consider only unsigned numbers!

Straightforward algorithm

```
  01010010  (multiplicand)
x 01101101  (multiplier)
```

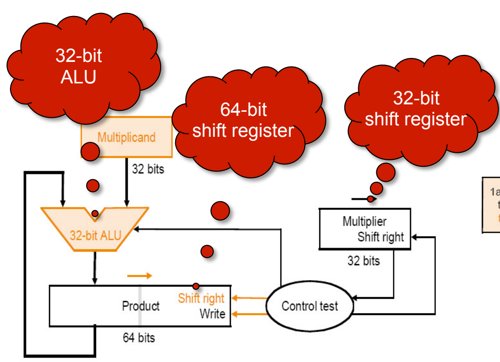
Hardware design 1

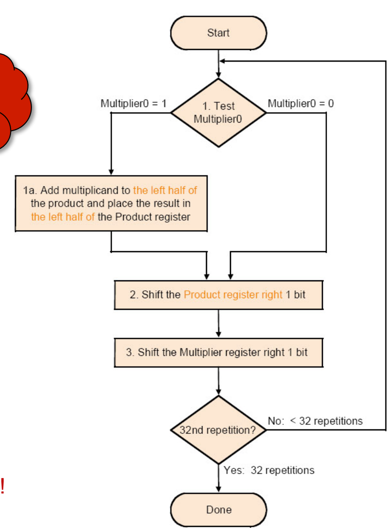




"Straightforward but naïve design"

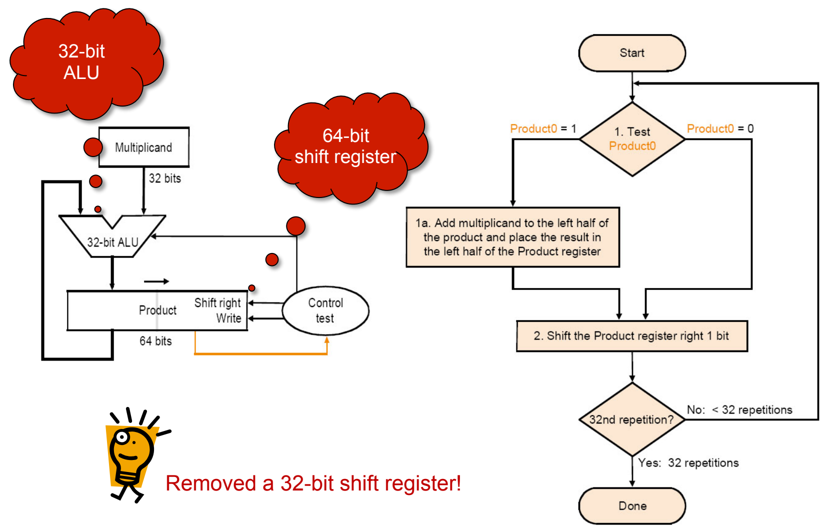
Hardware design 2





32-bit ALU instead of 64-bit ALU!

Hardware design 3



Removed a 32-bit shift register!

Example

- Let's do 0010×0110 (2×6), unsigned

Iteration	Multiplicand	Implementation 3	
		Step	Product
0	0010	initial values	0000 0110
1	0010	1: 0 -> no op	0000 0110
		2: shift right	0000 0011
2	0010	1: 1 -> product = product + multiplicand	0010 0011
		2: shift right	0001 0001
3	0010	1: 1 -> product = product + multiplicand	0011 0001
		2: shift right	0001 1000
4	0010	1: 0 -> no op	0001 1000
		2: shift right	0000 1100

Booth's encoding

- Three symbols to represent a binary number: {1,0,-1}
- Examples (8-bit encoding)
  - -1
    - 11111111 (two's complement)
    - 0000000-1 (Booth's encoding)
  - 14
    - 00001110 (two's complement)
    - 000100-10 (Booth's encoding)

- Bit transitions in number (in two's complement encoding) show how Booth's encoding works
  - 0 to 0 (from right to left): 0
  - 0 to 1: -1
  - 1 to 1: 0
  - 1 to 0: 1

Booth's encoding

- Key point
  - A "1" in the multiplier implies an addition operation
  - If you have many "1"s – that means many addition operations

- Booth's encoding is useful because it can reduce the number of addition operations you have to perform

- With Booth's encoding, partial results are obtained by
  - Adding multiplicand
  - Adding 0
  - Subtracting multiplicand

Booth's algorithm in action

- Let's do 0010×1101 (2×-3)

Iteration	Multiplicand	Booth's algorithm	
		Step	Product
0	0010	initial values	0000 1101 0
1	0010	10 -> product = product – multiplicand	1110 1101 0
		shift right	1111 0110 1
2	0010	01 -> product = product + multiplicand	0001 0110 1
		shift right	0000 1011 0
3	0010	10 -> product = product – multiplicand	1110 1011 0
		shift right	1111 0101 1
4	0010	11 -> no op	1111 0101 1
		shift right	1111 1010 1